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Mathematics

Numerical Analysis of the Stress Distribution by the Boundary Elements Method in Continuous Body with a Hole

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ABSTRACT. The stress-deformed state of tunnels has been studied. Numerical solutions of the corresponding boundary value problem are obtained by the boundary elements method. The corresponding curves are constructed.

Key words: *boundary elements, fictitious load.*

The stress-deformed state of an underground tunnel is studied, particularly, the main stresses initiated on the tunnel walls are investigated, depending on the depth of its location and on the distance of overground structures from the tunnel.

We consider a boundary value problem as a mathematical model of the mentioned practical problem for a semi-infinite space with a hole. We take a homogeneous isotropic body in the plane deformed state. Its interior boundary surface is stress-free. Nonzero normal loading on one part of the plane boundary surface and zero normal loading on the other are given. Zero tangent loading is given on the whole boundary surface.

Since the body is in plane deformation state, we consider a two-dimensional boundary value problem on the half-space with a hole (Fig. 1).

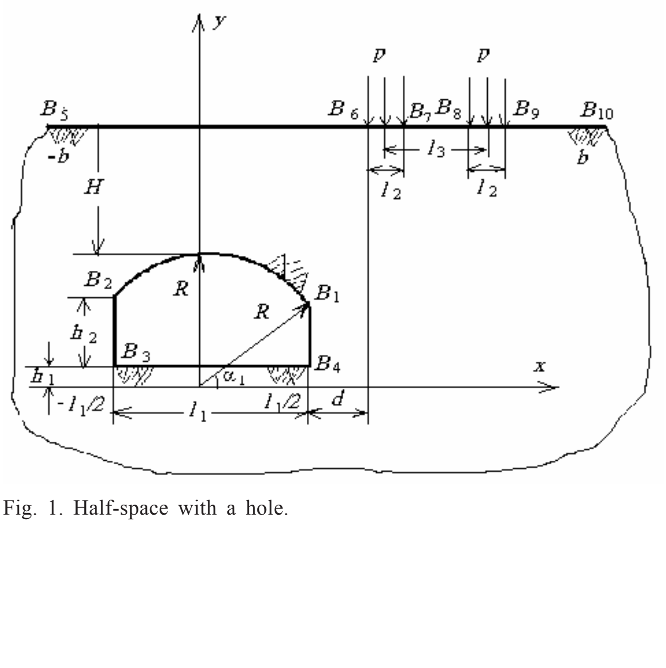


Fig. 1. Half-space with a hole.

Setting of the problem. Let us have a system of equilibrium equations with respect to unknown D, K, u, v on the infinite domain presented in Fig. 1.

$$\frac{\partial D}{\partial r}-\frac{1}{r}\frac{\partial K}{\partial \alpha}=0,\quad \frac{1}{r}\frac{\partial (ru)}{\partial r}+\frac{1}{r}\frac{\partial v}{\partial \alpha}=\frac{D}{\lambda+2\mu},$$
$$\frac{1}{r}\frac{\partial D}{\partial \alpha}+\frac{\partial K}{\partial r}=0,\quad \frac{1}{r}\frac{\partial (rv)}{\partial r}-\frac{1}{r}\frac{\partial u}{\partial \alpha}=\frac{K}{\mu} \tag{1}$$

Let us find the solution satisfying the following boundary conditions:

$$\begin{gathered}
r=R,\ \alpha_1<\alpha<\pi-\alpha_1:\ \sigma_{rr}=0,\ \sigma_{r\alpha}=0,\\
x=\pm l_1/2,\ h_1<y<h_1+h_2:\ \sigma_{xx}=0,\ \sigma_{xy}=0\Leftrightarrow\sigma_{rr}=0,\sigma_{r\alpha}=0,\\
-l_1/2<x<l_1/2,\ y=h_1:\ \sigma_{yy}=0,\ \sigma_{xy}=0\Leftrightarrow\sigma_{rr}=0,\sigma_{r\alpha}=0,\\
-b<x<l_1/2+d,\ y=R+H:\ \sigma_{yy}=0,\ \sigma_{xy}=0\Leftrightarrow\sigma_{rr}=0,\sigma_{r\alpha}=0\\
l_1/2+d<x<l_1/2+d+l_2,\ y=R+H:\ \sigma_{yy}=-p,\sigma_{xy}=0\Leftrightarrow\\
\Leftrightarrow\sigma_{rr}=-p\sin^2\alpha_1,\sigma_{r\alpha}=-p\sin\alpha\cos\alpha,\\
l_1/2+d+l_3<x<l_1/2+d+l_2+l_3,\ y=R+H:\ \sigma_{yy}=-p,\sigma_{xy}=0\\
\Leftrightarrow\sigma_{rr}=-p\sin^2\alpha_1,\sigma_{r\alpha}=-p\sin\alpha\cos\alpha,\\
l_1/2+d+l_2<x<l_1/2+d+l_3,\ y=R+H:\sigma_{yy}=0,\sigma_{xy}=0\Leftrightarrow\sigma_{rr}
\end{gathered} \tag{2}$$

where u, v are the components of the displacement vector, σ_{aa}, σ_{rr}, σ_{ra} are the components of the stress tensor in the polar coordinate system, σ_{xx}, σ_{yy}, σ_{xy} are those in Cartesian coordinates. $D/(\lambda+2\mu)$ is a divergence of the displacement vector, K/μ is a rotor of the displacement vector, $\lambda=E\nu/((1+\nu(1-2\nu))$, $\mu=E/(2(1+\nu))$ are known constants, E – Young modulus, ν - Poisson coefficient. To obtain a numerical solution of the problem we use one of the methods of boundary elements, in particular, the method of fictitious loadings [1, 2].

