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Mechanics

Engineering Methods of Modeling of Oscillation Processes and Rational Design of Mechanical Systems

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ABSTRACT. Methodology of optimization synthesis of linear multimass mechanical systems oscillation processes and rational design of structural schemes is offered, allowing to choose rational relationship of machine transmission elastic-mass (inertial and stiffness) parameters ensuring their steady functioning with minimum dynamic coefficient at transition regimes. *© 2007 Bull. Georg. Natl. Acad. Sci.*

Key words: *oscillation beatings, dimensionless generalized parameters, dynamic loads, vibration processes*

The basic scientific-technical task of modern mechanical engineering is connected with the creation of high productive and durable machines and constructions with low metal-intensivity.

The productivity of the machines and mechanisms depends upon the speed of their working parts and solving actual task of minimizing duration of transitional regimes and similar characteristics of technological cycles.

Unfortunately calculating and designing principal link parameters are mostly carried out on the basis of energy-force parameters and safety margin value without sound theoretical grounding and without taking into account dynamic characteristics of machines and peculiarity of transitional regimes.

If safety margin is taken more than is necessary it causes manufacturing of too heavy, massive machines and hence unjustified expenditure of metals. If safety margin is less than necessary, it leads to machine failure.

Modern industry (especially heavy engineering industry) is at present subjected to great losses due to fatigues failure of important parts of machines. The same is the case with mobile means of civil and defence purpose (units of transmission and lorries, trucks, tanks, vessel and aircraft engines of rotor type etc.).

The main reasons of failure are mistakes made at the initial stage of designing machines and incorrect selection of design parameters (mass and rigidity of links and quantity of their relationship), neglecting dynamic vibrating processes.

According to statistics, heavy engineering industry (particularly metallurgy) is subjected to considerable damage because of machine failure.

According to experimental data, 70% of failure is caused by internal resonance regimes in the mechanical and electromechanical systems, which has the character of oscillation beatings of elastic force amplitudes, and even at small technological loading fast fatigues breakage of important parts (especially transmission) is caused.

It is known that oscillation beatings arise at the approach of lowest values of natural oscillation frequencies, and the nearer these frequencies, the more is the negative effect. Coincidence of these natural frequencies causes the fastest breakage of a construction.

The picture of oscillation beating at nearness of frequency of natural oscillations ($\beta \approx \beta_2$) in torsion mechanical systems with two degrees of freedom is expressed in Fig. 1.

Unfortunately, this negative effect of oscillation beating is unknown to the majority of engineer-designers and they neglect this effect during designing.

According to the result of investigations, oscillation frequency is functionally connected with the values of the system constructive parameters, especially with the ratio of elastic and mass parameters.

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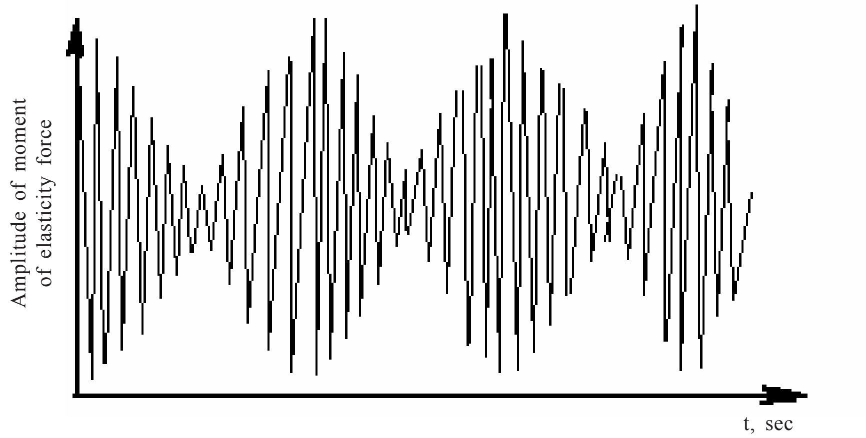

Fig. 1.

To determine natural oscillation frequencies, it is necessary to solve a characteristic equation of differential equations.

At high order equation (at design of real machine-aggregates), it is necessary to use computing technique. In this way a designer can determine the value of natural oscillations of the designed machines, but he cannot solve the problem of optimization synthesis.

Difficulties with these problems can easily be solved by an engineer method of optimizing synthesis of dynamic processes suggested by the author. The method is not to be solved with high order characteristic equation, and using easily attainable mathematics body, designers are able to determine systems generalized dimensionless parameters, proportional to the mechanical system and to construction parameters of the examined system.

