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Astronomy

On the Probability of Collisions of Interplanetary Space Vehicles with Meteors

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ABSTRACT. On the average once in 4 years uncontrolled geostationary satellites suffer small sudden changes of speed, which is connected with their collisions with fine space debris. Most of these events are caused by collisions with meteoric bodies. Such collisions threaten space vehicles as well, sent to planets of the solar system to study their physical nature.

The present paper is devoted to determining the degree of risk of collision of a space vehicle with meteors at different possible variants of its interplanetary orbit. The study allows to select - out of several possible variants – the least hazardous trajectory of interplanetary flight of a space vehicle from the point of view of meteoric danger.
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1. Introduction

Since the 1990s, in connection with the increased number of space vehicles (SV) investigating interplanetary space, the need has arisen of a careful calculation of their orbits from the viewpoint of minimization of expenditure of power resources, as well of the selection of the minimally hazardous orbits in space.

In the 21st century interplanetary flights of SV with people on board are also planned and practically preparations are under way for expeditions to the nearest planets. Therefore, careful choice of trajectories of flights is not only important but necessary as well.

Our long-term work with uncontrolled geostationary satellites shows that from time to time (on the average, once in 4 years) they suffer small - within the limits of several mm/s - sudden changes of speed, which is connected with their collisions with fine space debris. Most of these events are caused by collisions with meteoric bodies [1-5].

Obviously, SVs, sent to planets of the solar system to study their physical nature, are threatened with such collisions, entailing disaster.

The present study is devoted to determining the degree of risk of collision with meteors for different possible variants of SV interplanetary orbit.

As is known, the bulk of meteoric bodies moves in the form of streams created by comets. These bodies move along the orbit of the comet generating them, at speeds of some dozens of km/s, and at a relatively short (within one million kilometers) distance from it. Sporadic meteors are smaller in number and they move chaotically.

The task of the present work is to choose from a set of possible heliocentric orbits of a SV the optimum, allowing to avoid the crossing of the orbits of comets (the orbits being the sites of congestion of meteoric bodies) by it and ensuring maximal remoteness from them.

It is necessary to note that the details of the distribution of meteoric bodies in streams are more or less known

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only for those few objects that cross the Earth's orbit during their annual revolution round the Sun.

Owing to this, it is necessary to assume a uniform distribution of meteoric bodies along orbits. For different streams the identicalness of their spatial density and the laws of its decrease with distance from the orbit of a comet are also postulated.

2. Calculation of the risk-factor of collision of a space vehicle with meteors

We shall further assume that the spatial density of meteoric bodies changes by the Gauss formula according to the distance from the orbit of the comet that has generated meteoric stream:

$$\rho_\Delta = \rho_0 e^{-\left(\frac{\Delta}{\Delta_0}\right)^2}, \tag{1}$$

where ρ_D is the spatial density of meteoric bodies at distance Δ from the comet's orbit, r_0 - spatial density of meteoric bodies on the orbit (constant for all streams). The parameter Δ_0 is accepted to equal 0.01 AU, i.e. 1.5×10^6 km.

To calculate the value of Δ let us introduce a rectangular system of coordinates connected with the orbit of the comet. The origin of the coordinate system coincides with the Sun, X-axis shall be directed to the perihelion of the orbit and Y-axis - to the plane of this orbit, in the direction of the movement of the comet.

The position of a SV at some moment t in the selected system of coordinates will be designated through x_0, y_0, z_0.

The problem of finding the value of Δ is thus reduced to finding the minimal distance between the SV and the orbit of the comet, i.e. to the search of the minimum of the function:

$$\Delta^2 = (x - x_0)^2 + (y - y_0)^2 + z_0^2, \tag{2}$$

where the points on the orbit of the comet are designated through the x and y coordinates.

From theoretical astronomy it is known that:

$$x = \frac{p\cos v}{1 + e\cos v}, \quad y = \frac{p\sin v}{1 + e\cos v}, \tag{3}$$

where p is the parameter of the orbit, e - its eccentricity, v - true anomaly.