Numerical procedure. To conduct a numerical procedure of solving the boundary value problem (1), (2) let us divide the boundaries B_1B_2, B_2B_3, B_3B_4, B_4B_5, B_5B_6, B_6B_7, B_7B_8, B_8B_9, B_9B_{10} into elements N_1, N_2, N_3, N_2, N_4, N_5, N_6, N_5, N_7. If we take into account the expressions of stresses written by means of boundary coefficients of the stress influence [2], and satisfy the boundary conditions (2), one can obtain the following system of $2N$ equations with $2N$ unknowns ($N=N_1+2N_2+N_3+N_4+2N_5+N_6+N_7$):

$$\begin{cases}
\left.\begin{aligned}
&\sum_{j=M_1+1}^{M_2}\left(A_{ss}^{ij}P_s^j+A_{sn}^{ij}P_n^j\right)+\sum_{j=M_3+1}^{M_4}\left(A_{ss}^{ij}P_s^j+A_{sn}^{ij}P_n^j\right)=0\\
&\sum_{j=M_1+1}^{M_2}\left(A_{ns}^{ij}P_s^j+A_{nn}^{ij}P_n^j\right)+\sum_{j=M_3+1}^{M_4}\left(A_{ns}^{ij}P_s^j+A_{nn}^{ij}P_n^j\right)=-p
\end{aligned}\right\},i=M_1+1,\ldots,M_2,M_3+1,\ldots,M_4,\\
\left.\begin{aligned}
&\sum_{j=1}^{M_1}\left(A_{ss}^{ij}P_s^j+A_{sn}^{ij}P_n^j\right)+\sum_{j=M_2+1}^{M_3}\left(A_{ss}^{ij}P_s^j+A_{sn}^{ij}P_n^j\right)+\sum_{j=M_4+1}^{M_4+N_7}\left(A_{ss}^{ij}P_s^j+A_{sn}^{ij}P_n^j\right)=0\\
&\sum_{j=1}^{M_1}\left(A_{ns}^{ij}P_s^j+A_{nn}^{ij}P_n^j\right)+\sum_{j=M_2+1}^{M_3}\left(A_{ns}^{ij}P_s^j+A_{nn}^{ij}P_n^j\right)+\sum_{j=M_4+1}^{M_4+N_7}\left(A_{ns}^{ij}P_s^j+A_{nn}^{ij}P_n^j\right)=0
\end{aligned}\right\},i=1,\ldots,M_1,\\
\qquad M_2+1,\ldots,M_3,M_4+1,\ldots,M_4+N_7,
\end{cases} \tag{3}$$

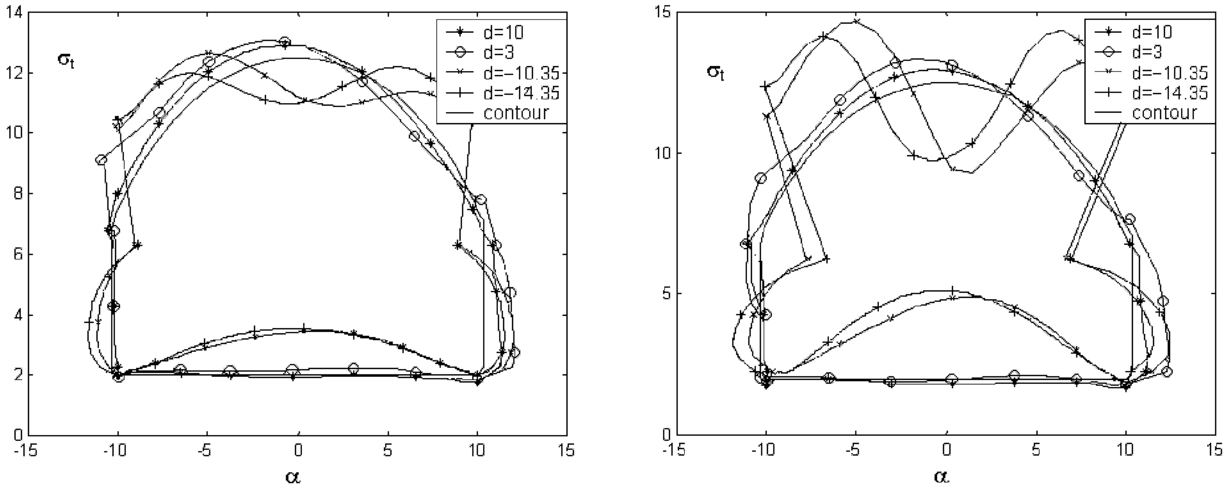
where $M_1=N_1+2N_2+N_3+N_4$, $M_2=M_1+N_5$, $M_3=M_2+N_6$, $M_4=M_3+N_5$; A^{ij}_{ss}, A^{ij}_{sn}, A^{ij}_{ns}, A^{ij}_{nn} are boundary coefficients of the influence of stresses for the problem under consideration. P^j_s and P^j_n are fictitious unknown values. They have no physical sense and are introduced only as a tool for solving the particular problem, but with their linear combination we can express regular tangent and normal stresses [1, 2] which are used to satisfy the boundary conditions.