To solve the problem it is necessary to consider high order differential equation [1,2]

$$\left.\begin{aligned} & X^{(2n)} + a_0 X^{[2(n-1)]} + \ldots + a_{n-2}\ddot{X} + a_{n-1}X = 0 \\ & (n = 1,2,3,\ldots), \end{aligned}\right\} \quad (1)$$

where a_1, a_2,…a_{n-1} are determined by the value of system parameters.

If argument t is substituted by value $t = \tau / a_0$, the following equation will be obtained:

$$X^{(2n)} + X^{[2(n-1)]} + C_1 X^{[2(n-2)]} + \ldots + C_{n-1}X = 0, \quad (2)$$

where C_k are generalized dimensionless parameters:

$$\left.\begin{aligned} & C_1 = \frac{a_1}{a_0^2};\ C_2 = \frac{a_2}{a_0^3};\ \ldots C_{\cdot} = \frac{a_{n-1}}{a_0^n}; \\ & (n = 1,2,3,\ldots). \end{aligned}\right\} \quad (3)$$

It is determined that the C_k parameter can alter in the limits:

$$0 \le C_k \le \frac{n-1}{2n}(k = 1,2 \ldots n-1). \quad (4)$$

Passing on from equation (1) to equation (2) an important practical significance for simplified the problem of optimizing synthesis of mechanical systems.

In equation (1), the range of alteration of a_1, a_2,…, a_3 determined by construction parameters, is very wide and indefinite. That is why, using this equation to solve the problem of optimizing synthesis, the designer would have to calculate infinite variants by using computer, which is uneconomic and unjustified.

Passing to generalized dimensionless parameters, C_k significantly narrows the range of alteration of differential equation coefficients and contracts the volume of computing.

For example, in the case of a differential equation of the eighth order, generalized parameters have the following maximum values:

C_1=3/8, C_2=1/16, C_3=1/256.

The values of coefficients a_i are determined by elastic and inertia parameters of the system.

For example, in the case of a three mass torsional system, values of a_0 and a_1 are calculated according to the formulas:

$$\left.\begin{aligned} & a_0 = C_{12}\frac{\theta_1 + \theta_2}{\theta_1\theta_2} + C_{23}\frac{\theta_2 + \theta_3}{\theta_2\theta_3}; \\ & a_1 = C_{12} \cdot C_{23}\frac{\theta_1 + \theta_2 + \theta_3}{\theta_1\theta_2\theta_3}, \end{aligned}\right\} \quad (5)$$

where C_{12} and C_{23} are rigidities of corresponding elastic shafts and – $\theta_{(i=1,2,3)}$ inertia moment of corresponding masses.

Substituting (5) values in equation (3) generalized dimensionless parameters can be determined, which, in turn, define the degree of nearness of natural oscillation frequencies, and hence, actual possibilities of the origination of oscillation beatings.

It follows from equation (4) that the value of generalized parameters for any degree of freedom will not exceed – 0.5.

Thus, if the degree of freedom n=100, which means that we are dealing with a differential equation of 200 degree, then:

$$\max C = \frac{n-1}{2n} = \frac{100-1}{200} \approx 0.5.$$

It follows from equation (4) that in the case of a differential equation of the fourth order (n = 2), variation of the generalized parameters takes place in the range

$$0 \le C_1 \le 0.25. \quad (6)$$

In the case of differential equation of the sixth order (n = 3)

$$0 \le C_1 \le 0.33, \;\; 0 \le C_2 \le 1/27. \quad (7)$$

For equation of the eighth order

$$0 \le C_1 \le 3/8, 0 \le C_2 \le 1/16, 0 \le C_3 \le 1/256. \quad (8)$$

As a result of numerous experiments and a lot of computing, it has been ascertained that the higher the value of C_i, (which in turn attests to a high degree of nearness of low frequencies of natural frequency of oscillations of the system) depending on the ratio of construction parameters of mechanical systems of any degree of freedom, the sharper the oscillation beating in mechanical systems.

The maximum value of C_{1max}=0.25 – for a differential equation of the fourth order, C_{1max}=1/3 – for a differential equation of sixth order, causes the pure oscillation beating with the highest value of elastic force amplitudes in mechanical systems. In this case the smallest natural frequencies of the system become equal to each other ($\beta_1 \approx \beta_2$), that is, internal resonance has originated.

