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Physics

Drift Effects in the Theory of Instabilities in a Rotating Plasma

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ABSTRACT. The drift effects are incorporated into the theory of instabilities in a rotating magnetized plasma. It is shown that these effects suppress the rotational-convective instability.

Key words: *magneto-rotational instability (MRI), axisymmetric perturbations.*

Initial papers on the theory of magneto-rotational instability (MRI) in a rotating medium in axial magnetic field [1, 2] dealt with the axisymmetric perturbations. The same concerns paper [3] where, in addition to the MRI, the rotation-convective instability (RCI) was considered. As is known [4], the drift effects in such perturbations are not revealed. On the other hand, according to [5], the MRI and RCI in the medium with the above geometry of magnetic field can be realized in the form of nonaxisymmetric perturbations. Then, basing on the general notions [4], one can suggest that in this case the drift effects can play the role. The present paper is addressed to the problem of allowance for the drift effects in the instabilities of rotating plasma in the mentioned geometry of magnetic field.

The parameter β characterizing the ratio of plasma pressure to the magnetic field pressure is assumed to be small, β<<1. The perturbation of the axial magnetic field, $\tilde{B}_z$, is neglected. In addition, the parallel motion of ions is also neglected. For such assumptions the MRI is not revealed, so that our problem reduces to elucidation of the role of drift effects in the RCI.

We start from the plasma motion equation of the form

$$\rho \frac{d_i \mathbf{V}_i}{dt} + \nabla \cdot \boldsymbol{\pi}_i^{\wedge} = \rho \mathbf{g} - \nabla \left(p + \frac{\mathbf{B}^2}{8\pi} \right) + \frac{1}{4\pi} \left(\mathbf{B} \cdot \nabla \mathbf{B} \right). \quad (1)$$

Here ρ is the plasma mass density, $\mathbf{V}_i$ is the plasma velocity, $d_i / dt = \partial / \partial t + \mathbf{V}_i \cdot \nabla$, $\boldsymbol{\pi}_i^{\wedge}$ is the tensor of oblique (magnetic) ion viscosity, p is the plasma pressure, $\mathbf{B}$ is the magnetic field, $\mathbf{g} = (g,0,0)$ is the gravitation force. We take the equilibrium magnetic field $\mathbf{B}_0$ in the form $\mathbf{B}_0 = (0,0,B_0)$. The equilibrium is cylindrically symmetric. We use the cylindrical coordinate system r, θ, z. It is assumed that plasma rotates around the cylinder axis with the angle velocity $\Omega = \Omega(r)$. The equilibrium plasma parameters satisfy the condition (viscosity effect on the equilibrium is neglected)

$$p_0' = \rho_0 \left(g + r\Omega^2 \right), \quad (2)$$

where p_0 and ρ_0 are the equilibrium pressure and mass density, the prime is the radial derivative. We assume the equilibrium magnetic field homogeneous, $B_0' = 0$.

Taking the curl of both parts of (1), we obtain

$$\left[\nabla \times \left(\rho \frac{d_i \mathbf{V}_i}{dt} + \nabla \cdot \boldsymbol{\pi}_i^{\wedge} - \rho \mathbf{g} \right) \right]_z^{\sim} = \frac{i k_z B_0}{4\pi} \left(\frac{\partial \tilde{B}_\theta}{\partial r} - i k_y \tilde{B}_r \right). \quad (3)$$

Here $\widetilde{B}_r, \widetilde{B}_\theta$ - r, θ -th projections of the perturbed magnetic field. We take the perturbations in the form $\exp(-i\omega t + ik_z z + im\theta)\tilde{F}(r)$, where ω is the oscillation frequency, k_z - parallel wave number, m - azimuthal mode number, $k_y = m/r$ - azimuthal wave number, $\widetilde{F}(r)$ - a function close to $\exp(ik_r r)$, where k_r is the radial wave number. The definition $(...)^{\sim}$ means the perturbed part of the value (...).

