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*Physics*

# On Vortex Model of Planet Formation in Keplerian Disks

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**ABSTRACT. Invoking a hypothesis that long-lived vortices in the protoplanetary nebula play an important role in the three-phase model of planet formation, we analyze the first phase and present results on the stability and nonlinear development of vortices against the background of shearing Keplerian flows. We discuss the conditions under which vortical perturbations evolve into long-lived self-sustained structures and describe the properties of these equilibrium vortices. The properties of equilibrium vortices appear to be independent of the initial conditions and depend only on the local disk parameters. In particular, we find that the ratio of the vortex size to the local disk scale height increases with the decrease of the sound speed, reaching values well above the unity. The process of spiral density wave generation by the vortex leads to the formation of spiral shocks attached to the vortex. These shocks may have important consequences on the long term vortex evolution and possibly on the global disk dynamics. Our study strengthens the arguments in favor of anticyclonic vortices as the candidates for the promotion of planet formation. Hydrodynamic shocks that are an intrinsic property of persistent vortices in compressible Keplerian flows are an important contributor to the overall balance. These shocks support vortices against viscous dissipation by generating local potential vorticity and should be responsible for the eventual fate of the self-sustained anticyclonic vortices.** 

***Key words:*** *planet formation, protoplanetary disks.*

## 1. Introduction

The modern trend of investigations on the formation of planetary systems may be attributed to Laplace [1], who introduced the nebular hypothesis to rationalize the distribution and the motion of planets and their satellites in the solar system. The conjecture is based on the fact that the solar system is formed from a rotating flattened gas cloud. In fact the astronomical nebular theory advanced by Laplace is a root of many astrophysical investigations as much as disk-like rotating bodies represent widely spread and important structural elements of our Universe. Among them are galaxies, quasars, accretion disks surrounding compact stellar objects (white dwarfs, neutron stars and black holes) in binary systems, protostellar and protoplanetary disks. The formation of protoplanetary disks proceeds by the following scenario: Stars condense from an interstellar medium consisting mostly of gas with an admixture of solid particles called interstellar dust. Both observational and numerical studies suggest that, as the central star contracts, it leaves around material that contains a sufficiently great share of the initial angular momentum of the whole system. In this nebula, the centrifugal force balances the stellar gravity in the radial direction and a protoplanetary disk is formed. Such disks represent initial material for planet formation.

It is just over ten years since the first planet outside our solar system was detected. Since then, much work has focused on understanding the formation of planetary systems in general and our solar system in particular. It is investigation of the dynamics of coherent vortices in protostellar and protoplanetary disks that has received increasing attention from the end of the last century [2]. However, it was von Weiszacker who proposed the hypothesis that long-lived vortices in protoplanetary nebula can play an important role in planet formation [3]. Indeed, it has been shown that vortices, if sustained long enough, lead to particle aggregation in their core and to the formation of protoplanets [4-10]. Invoking a three-phase model for the planetary formation [11-12], one can consider the process of vortex formation as the first phase. The second phase is characterized by the accumulation of solids in the center of vortices and by the growth of a planetary core, and the third phase by the accretion of gas onto the core. Our research is related to the first phase of the planet formation – development of long-lived vortices.