To avoid that phenomenon, the designer should try to remove the smallest frequencies of natural oscillation maximally from each other, that is, to withdraw the system from the zone of maximum values of generalized parameters.

The investigation has determined that the upper nonoptimal range for C_1 value exists for the mechanical systems with the degree of freedom two (n=2) under the range characterized by

$$C_1 = (0.18 \div 0.25). \quad (9)$$

For the system with the degree of freedom three (n=3)

$$C_1 = (0.25 \div 1/3). \quad (10)$$

For the system with the degree of freedom four (n=4)

$$C_1 = (0.33 \div 0.375). \quad (11)$$

In general, maximum permissible value of the generalized parameter C_1 should be 20% less than its maximum value, determined according to expression (4). Otherwise, at the initial stage of designing a designer must change the construction parameters ratio, so that the newly selected parameter values would ensure the withdrawing of C_1 from the range of nonoptimal values (9¸11).

The author has determined that there exist the second (lower) range of nonoptimal values of parameters C_1 and for the systems with the degree of freedom two it is defined with the range

$$0 \le C_1 \le 0.04. \quad (12)$$

For the systems with the degree of freedom three (n=3)

$$0 \le C_1 \le 0.07. \quad (13)$$

For the systems with the degree of freedom four (n=4)

$$0 \le C_1 \le 0.09. \quad (14)$$

It is found that mechanical systems, whose ratio of construction parameters stipulates the lower nonoptimal value range of the generalized parameter C_1, are characterized by high sensibility (reaction) towards external, particularly impact forces. Minimum value of C_1 ensures the maximum removal of natural oscillation frequencies, caused by the increased characteristics of transmission elastic link stiffness, but such mechanical systems, at the same time, are characterized by high clearances which represent the impact type.

The range of lower nonoptimal values of generalized parameters is particularly dangerous for machines with reverse working regimes and are characterized by periodical opening and closing of clearances.

It is clear from the foregoing that there occurs optimal alteration of the range of generalized parameter, practical realization of which ensure a minimal reaction of mechanical systems to the influence of external forces of any type.

For mechanical systems with the degree of freedom two this range

$$0.05 \le C_1 \le 0.18. \quad (15)$$

In the case of a differential equation of the sixth order (n=3)

$$0.08 \le C_1 \le 0.25. \quad (16)$$

For an equation of the eighth order

$$0.12 \le C_1 \le 0.3. \quad (17)$$

Thus, the simplest method of engineer solving of the problem of synthesis of mechanical systems for dynamic processes optimization, ensuring minimal dynamic loading of machine aggregates, reliability and decreasing of metal-intensivity, consists in the algorithm based on the simplest arithmetic operations, demanding definition of the elastic-mass parameters of the designed objects and calculation of generalized dimensionless parameters.

It should be noted that this method of optimization synthesis of dynamic processes of mechanical system is true for linear systems, requiring some corrections for objects with nonlinear characteristics.

The author has defined the values of elastic and inertia parameters for two hundred rolling mills and other metallurgical equipment in the former USSR Republics and calculated the values of generalized parameters.

Analysis shows that 23 out of 200 objects examined are characterized by nonoptimal values of generalized parameters. 21 objects have been found to be subjected to fatigue failure of responsible links of transmissions, leading to great economic losses.

Technological loading (rolling moments) of some of them was so small that the failure of basis parts of transmissions was paradoxical.

The suggested approach to solving the problem enables us to throw light upon an analogous phenomenon and carry out some modernization of many metallurgical and mobile objects. Among them Rustavi plate rolling mill 2100, Novolipetsk metallurgical plants 120 ton converter, plate rolling mill – 2000, working stand No 5 at Karaganda metallurgical plant, transmissions of the Kutaisi lorries, etc.

This reconstruction eradicated the emergency working regimes at the above-mentioned objects and increased the stability and the speed of working regimes.

It is worth noting that to realize the aim discussed above it turned out to be enough to decrease the rigidity of engine shafts (not to increase?!). This fact seems paradoxical from the traditional point of view.

Had the mentioned objects been designed according to the method suggested by the author, their metalintensivity would have been decreased by 30%, and, at the same time, their productivity and stability would have increased considerably.

On the basis of scientific analysis of the obtained data that engineering method of the synthesis of structural schemes of mechanical systems was worked out.

The relationship between the ratio of a system's construction parameters and generalized parameters have been determined.

