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Mathematics

On the Approximation of Inverse Functions of the Conformal Mapping Functions and their Derivatives on the Circle of Close Domains

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ABSTRACT. An order of nearness of inverse functions of conformally mapping functions and their derivatives is estimated on the unit circle of the neighbouring domains according to nearness of these domains. *© 2008 Bull. Georg. Natl. Acad. Sci.*

Key words: *mapping, conformal mapping functions, neighbouring domains*

Suppose we have a finite single bounded domain G in the plane $\boldsymbol{C}$ of a complex variable $z \equiv x+iy$. It is assumed that the bound Γ of the domain G is described by parameter equation $t = g(\tau)$ $(0 \le \tau \le 2\pi;\ g(0) = g(2\pi))$ and it presents a close line of the class C_α^1 $(0 < \alpha \le 1)$.

Let us consider a domain $\tilde{G}$, similar to the domain G, with the parameter equation $\tilde{t} = \tilde{g}(\tau)$ $(0 \le \tau \le 2\pi;\ \tilde{g}(0) = \tilde{g}(2\pi))$. Domains G and $\tilde{G}$ are called neighbouring domains if the following conditions are fulfilled:

$$|\,g(\tau) - \tilde{g}(\tau)\,| < \varepsilon, \quad (0 \le \tau \le 2\pi); \qquad \|\,g - \tilde{g}\,\|_{C_\alpha} < \varepsilon. \tag{1}$$

It is clear that an infinite set $\{G_\varepsilon\}$ of neighbouring domains of the domain G corresponds to any number $\varepsilon > 0$.

By the integral equation method the following functions are constructed [1]

$$f : G \to D, \tag{2}$$

$$\tilde{f} : \tilde{G} \to D, \tag{3}$$

which transform conformally G and $\tilde{G}$ domains in the circle $D=\{W\colon |W|<1\}$, provided that $f(0) = \tilde{f}(0) = 0$ and $f'(0) > 0,\ \tilde{f}'(0) > 0.$

If we use the approach presented in [2] and [3], it can be easily verified that for the functions, mapping conformally G and $\tilde{G}$ domains, and for their derivatives the following estimations take place:

$$|f(g(\tau))-\widetilde{f}(\widetilde{g}(\tau))|<M_1\varepsilon, \qquad (0<\varepsilon<1),$$
$$|f'(g(\tau))-\widetilde{f}'(\widetilde{g}(\tau))|<M_1\varepsilon, \qquad (0<\varepsilon\le\varepsilon_0),$$

where M_1, M_2 and ε_0 are positive numbers which are defined by the initial domain G. Our purpose is to make analogous estimates of the functions $f^{-1}:D\to G$ and $\widetilde{f}^{-1}:D\to\widetilde{G}$ and their derivatives. For that let us find these functions in the following manner:

$$f^{-1}(W)=\frac{1}{\pi i}\int_L\frac{\mu(\omega)d\omega}{\omega-W}, \qquad (W\in D), \tag{4}$$

$$\widetilde{f}^{-1}(W)=\frac{1}{\pi i}\int_L\frac{\widetilde{\mu}(\omega)d\omega}{\omega-W}, \qquad (W\in D), \tag{5}$$

where $L=\{W: |W|<1\}$, and μ and $\widetilde{\mu}$ are desired functions with the real values of the C^1_α class. If we consider that $\omega=f(t)$, $t\in\Gamma$ and $\omega=\widetilde{f}(\widetilde{t})$, $\widetilde{t}\in\widetilde{\Gamma}$, the (4) and (5) functions can be presented as follows:

$$f^{-1}(W)=\frac{1}{\pi i}\int_\Gamma\frac{\varphi\ (t;z)\mu\,(t)}{t-z}dt, \qquad (z=f^{-1}(W)\in G), \tag{6}$$

$$\widetilde{f}^{-1}(W)=\frac{1}{\pi i}\int_{\widetilde{\Gamma}}\frac{\widetilde{\varphi}(\widetilde{t};\widetilde{z})\widetilde{\mu}(\widetilde{t})}{\widetilde{t}-\widetilde{z}}d\widetilde{t}, \qquad (\widetilde{z}=\widetilde{f}^{-1}(W)\in\widetilde{G}), \tag{7}$$

where

$$\varphi(t;z)=\frac{f'(t)(t-z)}{f(t)-f(z)},$$
$$\widetilde{\varphi}(\widetilde{t};\widetilde{z})=\frac{\widetilde{f}'(\widetilde{t})(\widetilde{t}-\widetilde{z})}{\widetilde{f}(\widetilde{t})-\widetilde{f}(\widetilde{z})},$$

If we take into account the properties of the Cauchy generalized integral [4:128], we can conclude, based on (6) and (7), that μ and $\widetilde{\mu}$ are the only solutions of the integral equations

$$\mu(t_0)+\operatorname{Re}\frac{1}{\pi i}\int_\Gamma\frac{\varphi(t;t_0)\mu(t)}{t-t_0}dt=\operatorname{Re}t_0, \qquad (t_0\in\Gamma), \tag{8}$$

$$\widetilde{\mu}(t_0)+\operatorname{Re}\frac{1}{\pi i}\int_{\widetilde{\Gamma}}\frac{\widetilde{\varphi}(\widetilde{t};\widetilde{t}_0)\widetilde{\mu}(\widetilde{t})}{\widetilde{t}-\widetilde{t}_0}d\widetilde{t}=\operatorname{Re}\widetilde{t}_0, \qquad (\widetilde{t}_0\in\widetilde{\Gamma}), \tag{9}$$

Using the method of proof [5, 3], it can be easily verified that the following estimations take place for the solutions of integral equations (8) and (9):

$$\|\mu-\widetilde{\mu}\|_{C_{\alpha-\beta}}\le M_3\|\mu\|_{C_\alpha}\ \varepsilon,$$
$$\|\mu'-\widetilde{\mu}'\|_{C_{\alpha-\beta}}\le M_4\|\mu'\|_{C_\alpha}\ \varepsilon, \qquad (0<\varepsilon\le\varepsilon_0),$$

where M_3, M_4 and ε_0 are positive constants defined by means of the initial domain G; β is any positive number, $\beta<\alpha$. Then using (4) and (5) we get

$$|f^{-1}(\omega)-\widetilde{f}^{-1}(\omega)|<K\varepsilon, \qquad (\omega\in L), \qquad (0<\varepsilon\le\varepsilon_0), \tag{10}$$

$$|[f^{-1}(\omega)]' - [\widetilde{f}^{-1}(\omega)]'| < P\varepsilon, \qquad (\omega \in L), \quad (0 < \varepsilon \le \varepsilon_0), \tag{11}$$

where K and P are constants depending on the G domain.

Because of the boundedness of the domain G we can suppose that G and its neighbouring domains are put in any fixed circle. Also, if we remember that Cauchy type singular integral is a limited integral operator in the space C_α, we can conclude that estimations (10) and (11) with other constants will also be easily applied in the case when $0 < \varepsilon < 1$.

მათემატიკა

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