The vortex scenario for planet formation encounters an apparent obstacle: any structure in a protoplanetary/Keplerian disk is subject to strong shearing that may eventually lead to its decay. The only mechanism for sustaining a stable vortex in such flows is nonlinearity. Hence, vortices that may start the process of planet formation should exceed a critical threshold in their amplitude. Direct numerical simulations are therefore an important tool in these studies. Several works have been devoted to the analysis of the possibility of forming and maintaining coherent vortex structures in the strongly sheared flow pertaining to a Keplerian disk, both in barotropic configurations, where the perturbations of potential vorticity are conserved [6,7,13- 15], and in baroclinic situations, where potential vorticity can be generated [11-12]. In the incompressible case it has been shown that coherent vortex structures can indeed survive nonlinearly (with conserved potential vorticity). In this case anticyclonic vortices can survive longer than cyclonic ones [13] and give rise to the Rossby waves in the system [15]. The effects of compressibility have not yet been fully analyzed and require rigorous study. Two-dimensional time-dependent numerical simulations of vortices in viscous compressible Keplerian disks can be found in [6]. Vorticity waves are considered as one of the constituents of (anticyclonic) vortex dynamics, but without description of the wave properties and any analysis of their genesis and dynamics (the subject of the study was the stability and lifetime of vortices). Detailed analytical and numerical study of the dynamics of perturbations (vortex/aperiodic mode, Rossby and spiral-density waves) in 2D compressible disks with a Keplerian law of rotation has been performed in [16]. The study has shown that small amplitude coherent circular vortex structures are capable of generating density-spiral waves linearly. The main features of this generation phenomenon have been studied, using global direct numerical simulations.

Here we present the results of a numerical study of the nonlinear dynamics of vortices and spiral-density waves in compressible Keplerian disk flows. We show that nonlinear anticyclonic vortices undergo direct nonlinear adjustment to the long-lived self-sustained coherent structure. Vortices generate density-spiral waves under the influence of Keplerian shear. We show the nonlinear development of spiral-density waves that result in the formation of spiral shocks with a steady spatial pattern. These shocks may increase the stability of anticyclonic vortices by slowing down their decay and may also affect the global disk dynamics.

## 2. Physical model and numerical setup

The following are the basic equations governing the dynamics of disk flows rotating around the gravitating center in cylindrical co-ordinates:

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho V_r\right)+\frac{1}{r}\frac{\partial}{\partial\varphi}\left(\rho V_\varphi\right)=0\,, \qquad (1)$$

$$\frac{\partial V_r}{\partial t}+(V\nabla)V_r-\frac{V_\varphi^{\ 2}}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}-\frac{\partial\Phi}{\partial r}\,, \qquad (2)$$

$$\frac{\partial V_\varphi}{\partial t}+(V\nabla)V_\varphi+\frac{V_\varphi V_r}{r}=-\frac{1}{\rho r}\frac{\partial P}{\partial\varphi}\,, \qquad (3)$$

$$\left(\frac{\partial}{\partial t}+(V\nabla)V\right)P=\gamma\frac{P}{\rho}\left(\frac{\partial}{\partial t}+(V\nabla)V\right)\rho\,, \qquad (4)$$

where

$$(V\nabla)=V_r\frac{\partial}{\partial r}+\frac{V_\varphi}{r}\frac{\partial}{\partial\varphi}\,.$$

Our equilibrium state corresponds to the Keplerian disk flow with $V_0=(0,r\Omega(r))$ and $\Omega(r)\sim r^{-3/2}$. The central gravitational potential balances the centrifugal force: $\Phi(r)\sim r^{-1}$. Equilibrium pressure and density are set to be constants.

Initial conditions for our simulations are composed by Keplerian equilibrium flow and local vortical perturbations (denoted with prime) with the following geometry:

$$V_x'(0)=\pm\frac{\varepsilon(y-y_0)}{q}\exp\left[-\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{q^2a^2}\right], \qquad (5)$$

$$V_y'(0)=\mp\frac{\varepsilon(x-x_0)}{q}\exp\left[-\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{q^2a^2}\right], \qquad (6)$$

$$\rho'(0) = 0 . \quad (7)$$

Here $\varepsilon$ defines the amplitude of the initial perturbation and its sign determines the vortex polarity (positive in the case of anticyclonic vortex). The parameters $a$ and $q$ describe, respectively, the size (in the radial direction) and aspect ratio of an elliptic vortex. A circular vortex corresponds to $q = 1$, and $q > 1$ refers to a vortex elongated in the azimuthal direction. $(x_0, y_0)$ so that it corresponds to the radial location $r_0 = 1$.