The range of rational alternation of the mentioned parameters has been determined. Their practical realization ensures maximal removal of lowest natural oscillation frequencies of mechanical systems, leading to complete elimination of internal resonance (oscillation beating) and accordingly, minimization of dynamic loading.

With numerical examples the influence will be shown of the correlation of construction parameters and structural schemes of mechanical systems on the nearness of natural oscillation frequencies and consequently on the values of the generalized parameter C_1.

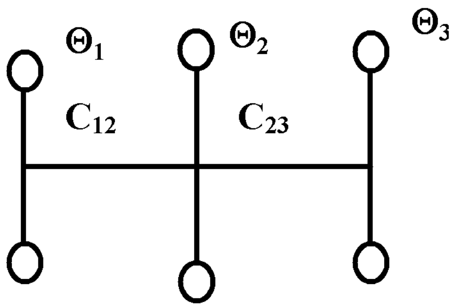

Fig. 2.

Let us consider several examples of different disposition and correlation of masses and rigidity of three mass equivalent torsion mechanical systems

1. $\theta_1 = \theta_2 = \theta_3 = 100$; $C_{12} = C_{23} = 10^4$;
2. $\theta_1 = \theta_3 = 100$; $\theta_2 = 1000$; $C_{12} = C_{23} = 10^4$;
3. $\theta_1 = \theta_3 = 100$; $\theta_2 = 10^4$; $C_{12} = C_{23} = 10^4$;
4. $\theta_1 = \theta_3 = 100$; $\theta_2 = 1000$; $C_{12} = 5\times10^4$; $C_{23} = 10^4$;
5. $\theta_1 = 10$; $\theta_2 = \theta_3 = 100$; $C_{12} = C_{23} = 10^4$.

It is known that the indicated three-mass system is described by a differential equation of the fourth order

$$X^{IV} + a_0 X^{II} + a_1 X = 0. \qquad (18)$$

a_0 and a_1 are determined by expression (5)

In generalized dimensionless parameters, according to (2), expression (18) takes the form

$$X^{IV} + X^{II} + .C_1 X = 0. \qquad (19)$$

Then for case 1 we have:

$$a_0 = C_{12}\frac{\theta_1+\theta_2}{\theta_1\theta_2} + C_{23}\frac{\theta_2+\theta_3}{\theta_2\theta_3} = 400\,;$$

$$a_1 = C_{12}\cdot C_{23}\frac{\theta_1+\theta_2+\theta_3}{\theta_1\theta_2\theta_3} = 30000\,;$$

$$C_1 = \frac{a_1}{a_0^2} = \frac{30000}{160000} = 0.188\,.$$

The characteristic equation for case 1 in numerical form will be:

$$\lambda^4 - 400\cdot\lambda^2 + 30000 = 0;$$

$$\lambda^2 = 200 \pm \sqrt{10000} = 200 \pm 100;$$

$$\lambda_1^2 = \beta_1 = 100; \quad \lambda_2^2 = \beta_2 = 300.$$

For case 2 we have

$$a_0 = 220; \quad a_1 = 12000; \quad C_1 = 0.248.$$

The characteristic equation

$$\lambda^4 - 220 \cdot \lambda^2 + 12000 = 0;$$
$$\lambda^2 = 110 \pm \sqrt{100};$$
$$\lambda_1^2 = \beta_1 = 100; \quad \lambda_2^2 = \beta_2 = 120.$$

Hence at increasing the generalized parameter C_1 natural frequencies of the oscillations of the system become close to each other, pointing to origination of beating of oscillations.

It will be recalled that for differential equations of the fourth order the maximum value of C_{1max}=0.25.

For the case 3 we have

$$a_0 = 202; \quad a_1 = 10200; \quad C_1 = 0.24998$$
$$\lambda^4 - 202 \cdot \lambda^2 + 10200 = 0;$$
$$\lambda^2 = 101 \pm \sqrt{1};$$
$$\lambda_1^2 = \beta_1 = 100; \quad \lambda_2^2 = \beta_2 = 102.$$

Thus, when the structural scheme is chosen unsuccessfully, i.e. when the middle mass is considerably (100 times) greater than extreme masses, the generalized parameter C_{1max}=0.24998≈0.25. Therefore natural frequencies become equal to β_1=100; β_2=102, which is the cause of origination of pure oscillations beating (internal resonance).