After solving the system of equations (3) by an arbitrary method, displacements or stresses may be expressed by another linear combination of fictitious loadings P^j_s and P^j_n ($j=1, \ldots, N$) at any point of the body.

Numerical realization of boundary-value problem (1), (2) is made. Numerical values of tangent stress s_t are obtained on contour $B_1B_2B_3B_4$ (Fig.1) and the corresponding curves are constructed for the following data: $R=12.5$, $\nu=0.33$, $E=2.5\times10^3$, $\rho=-500$ ton/m^2, $h_1=2$ m, $h_2=5.015$ m, $l_1=20.7$ m, $l_2=2$ m, $l_3=6$ m, $H=5$, 10, 20, 40 m and $d=10$, 3, -10.35, -14.35 m. In Figs. 2 and 3 the diagrams of the distribution of stress s_t on the interior boundary contour are presented.

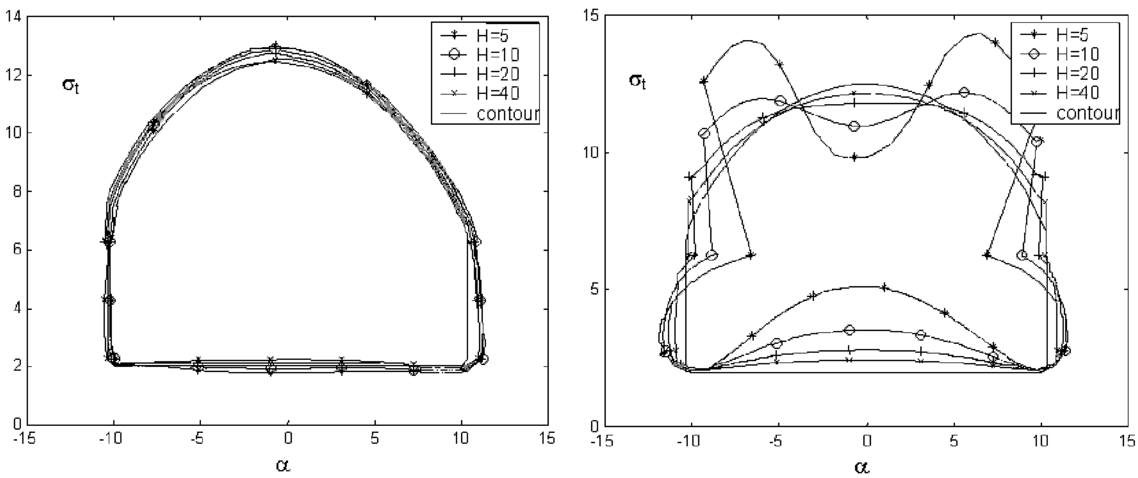
Fig. 2a) shows diagrams of the stress distribution on the interior contour for four values of d at H=10m. H is the depth of the location of tunnel and d is distance between overground constructions and tunnel wall (see Fig.1). As expected, the farther overground the constructions from the tunnel are, the less is the stress σ_t on the walls of the tunnel.

Fig. 2b) presents the diagrams of the stress distribution on the interior contour for four values of d at H=40 m. In the case when overground constructions are far from the wall of the tunnel the effect is the same as in the previous case. When constructions are quite near to the tunnel (located directly over it), then the stress σ_t increases much more than in the previous case.



a) b)

Fig. 2. Distribution of the stress σ_t on the hole outline when loading ρ=-500t/m^2 ; a) H=10m, b) H=40m.



a) b)

Fig. 3. Distribution of the stress σ_t on the hole outline, when loading ρ=-500t/m^2 ; a) d=10m, b) d=-14.35m.

In Fig. 3a) diagrams of the stress distribution on the interior contour are given for four values of *H,* when d=10 m. As seen from the obtained data, when overground constructions are far from the tunnel, then stress σ_t is small and varies little at different depths of the tunnel. Fig.3b) shows distribution of the stress σ_t on the interior contour for four values of *H* when d=14.35 m. In this case overground constructions are located directly over the tunnel. The obtained data show that the deeper the tunnel is located, the less the tangent stress σ_t is.

მათემატიკა

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