We perform direct numerical simulations (DNS) based on the nonlinear set of Eqs. (1-4) using initial conditions corresponding to the sum of equilibrium Keplerian flow and a local vortex perturbation shown in Eqs. (5-7). Numerical simulations are based on the DNS code PLUTO [17] with implemented FARGO scheme [18].

## 3. Nonlinear development of vortices

The evolution of the perturbations depends on three main parameters: the amplitude and size of the perturbation (respectively $\varepsilon$ and $a$) and the sound speed in the disk, $Cs$. Our first aim is to determine the region, in the parameter space described by $\varepsilon$, $a$ and $Cs$, in which the evolution of the initial perturbation leads to a stable, long-lived equilibrium configuration. As we shall see, before reaching this final state the system undergoes, in the course of several disk revolutions, a transition phase that we call *nonlinear adjustment*. Our second aim is to provide a detailed description of the equilibrium vortex configuration. With these purposes we have performed runs with different values of the three parameters and different numerical setups, using Eqs. 5-7 with $q = 1$ (circular vortices). In particular, by increasing the value of the sound speed ($Cs = 0.001, 0.01, 0.1$), we have explored the behavior of the vortex changing $a$ at fixed $e$ and changing $e$ at fixed $a$. Additional calculations have been performed for the purpose of exploring in more detail particular regions of the parameter space. For instance, additional values of $Cs$ have been used for a better understanding of the scaling behaviors of some of the vortex properties. Moreover, in order to investigate whether the general behavior is changed by varying the shape and structure of the initial perturbation, we have performed computations with different values of the ellipticity parameter.

We found that there are two threshold parameters for the vortex amplitude that control the fate of initially imposed vertical perturbation in Keplerian flow. The first threshold parameter $\varepsilon$=0.1. When the vortex amplitude exceeds the first nonlinear threshold, a two-stage process occurs. First, the vortex is sheared into a narrow vortex layer, which then undergoes local instabilities. We then observe the formation of small-scale weak anticyclonic vortices at different azimuthal locations.

Vortices with $\varepsilon > \varepsilon^{**}$ experience direct adjustment from the initial to the final self-sustained structure, *i.e.,* a strong anticyclonic vortex is developed. The final equilibrium configuration appears to be a non-linear attractor reached by the system if the initial amplitude exceeds $\varepsilon^{**}$ and if the initial spatial scale falls in a range discussed in the next subsection. Indeed, the same nonlinear state is developed from all initial vortices satisfying these conditions, independently of the details of the initial potential vorticity distribution (exponential or algebraic, circular or elliptic). Fig. 1 shows the nonlinear vortex configuration developed from the initially imposed anticyclonic vortex perturbations with amplitude exceeding the second nonlinear threshold.

The result of nonlinear adjustment strongly depends also on the initial vortex size. In order to quantify this dependence, we have carried out computations with vortices of different initial size $a$. Our numerical results show that vortices undergoing direct nonlinear adjustment have initial size in the range $a_{\min} < a < a_{\max}$.

When the spatial scale of the initial vortex exceeds $a_{\max}$, the evolution is quite complex. We observe a radial