For case 4 we have

$$a_0 = 660; \quad a_1 = 60000; \quad C_1 = 0.137$$
$$\lambda^4 - 660 \cdot \lambda^2 + 60000 = 0;$$
$$\lambda^2 = 300 \pm \sqrt{48900};$$
$$\lambda_1^2 = \beta_1 = 109; \quad \lambda_2^2 = \beta_2 = 551.$$

For this example we see that with a 5-fold increase of rigidities C_{12} optimization of oscillation processes is ensured at the expense of removal of natural frequencies from each other to a considerable degree.

The criterion for the evaluation of the removal of natural frequencies from each other is a quantity of generalized parameter C_1, which for the case under consideration, is equal to C_1=0,137; i.e. it satisfies the condition of optimality determined by expression (15).

For case 5 we have

$$a_0 = 310; \quad a_1 = 12000; \quad C_1 = 0.124$$
$$\lambda^4 - 310 \cdot \lambda^2 + 12000 = 0;$$
$$\lambda^2 = 155 \pm \sqrt{11025};$$
$$\lambda_1^2 = \beta_1 = 50; \quad \lambda_2^2 = \beta_2 = 260.$$

It is clear that another effective means of obtaining the desirable value of generalized parameter, hence re-

Table 1.

Values of generalized parameter for different variants of fulfilment of three-mass design schemes

Variants	Correlation (ratio) of mass and rigid parameters (dimensionless values)					Generalized parameter C_1
	θ_1	θ_2	θ_3	c_{12}	c_{23}	
1	1	1	1	1	1	0.1875
2	1	2	1	1	1	0.22
3	1	5	1	1	1	0.243
4	1	10	1	1	1	0.248
5	5	1	1	1	1	0.135
6	2	1	1	1	1	0.164
7	3	1	1	1	1	0.150
8	10	1	1	1	1	0.125
9	2	1	2	1	1	0.140
10	3	1	3	1	1	0.110
11	5	1	5	1	1	0.076
12	10	1	10	1	1	0.043
13	1	1	1	2	1	0.090
14	1	1	1	3	1	0.062
15	1	1	1	2	2	0.187
16	1	1	1	5	5	0.187
17	2	1	1	2	1	0.040
18	2	1	1	2	2	0.163
19	1	10	2	2	1	0.220
20	1	10	1	5	1	0.135
21	1	10	1	10	1	0.082
22	1	5	1	3	1	0.082
23	1	5	1	5	1	0.134

moval of natural frequencies of oscillation, is to increase the correlation of the moments of mass inertia.

A more acceptable means of obtaining the necessary value of generalized parameter C_1, from the viewpoint of practical realization, is to increase the correlation of the rigidities of elastic links.

Numerous examples of calculations have shown that to prevent oscillation beatings it is enough to choose a correlation of rigidities equal to two and more. In such a system no internal resonance will arise.

Numerical examples of calculations of three-mass torsion mechanical systems have shown that symmetrical schemes are nonoptimal; extremely dangerous are structural schemes with middle greater mass. However, such schemes are optimal for the designer of vibromachines.

Values of generalized dimensionless parameters defining the level (values of amplitude) and the character of change of dynamic loadings (moments of elastic force) at different magnitudes of moments of inertia of mass and rigidity are given in Table 1.

Consideration of Table 1 shows variants of relationship of parameters that satisfy the conditions of optimality (15).

For four-mass torsion mechanical systems (Fig. 3) change of moments of elastic force is described by the differential equation of the sixth order

$$M_{i,i+1}^{VI} + a_0 M_{i,i+1}^{IV} + a_1 M_{i,i+1}^{II} + a_2 M_{i,i+1} = 0 \qquad (20)$$
$$(i = 1,2,3)$$

In generalized dimensionless parameters, according to (2), expression (20) takes the form

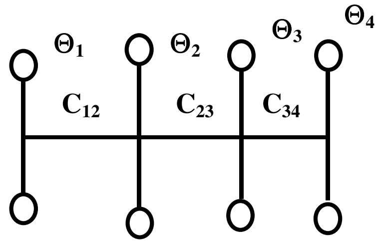

Fig. 3.

$$M_{i,i+1}^{VI} + M_{i,i+1}^{IV} + C_1 M_{i,i+1}^{II} + C_2 M_{i,i+1} = 0 \qquad (21)$$
$$(i = 1,2,3)$$