Equating to zero the result of differentiation (2) with respect to v and substituting the values x and y, determined from (3), we receive:

$$(ex_0 \cos v_m + x_0 + ep)\sin v_m = y_0[e\cos^2 v_m + (1 + e^2)\cos v_m + \mathrm{e}], \tag{4}$$

where the value of the true anomaly for a point on the comet's orbit, being at the minimal distance from the SV, is designated through v_m.

Inputting a new variable

$$q = tg\frac{v_m}{2}, \tag{5}$$

the equation (4) can be given in a simpler form:

$$y_0(1-e)^2 q^4 + 2(ep + x_0 - ex_0)q^3 + 2(ep + x_0 + ex_0)q - y_0(1+e)^2 = 0. \tag{6}$$

The fourth degree of the equation (6) relative to q reflects the fact that generally from one point on an ellipse it is possible to drop four perpendiculars, satisfying the condition of the extremum (4).

3. Solution of the equation for true anomaly

The equation (6) is solved most conveniently by way of consecutive approximations.

It is possible to find geometrically, the first approximation to the root of this equation, corresponding to minimal Δ, by crossing the ellipse of the comet's orbit with the bisector of an angle formed by straight lines, connecting the SV with the focuses of the ellipse.

These lines (the sides of an angle) form some angles with X-axis, whose values we shall designate through α and β. They are determined by the following equations:

$$tg\alpha = \frac{y_0}{x_0}, \quad tg\beta = \frac{(1-e^2)y_0}{(1-e^2)x_0 + 2ep}, \tag{7}$$

whence the factor k of the inclination of the bisector is equal to:

$$k = tg\frac{\alpha+\beta}{2}. \tag{8}$$

Hence, the equation for the point of intersection of the bisector with the comet's orbit is as follows:

$$\frac{p\sin v_0}{1+e\cos v_0} - y_0 = k\left(\frac{p\cos v_0}{1+e\cos v_0} - x_0\right), \tag{9}$$

where the preliminary value of v_m is designated through v_0.

Substituting in (9) the value of v_0, determined by means of (5), for the initial (zero) approximation of q we obtain the equation:

$$[(1-e)(y_0 - kx_0) - kp]q_0^2 - 2pq_0 + (1+e)(y_0 - kx_0) + kp = 0, \tag{10}$$

easily solved relative to q_0.

Because the eccentricities of the comets' orbits are close to unity, the member of the fourth degree in the equation (6) is small, allowing to consider a third degree equation, instead of (6):

$$q^3 + 3Aq - 2B = 0, \tag{11}$$

where

$$A = \frac{ep + (1+e)x_0}{3[ep + (1-e)x_0]}, \quad B = \frac{y_0[(1+e)^2 - (1-e)^2 q^4]}{4[ep + (1-e)x_0]}. \tag{12}$$

The value of B is specified by consecutive approximations.

If the inequality takes place:

$$B^2 + A^3 > 0, \tag{13}$$

then the equation (11) has one real root, according to the Kardano formula, equal to

$$q = \left(B + \sqrt{B^2 + A^3}\right)^{1/3} + \left(B - \sqrt{B^2 + A^3}\right)^{1/3} \tag{14}$$

If the inequality (13) does not take place, the equation (11) has three real roots. In that case we shall use iterations in the process of approximations:

$$q_{k+1} = (2B - 3Aq_k)^{1/3}. \tag{15}$$

Here it is useful to apply the following routine allowing to essentially improve the convergence of the iteration process. If we have three consecutive approximations q_1, q_2 and q_3, the following expression may be derived:

$$q_4 = \frac{q_1 q_3 - q_2^2}{q_1 + q_3 - 2q_2}, \tag{16}$$

then q_4 will essentially prove more precise than the previous approximations. With a view to accelerating the convergence of the process of iterations the formula (16) can be applied repeatedly.