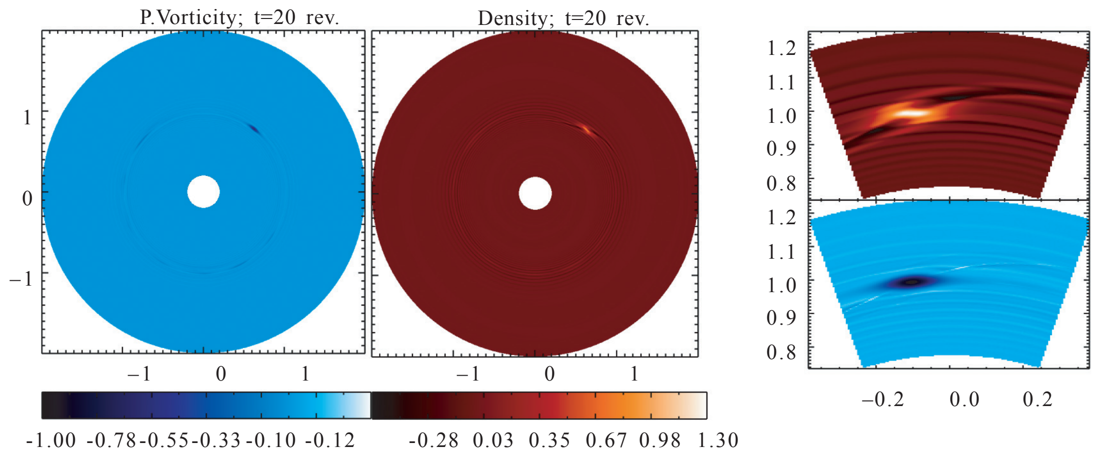


Fig. 1. Result of the nonlinear adjustment of the initially imposed anticyclonic vortex 20 local disk revolutions. Surfaces of the potential vorticity (left) and density (center) are shown in blue and red color, respectively. The right panel zooms on the vortex structure. Density maximum in the center of the vortex corresponds to the potential vorticity minimum.

transfer of potential vorticity in both directions, caused by the combined action of shocks (radiated from the initially imposed supersonic vortex) and of flow curvature (inducing Rossby wave variations). When the size of the vortex is smaller than $a_{\min}$ and the initial vortex amplitude exceeds the second nonlinear threshold, we observe nonlinear adjustment to a final configuration with sizes of the order of the initial value.

One of the main goals of the present study is to describe the stability and structure of long-lived self-sustained vortices in Keplerian disks. For this purpose, we selected cases undergoing direct adjustment to a single vortex, and we followed their long-time behavior. Fig. 1 shows radial profiles of potential vorticity and density at the center of a vortex.

Interestingly, the self-sustained vortex seems to be temporarily able to oppose viscous dissipation, exhibiting for some time an increase of the maximum potential vorticity. This effect implies production of potential vorticity. On the other hand, potential vorticity is a nonlinearly conserved quantity in barotropic flows. Thus, the spiral shock waves are the sources of the coherent generation of potential vorticity necessary to support or even enhance the vortex.