In these expressions the coefficients a_i (i=0,1,2) and C_i (i=0,1,2) are defined

$$a_0 = C_{12}\frac{\theta_1+\theta_2}{\theta_1\theta_2} + C_{23}\frac{\theta_2+\theta_3}{\theta_2\theta_3} + C_{34}\frac{\theta_3+\theta_4}{\theta_3\theta_4};$$

$$a_1 = C_{12}\cdot C_{23}\frac{\theta_1+\theta_2+\theta_3}{\theta_1\theta_2\theta_3} + C_{23}\cdot C_{34}\frac{\theta_2+\theta_3+\theta_4}{\theta_2\theta_3\theta_4} + $$
$$+ C_{12}\cdot C_{34}\frac{\theta_1\theta_4+\theta_2\theta_3+\theta_1\theta_3+\theta_2\theta_4}{\theta_1\theta_2\theta_3\theta_4};$$

$$a_2 = C_{12}C_{23}C_{34}\frac{\theta_1+\theta_2+\theta_3+\theta_4}{\theta_1\theta_2\theta_3\theta_4}, \qquad (22)$$

$$C_1 = \frac{a_1}{a_0^2}; \qquad C_2 = \frac{a_2}{a_0^3}.$$

Table 2.

Values of generalized parameter for different variants of fulfilment of four-mass design schemes

Variants	Dimensionless parameter							Generalized parameter	
	θ_1	θ_2	θ_3	θ_4	c_{12}	c_{23}	c_{34}	c_1	c_2
1	1	1	1	1	1	1	1	0.277	0.018
2	10	1	1	1	1	1	1	0.250	0.010
3	1	10	1	1	1	1	1	0.260	0.015
4	10	10	1	1	1	1	1	0.240	0.006
5	1	10	10	1	1	1	1	0.280	0.016
6	10	1	1	10	1	1	1	0.150	0.003
7	5	1	1	5	1	1	1	0.180	0.0056
8	1	1	1	1	3	1	1	0.240	0.0120
9	1	1	1	1	5	1	1	0.190	0.0073
10	1	1	1	1	10	1	1	0.120	0.003
11	1	1	1	1	1	3	1	0.220	0.0120
12	1	1	1	1	1	5	1	0.170	0.0036
13	1	1	1	1	1	10	1	0.110	0.003
14	1	10	10	1	1	1	1	0.260	0.010
15	1	10	10	1	1	1	1	0.145	0.0035
16	1	10	10	1	1	1	2	0.275	0.0090
17	1	10	10	1	1	1	5	0.268	0.0071

Values of generalized dimensionless parameters C_1 and C_2, defining the level and the character of elastic force (M_{12}, M_{23}, M_{34}) in a four-mass torsion mechanical system are given in Tab. 2.

After using data given in the table the engineer easily ascertains what kind of relationship of moments of inertia and rigidity satisfy the conditions of optimality (16), at keeping of which minimization of dynamic loads and prevention of inner resonances will be provided.

In Tables 1 and 2 the values of digits 1…10 denote not absolute values of inertia masses and rigidity but their relationship. That is why the digits have no dimention.

One can conclude that the level and the character of change of dynamic loads in mechanical and electromechanical systems (in transmission of machines and mechanisms) depends not on real meanings of the parameters of the system but on their relationship, i.e. on the structure (by structure and order of position inertia and elastic parameters) of the objects under study.

მექანიკა

მექანიკური სისტემების რხევითი პროცესების და რაციონალური დაპროექტების მოდელირების ინჟინრული მეთოდები

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ნაშრომში ანალიზური და ორიგინალური საინჟინრო მიდგომებისა და მეთოდების შემუშავებით გადაწყვეტილია მოძრაობის მრავალი თავისუფლების ხარისხის მქონე როტორული ტიპის მექანიკური დინამიკური სისტემების (სამანქანო აგრეგატების) გარდამავალი (რხევითი) პროცესების ოპტიმიზაციისა და რაციონალური დაპროექტების სამეცნიერო-ტექნიკური ამოცანები, რაც უზრუნველყოფს ანალოგიურ სისტემებში დინამიკური დატვირთვების მინიმიზაციას, ვიბრაციულ-რეზონანსული (რხევათა ცემის) მოვლენების აღმოფხვრას და ამით მათი საიმედოობისა და მედეგობის მნიშვნელოვან ამაღლებას.

REFERENCES

1. *R. Adamia* (1999), Dynamics of Machines. Tbilisi, 400 p. (Russian).
2. *R. Adamia, F. Mikhailenko* (1989), Limitations of Dynamic Loading in Machines. Tbilisi, 180 p. (Russian).

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