4. Transition from the system of coordinates connected with the SV orbit to the system of the comet's orbit

In the system of coordinates connected with an SV orbit (if X-axis is directed to its perihelion) the coordinates of the SV x_1, y_1, z_1 are expressed similarly to (3):

$$x_l = \frac{p_l \cos v_l}{1+e_l \cos v_l}, \quad y_l = \frac{p_l \sin v_l}{1+e_l \cos v_l}, \quad z_l = 0. \tag{17}$$

The index l corresponds to the system of coordinates related to the SV orbit and the value v_l for each separately taken moment t_l is determined through the solution of Kepler's equation:

$$tg\frac{v_l}{2} = \sqrt{\frac{1+e_l}{1-e_l}}tg\frac{E_l}{2}, \tag{18}$$

where

$$E_l - e_l \sin E_l = M_l. \tag{19}$$

The duration of the flight of the SV is equal to

$$\Delta T = a_L^{3/2}(M_L - M_0), \tag{20}$$

where the initial and final values of the mean anomaly of the SV are designated through M_0 and M_L respectively; a_L is a semimajor axis of the SV orbit.

To calculate the values of v_l by means of the expressions (19) and (20), we shall divide the interval DT into L equal parts.

Transition from the coordinates x_l, y_l, z_l to the coordinates x_0, y_0, z_0 in the system connected with the orbit of the comet is effected by the formulas of spherical trigonometry:

$$x_0 = x_l(\cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i) - y_l(\sin\omega\cos\Omega + \cos\omega\sin\Omega\cos i),$$

$$y_0 = x_l(\cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i) - y_l(\sin\omega\sin\Omega - \cos\omega\cos\Omega\cos i), \tag{21}$$

$$z_0 = \sin i(x_l \sin\omega + y_l \cos\omega),$$

where the inclination, argument of perihelion and the longitude of the Node of the SV orbit relative to the comet's orbit are designated through *i,* ω and Ω respectively. These values are calculated by means of the expressions:

$$\cos i = \cos i_n \cos i_l + \sin i_n \sin i_l \cos(\Omega_n - \Omega_l),$$

$$\omega = \omega_l - arctg\frac{\sin i_n \sin(\Omega_n - \Omega_l)}{-\cos i_n \sin i_l + \sin i_n \cos i_l \cos(\Omega_n - \Omega_l)}, \tag{22}$$

$$\Omega = arctg\frac{\sin i_l \sin(\Omega_n - \Omega_l)}{\sin i_n \cos i_l - \cos i_n \sin i_l \cos(\Omega_n - \Omega_l)} - \omega_n.$$

where the index n designates the elements of an orbit of n^{th} comet relative to the ecliptic.

Finally, the non-normalized probability of collision of the SV with a meteor for the entire time of flight will be:

$$P_{LN} = \frac{\Delta T}{L+1}\sum_{i=0}^{L}\sum_{k=1}^{N} e^{-\left(\frac{\Delta_{ik}}{\Delta_0}\right)}, \tag{23}$$

where N designates the total number of the orbits of the comets used at calculations.

By the software realized on Fortran, in the case of using elements of the orbits of 500 comets and dividing the DT interval into 1000 times, it takes PC Pentium – 4, to calculate the probability of collision of an SV with meteors, 4s of machine time.

Thus, the present study allows choosing the safest, out of several possible variants of the trajectory of interplanetary flight of SV, from the point of view of collisions with meteors.

ასტრონომია

მეტეორებთან საპლანეტთაშორისო ხომალდის შეჯახების ალბათობის შესახებ

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უმართავი გეოსტაციონარული თანამგზავრები საშუალოდ ოთხ წელიწადში ერთხელ მყისიერად იცვლიან სიჩქარეს, რაც გამოწვეულია კოსმოსურ ნაგავთან მათი შეჯახებებით. ასეთი შემთხვევების უდიდესი ნაწილის მიზეზია შეჯახებები მეტეორებთან.

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წინამდებარე ნაშრომი მიზნად ისახავს კოსმოსური ხომალდების მეტეორებთან შეჯახების რისკის განსაზღვრას საპლანეტათაშორისო ორბიტის სხვადასხვა ვარიანტისათვის. ნაშრომი საშუალებას იძლევა, საპლანეტათაშორისო ხომალდის ტრაექტორიის რამდენიმე შესაძლო ვარიანტიდან არჩეული იქნეს მეტეორული საფრთხის თვალსაზრისით ყველაზე უხიფათო.

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