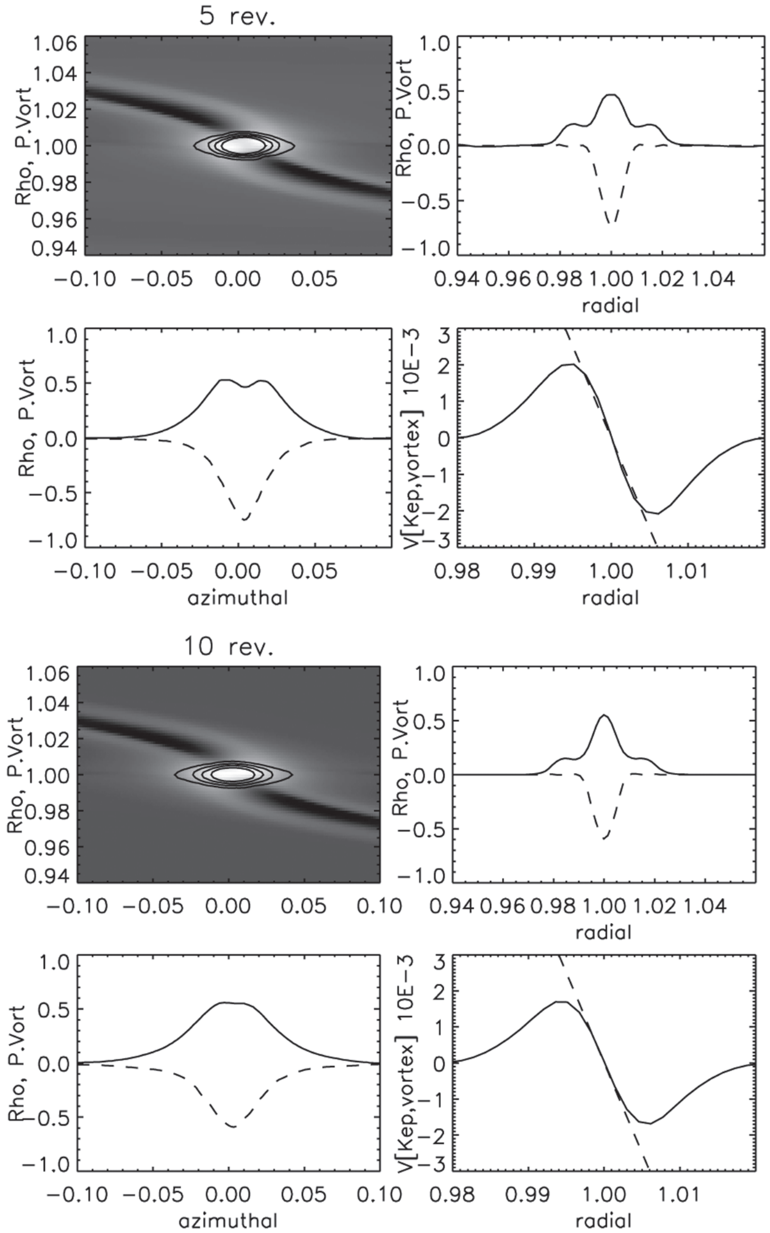


Fig. 2. The structure of the nonlinear long-lived anticyclonic vortex. The two panels whose vortex structure after 5 and 10 local disk revolutions, respectively. Graphs show grayscale visualisation of the density over potential vorticity contours (top left), together with radial and azimuthal cuts in density and potential vorticity. Radial velocity profiles of the background Keplerian flow and local nonlinear vortex are shown in bottom right graphs. Surfaces of the density perturbations reveal high density cores in the center of the vortex together with the well profound density-spiral waves beaming from the vortex center.

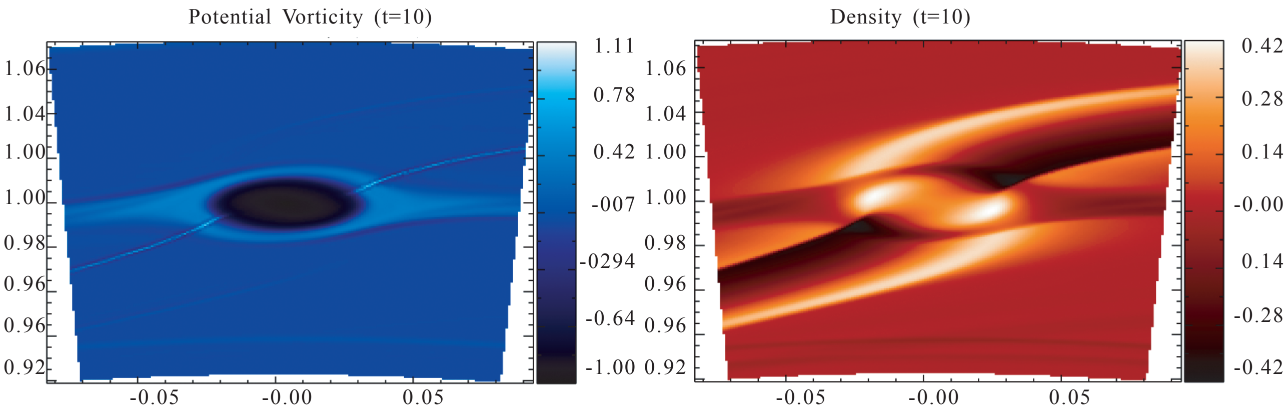


Fig. 3. Potential vorticity (left) and density (right) of the long-lived anticyclonic vortex after 10 local disk revolutions. Anticyclonic potential vorticity balances increased density within the vortex. Shock waves are revealed in jump lines beaming out of the central circulation. Density jumps associated with this steady pattern of shock waves are also readily seen.