In the case of two-dimensional motion one should complement (3) by another equation following from (1). As such an equation, we take (cf. above said)

$$\widetilde{B}_z = 0. \tag{4}$$

Now we represent the ion velocity as the sum of the electric and Larmor drifts, $\mathbf{V}_E, \mathbf{V}_L$ [4],

$$\mathbf{V}_i = \mathbf{V}_E + \mathbf{V}_L. \tag{5}$$

Here

$$\mathbf{V}_E = \frac{c}{\mathbf{B}^2}[\mathbf{E}\times\mathbf{B}] \ , \tag{6}$$

$$\mathbf{V}_L = \frac{c}{en\mathbf{B}^2}[\mathbf{B}\times\nabla p_i], \tag{7}$$

where p_i is the ion pressure, n - the plasma density, e - the ion charge, $\widetilde{n}$ is the light velocity. Then (3) reduces to

$$\hat{K} + \hat{L} + \hat{M} = \hat{N} \tag{8}$$

where

$$\hat{K} = \left[\nabla\times\rho\left\{\left(\frac{\partial}{\partial t} + \mathbf{V}_E\cdot\nabla\right)\mathbf{V}_E - \mathbf{g}\right\}\right]_z^{\sim}, \tag{9}$$

$$\hat{L} = \left[\nabla\times\rho\left\{\left(\frac{\partial}{\partial t} + \mathbf{V}_E\cdot\nabla\right)\mathbf{V}_L\right\}\right]_z^{\sim}, \tag{10}$$

$$\hat{M} = \left[\nabla\times\left\{\rho(\mathbf{V}_L\cdot\nabla)(\mathbf{V}_E + \mathbf{V}_L) + \nabla\cdot\boldsymbol{\pi}_i^{\wedge}\right\}\right]_z^{\sim}, \tag{11}$$

$$\hat{N} = \frac{ik_z B_0}{4\pi}\left(\frac{\partial\widetilde{B}_\theta}{\partial r} - ik_y\widetilde{B}_r\right). \tag{12}$$

We calculate the value $\hat{M}$ in the traditional approximation of the theory of instabilities. Then we obtain (see in detail [4])

$$\hat{M} = 0. \tag{13}$$

In the same approximation we calculate $\hat{L}$, which yields, in allowing for (6)

$$\hat{L} = -\frac{i\widetilde{\omega}}{\omega_{Bi}}\nabla_\perp^2\widetilde{p}_i, \tag{14}$$

where $\widetilde{\omega} = \omega - m\Omega$, ω_{Bi} is the ion cyclotron frequency, $\widetilde{p}_i$ is the perturbed ion pressure. For calculation of $\widetilde{p}_i$ we use the ion heat balance equation

$$\frac{3}{2}\frac{d_i p_i}{dt} + \frac{5}{2}p_i\nabla\cdot\mathbf{V}_i = -\nabla\cdot\mathbf{q}_i, \tag{15}$$

where $\mathbf{q}_i$ is "oblique" ion heat flux determined by

$$\mathbf{q}_i = \frac{5}{2}\frac{p_i}{M_i\omega_{Bi}}\mathbf{e}_z\times\nabla T_i, \tag{16}$$

and the ion continuity equation

$$\frac{\partial n}{\partial t} + \mathbf{V}_E\cdot\nabla n = 0, \tag{17}$$

where M_i is the ion mass, $\mathbf{e}_z$ is the unit vector along z. By means of (15)-(17) we find

$$-i\widetilde{\omega}\widetilde{p}_i + \widetilde{V}_{Er}p'_{0i} = 0, \tag{18}$$

where p_{0i} is the equilibrium ion pressure. Substituting (18) into (14), we arrive at

$$\hat{L} = -\frac{p'_{0i}}{\omega_{Bi}}\nabla_\perp^2\widetilde{V}_{Er}. \tag{19}$$

We introduce the ion drift velocity defined by the pressure gradient, V_{*i}, taking

$$V_{*i} = \frac{cp'_{0i}}{en_0 B_0}, \tag{20}$$

where n_0 is the equilibrium plasma density. Then (19) takes the form

$$\hat{L} = -\rho_0 V_{*i}\nabla_\perp^2\widetilde{V}_{Er}. \tag{21}$$

According to the frozen-in condition,

$$\widetilde{V}_{Er} = -\frac{\widetilde{\omega}}{k_z B_0}\widetilde{B}_r. \tag{22}$$

Substituting (22) into (21), we obtain

$$\hat{L} = -\rho_0\frac{\widetilde{\omega}V_{*i}}{k_z B_0}\nabla_\perp^2\widetilde{B}_r. \tag{23}$$

