საქართველოს მეცნიერებათა ეროვნული აკადემიის მოამბე, 175, №2, 2007
BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, 175, №2, 2007

Mathematics

Lebesgue Constants of Generalized Cesáro (C,α_n)-Means of Trigonometric Series

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ABSTRACT. The problem of the behavior of Lebesgue constants of generalized Cesáro (C, α_n)-means ($\alpha_n \in$ (-1, d)) of trigonometric series is considered. © *2007 Bull. Georg. Natl. Acad. Sci.*

Key words: *Lebesgue constants, trigonometric series, generalized Cesáro means.*

The important role played by Lebesgue constants in the theory of Fourier series is well known. For a trigonometric system they are defined as follows:

$$L_n = \frac{2}{\pi}\int_0^\pi \left|\frac{\sin(n+1/2)t}{2\sin(t/2)}\right| dt.$$

In 1910 Fejér [1] proved that

$$L_n = 4\pi^{-2}\ln n + O(1), \quad n \to \infty.$$

Subsequently, this result was repeatedly generalized by different authors (see, for example, E.Landau [2], A. Kolmogorov [3], S. Kaczmarz [4, 5], G. Alexits [6]).

In my paper [7] the behavior of generalized Cesáro (C, α_n)-means ($\alpha_n \in$(-1, d)) of trigonometric series of H^ω classes in the space of continuous functions is studied. The behaviour of Lebesgue constants is important for both traditional methods and my summability method. The present paper deals with this problem.

Let $\alpha_n \in(-1,1]$,

$$A_n^{\alpha_n} = (a_n+1)(a_n+2)\cdots(a_n+n)/n!$$

and

$$K_n^{\alpha_n}(t) = \left(\sum_{\nu=0}^{n} A_{n-\nu}^{\alpha_n-1}\frac{\sin(\nu+1/2)t}{2\sin(t/2)}\right) / A_n^{\alpha_n}.$$

Denote positive absolute constants by C_1 and C_2. Besides, the designations $C(\delta)$, $C_1(\delta)$, $C_2(\delta)$, $C_1(\delta_1, \delta_2)$, $C_2(\delta_1, \delta_2)$ are used for positive constants depending only on the indicated parameters.

The following statement is valid.

Theorem. *If* (α_n) *is an arbitrary sequence* ($\alpha_n \neq 0$) *from the interval* (–1,1], *then*

$$L_n^{\alpha_n} = \frac{1}{\pi}\int_{-\pi}^{\pi}\left|K_n^{\alpha_n}(t)\right| dt = \frac{2^{2-\alpha_n}}{\pi^{2+\alpha_n}A_n^{\alpha_n}}\left(2n+1+\alpha_n\right)^{\alpha_n}\left(\frac{n^{\alpha_n}-1}{\alpha_n n^{\alpha_n}} + O(1)\right).$$

Corollary 1. *Let* $0<|\alpha_n|\le \varepsilon_n/\ln n$, *where* $\lim_{n\to\infty}\varepsilon_n=0$, *then*

$$C_1\, ln\, n \le L_n^{\alpha_n} \le -C_2 ln\, n,\ n>1.$$

Corollary 2. *If* $-1<\delta_2\le\alpha_n\le\delta_1/\ln n<0$, *where* δ_1 *and* δ_2 *are constants, then*

$$-C_1(\delta_1,\delta_2)\,n^{-\alpha_n}/a_n\le L_n^{\alpha_n}\le -C_2(\delta_1,\delta_2)\,n^{-\alpha_n}/a_n.$$

Corollary 3. *In the case* $-1<\delta_2\le a_n\le\delta_1/lnn\le 0$ *we have*

$$C_1(\delta_1,\delta_2)\,n^{-\alpha_n}\le L_n^{\alpha_n}\le C_2(\delta_1,\delta_2)\,n^{-\alpha_n}.$$

Corollary 4. *Let* $-1<\alpha_n<\delta<0$, *where* δ *is a fixed number, then*

$$C_1(\delta)\frac{n^{-\alpha_n}}{1+\alpha_n}\le L_n^{\alpha_n}\le C_2(\delta)\frac{n^{-\alpha_n}}{1+\alpha_n}.$$

Corollary 5. *If* $-1<\delta_1/lnn\le a_n\le\delta_2<1$, *where* δ_1 *and* δ_2 *are constants, then*

$$C_1(\delta_1,\delta_2)/\alpha_n\le L_n^{\alpha_n}\le C_2(\delta_1,\delta_2)/\alpha_n.$$

Corollary 6. Let $0<\delta<\alpha_n<1$, where δ is a fixed number, then

$$L_n^{\alpha_n}\le C(\delta).$$

In the proof of the Theorem two lemmas are used which have self-dependent interest.
Lemma 1([7]). *If* (α_n) *is an arbitrary sequence from the interval* (-1,1], *then*

$$C_1(1+\alpha_n)n^{\alpha_n}\le A_n^{\alpha_n}\le C_2(1+\alpha_n)n^{\alpha_n}.$$

Lemma 2([7]). For $\alpha_n\in(-1,1]$

$$\left|K_n^{\alpha_n}(t)\right|\le\frac{2n}{1+\alpha_n}.$$

მათემატიკა

ტრიგონომეტრიულ მწკრივთა განზოგადებული ჩეზაროს (C,α_n)-საშუალოების ლებეგის კონსტანტები

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(წარმოდგენილია აკადემიის წევრის ლ. ჟიჟიაშვილის მიერ)

განხილულია ტრიგონომეტრიულ მწკრივთა განზოგადებული ჩეზაროს (C,α_n)-საშუალოების ლებეგის კონსტანტების ყოფაქცევა.

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Received November, 2006