The nonlinear dynamics of cyclonic vortices shows significant differences from anticyclonic ones. In the limit of small amplitudes, cyclonic vortices decay on the shearing timescale, related to the geometrical stretching induced by the background differential rotation.

Vortices in shear flows generate spiral density waves by a linear mechanism first described in [19] and further investigated numerically in Keplerian flows [16]. In the present context we study the characteristics of this process when nonlinear forces are also active. As clearly seen from Fig. 2 the dynamics of potential vorticity is accompanied by wave emission, not only during the transition time, but also afterwards, when the nonlinear self-sustained vortex is fully developed. This enforces the fact that nonlinearity does not suppress the wave emission. Moreover, after the adjustment, waves emitted by the coherent vortex structure appear to develop into spiral shocks. Coherent emission of acoustic waves leads to the density accumulation in compressible motions and to the development of shocks. The shock front naturally matches the ray trajectory of the generated waves, since these waves are responsible for the sustained pattern of dissipating shock waves.

Traces of shock generation by vortices may be found in [20]. In our simulations we verify the existence of a steady pattern of spiral shocks produced by a single self-sustained vortex. Moreover, shock waves appear to be an inherent property of vortices in sheared compressible flows, but they can be observed only at fairly high resolutions using shock capturing schemes. Our high resolution calculations allow to study these shock waves in fine details. Fig. 2 shows the distributions of potential vorticity and density for the vortex and the attached shock waves. One can distinctly recognize a wave-crest of the density-spiral wave developing into a double shock configuration, with the shock ahead of the vortex facing the outward region. A couple of much weaker shocks, parallel to the strong ones, appear to be present, although they remain barely visible. These shocks strongly affect the density structure of the developed vortex configuration, resulting in a splitting of the vortex core. Eventually, however, the shearing background leads to the merging of the cores, but the shocks persist.

Spiral shocks induced by a wake of planets are believed, in some situations, to be responsible for planet migration (see [21] and references therein). In our computations no radial variation of the vortex position has been observed. As seen in previous studies, spiral shocks affect dust accretion rates on the vortex core and thus promote the formation of a planetesimal. In this sense, they increase the importance of anticyclonic vortices in planetary formation. The presence of shocks has consequences for the vortex evolution. Nevertheless, the final fate of these structures cannot be easily foreseen and requires much longer simulations. We can here only sketch some possible scenarios. One possibility is the exhaustion of matter in the vortex bearing ring and the formation of an isolated planetesimal. Arguments that spiral shocks may lead to the gap formation can be found in [22]. On the other hand, shocks heat the ring at the radius where the vortex is sustained, which in turn may trigger the linear Rossby wave instability due to the unusual entropy gradient in the disk matter [23]. The instability will induce radial mixing and possibly the destruction of the coherent vortices. On the other hand, spiral shocks can be themselves unstable in three dimensions [24]. Hence, as we said, longer and possibly three-dimensional simulations can clarify this issue.

## 4. Summary

We have shown the possible existence of anticyclonic vortices with sizes exceeding the Keplerian disk height scales. We have followed the evolution of such vortices for 200 local revolutions, showing their persistence and stability.

We have found that the development of long-lived self-sustained nonlinear anticyclonic vortex configuration occurs only when the amplitudes of the initially imposed vortex perturbations exceed some threshold value. We have interpreted the latter process as the nonlinear vortex adjustment and studied the parameters that can describe this process.

The structure of the developed long-lived vortex does not depend on the initial vortex configuration, provided it exceeds the second threshold amplitude and its size does not exceed a limiting value. In this sense we found a nonlinear attractor that is the final configuration of a wide range of initial vortical perturbations.