Turning to [5] and allowing for the gravitation force there, we note that

$$\hat{K} = \frac{\partial}{\partial r}\left[\rho_0\left(-i\tilde{\omega}\tilde{V}_{E\theta} + \frac{\kappa^2}{2\Omega}\tilde{V}_{Er}\right)\right] -$$

$$-ik_y\left[\rho_0\left(-i\tilde{\omega}\tilde{V}_{Er} - 2\Omega\tilde{V}_{E\theta}\right) - \tilde{\rho}\frac{p_0'}{\rho_0}\right]. \qquad (24)$$

As $\tilde{V}_{Er}$ we substitute here Eq. (23), while as $\tilde{V}_{E\theta}$ - the value given in [5]:

$$\tilde{V}_{E\theta} = -\frac{\tilde{\omega}}{k_z B_0}\tilde{B}_\theta + \frac{i}{k_z B_0}\frac{d\Omega}{d\ln r}\tilde{B}_r. \qquad (25)$$

Instead of $\tilde{\rho}$, one should take here the expression following from (17) and (22):

$$\tilde{\rho} = \frac{ip_0'}{k_z B_0}\tilde{B}_r. \qquad (26)$$

Then (24) transits to

$$\hat{K} = \frac{1}{k_z B_0}\left\{\frac{\partial}{\partial r}\left[\rho_0\left(-i\tilde{\omega}^2\tilde{B}_\theta - 2\Omega\tilde{B}_r\right)\right] -\right.$$

$$\left.-ik_y\rho_0\left[i\left(\tilde{\omega}^2 - \frac{d\Omega^2}{d\ln r} - \frac{p_0'}{\rho_0}\frac{d\ln\rho_0}{dr}\right)\tilde{B}_r + 2\Omega\tilde{\omega}\tilde{B}_\theta\right]\right\}. \qquad (27)$$

Allowing for (4), it follows from the Maxwell equation $\nabla\cdot\tilde{\mathbf{B}} = 0$ that

$$\tilde{B}_\theta = \frac{i}{k_y}\frac{\partial\tilde{B}_r}{\partial r}. \qquad (28)$$

Substituting (28) into (27), we obtain

$$\hat{K} = \frac{\rho_0 k_\perp^2}{k_y k_z}\left(\tilde{\omega}^2 - \frac{k_y^2}{k_\perp^2}\frac{p_0'}{\rho_0}\frac{d\ln\rho_0}{dr}\right)\tilde{B}_r. \qquad (29)$$

where $k_\perp^2 = k_r^2 + k_y^2$.

Note that the term with $d\Omega^2/d\ln r$ in (27) describes the Velikhov effect [6]. However, it does not lead to the contribution into (29) due to its compensating with the effect due to the radial derivative of $2\Omega\tilde{\omega}$, which has been called in [6] the "anti-Velikhov" effect. In taken approximations, this compensation is complete (see in detail [6]). As a result, the MRI is not predicted.

In addition, allowing for (28), we reduce (12) to

$$\hat{N} = \frac{k_z B_0}{4\pi k_y}k_\perp^2\tilde{B}_r. \qquad (30)$$

By means of (13), (23), (29), (30), we arrive at the local dispersion relation

$$\tilde{\omega}^2 - \tilde{\omega}\omega_{*i} = \frac{k_y^2}{k_\perp^2}\frac{p_0'}{\rho_0}\frac{d\ln\rho_0}{dr} + k_z^2 v_A^2. \qquad (31)$$

Here the term with $\omega_{*i} \equiv k_y V_{*i}$ characterizes the contribution of the ion drift effects, while the term with $p_0' d\ln\rho/dr$ is responsible for generation of the RCI. It can be seen that for $p_0'\rho_0' < 0$ the RCI is not generated for

$$\frac{\omega_{*i}^2}{4} + k_z^2 v_A^2 > -\frac{k_y^2}{k_\perp^2}\frac{p_0'}{\rho_0}\frac{d\ln\rho_0}{dr}. \qquad (32)$$

Allowing for the equilibrium condition (2), we find that in the limiting cases of large and small g the condition (32) for $k_z = 0$ transits to the criteria of drift suppression of the flute (interchange) and centrifugal instabilities, respectively [4].

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ფიზიკა

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