Vortices generate density-spiral waves that rapidly develop into shocks. As a result, a long-lived nonlinearly balanced vortex is accompanied by two spiral compressible shock waves facing both radial directions.

We analyzed cyclonic vortices at nonlinear amplitudes. It seems that the linear decay due to the shearing deformation is accelerated by nonlinear effects. Our study contributes to the scenario of planetary formation inside the core of the long-lived vortices. We found that protoplanetary disks with lower sound speed can sustain vortices with a higher ratio of vortex size to disk thickness and create more favorable conditions for dust trapping and mass accumulation. In this context, we have also found a steady increase of density inside the nonlinearly balanced vortex, partly, due to the existence of persistent, steady, spiral shock waves that we showed to be an intrinsic property of stationary vortices in compressible Keplerian fows.

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*ფიზიკა*

# კეპლერულ დისკებში პლანეტების წარმოშობის გრიგალური მოდელის შესახებ

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**პლანეტების წარმოშობის სამფაზოვანი მოდელის მიხედვით პროტოპლანეტარულ ნისლოვანებაში ხანგრძლივად არსებულ გრიგალებს შეუძლიათ მნიშვნელოვანი როლი ითამაშონ პლანეტების ფორმირების პროცესში. ვიყენებთ რა ამ ჰიპოთეზას, ჩვენ ვაანალიზებთ პლანეტების ჩამოყალიბების პირველ ფაზას და წარმოვადგენთ არაერთგვაროვნად მბრუნავ კეპლერულ დისკებში გრიგალების მდგრადობისა და არაწრფივი დინამიკის კვლევის შედეგებს. ჩვენ ვმსჯელობთ იმ პირობებზე, რომლის დროსაც გრიგალური ტიპის შეშფოთებები ევოლუციის შედეგად აყალიბებენ სიცოცხლისუნარიან თვითშენარჩუნებად კოჰერენტულ სტრუქტურებს და აღვწერთ ამ წონასწორული გრიგალების თვისებებს. ამ არაწრფივი წონასწორული გრიგალების თვისებები არ არის დამოკიდებული საწყის პირობებზე და განისაზღვრება მხოლოდ დისკის ლოკალური პარამეტრებით. კერძოდ, კვლევებმა აჩვენეს, რომ გრიგალის ზომის შეფარდება დისკის მახასიათებელ ლოკალურ სისქესთან იზრდება ბგერის სიჩქარის კლებასთან ერთად და აღწევს ერთზე მნიშვნელოვნად უფრო მაღალ სიდიდეებს ცივ პროტოპლანეტარულ დისკებში. გრიგალების მიერ სპირალურ-გრავიტაციული ტალღების გენერაციის პროცესი იწვევს გრიგალიდან გამომავალი დარტყმითი ტალღების გენერაციას. ასეთი ტიპის დარტყმითმა ტალღებმა შეიძლება გამოიწვიონ მნიშვნელოვანი შედეგები როგორც გრიგალის ხანგრძლივი ევოლუციის პროცესში, ისე დისკის გლობალურ დინამიკაში. კეპლერულ დისკებში სიცოცხლისუნარიანი გრიგალების თანმდევი ჰიდროდინამიკური დარტყმითი ტალღები არიან ჯამური სურათის**

მნიშვნელოვანი მონაწილენი. დარტყმით ტალღებს შეუძლიათ ლოკალური პოტენციური ცირკულაციის გენერაცია, გრიგალების კვება დისიპაციური დანაკარგების ასანაზღაურებლად და მათი საბოლოო ბედის განსაზღვრა. ჩვენი კვლევები ამყარებენ არგუმენტებს იმ მოდელის სასარგებლოდ, რომლის მიხედვითაც ჰიდროდინამიკური გრიგალები წარმოადგენენ პლანეტების ფორმირების პროცესის ხელშემწყობ ფაქტორს, და პლანეტების ფორმირება იწყება ხანგრძლივად არსებული თვითშენარჩუნებადი გრიგალების ბირთვებში.

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