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Mechanics

On One Iteration Algorithm of Investigation of the Nonlinear Problem of Oscillation of a System of Constrained Particles under Strong Dynamic Load

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ABSTRACT. The solution of the problem mentioned in the title demands that the formation of a defining system of algebraic equations and its solution be repeated at each discrete time interval. The paper proposes a special iteration algorithm by which the formulated equation can be solved by a simple operation of multiplication of matrices and vectors. The conditions for the iteration process convergence are formulated. The proposed iteration algorithm can be considered as the description of the process of motion of a system of particles in a medium in which the position and motion of particles obey non-stationary geometrical conditions, in particular with time delay. *© 2009 Bull. Georg. Natl. Acad. Sci.*

Key words: *oscillation of a system of particles, iteration algorithm, convergence conditions, non-stationary geometrical conditions time delay.*

We consider a system of particles interconnected by constraints when the system is under the action of force or acts in kinematic conditions in which changes of the physical and mechanical characteristics of constraints, their damage or rupture may occur.

The algorithm proposed for the investigation of the above-mentioned problem was originally used successfully for the study of nonlinear oscillations of an arch dam under the action of a strong seismic wave that caused the opening of the construction joints in the dam [1].

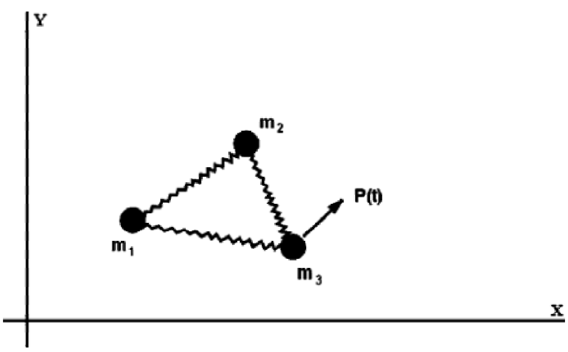

Fig. 1.

The application of the algorithm to the investigation of the dynamic behavior of the system of particles revealed a number of interesting phenomena which are discussed in [2, 3].

In the present paper, the question is posed whether it is possible to replace the stage of solution of a system of algebraic equations by a special iteration process.

For the clarity of the discussion, our proposition is illustrated by a simple example of the system of particles which contains three masses interconnected by constraints (by springs in our example) (Fig. 1).

The system of algebraic equations expressing the presence of constraints is shown below:

	$S_{12}=1$	$S_{13}=1$	$S_{23}=1$
δ_{12}	$\frac{\tau^2}{m_1}+\frac{\tau^2}{m_2}+\Delta_{12}$	$\frac{\tau^2}{m_1}\cos(13,12)$	$\frac{\tau^2}{m_2}\cos(23,12)$
δ_{13}	$\frac{\tau^2}{m_1}\cos(12,13)$	$\frac{\tau^2}{m_1}+\frac{\tau^2}{m_3}+\Delta_{13}$	$\frac{\tau^2}{m_3}\cos(23,13)$
δ_{23}	$\frac{\tau^2}{m_2}\cos(12,23)$	$\frac{\tau^2}{m_3}\cos(13,23)$	$\frac{\tau^2}{m_2}+\frac{\tau^2}{m_3}+\Delta_{23}$

=

δ^p
δ_{12}^p
δ_{13}^p
δ_{23}^p

(1)

In the system (1):

- τ is a time step (a real time interval),
- S_{ik} is the interaction force between the masses,
- δ_{ik} are the gaps between the springs and the masses,
- Δ_{ik} is the contraction or the extension of the spring (i,k) under the action of forces applied to its ends and equal to one,
- cos(kl,mn) is the cone between the directions (kl) and (mn).

The system (1) can be written in matrix form as follows:

$$D \cdot S = \delta^p.$$

Let us write the matrix D as the sum of two matrices

$$D = C + M =$$

=

Δ_{12}		
	Δ_{13}	
		Δ_{13}

+

$\frac{\tau^2}{m_1}+\frac{\tau^2}{m_2}$	$\frac{\tau^2}{m_1}\cos(13,12)$	$\frac{\tau^2}{m_2}\cos(23,12)$
$\frac{\tau^2}{m_1}\cos(12,13)$	$\frac{\tau^2}{m_1}+\frac{\tau^2}{m_3}$	$\frac{\tau^2}{m_3}\cos(23,13)$
$\frac{\tau^2}{m_2}\cos(12,23)$	$\frac{\tau^2}{m_3}\cos(13,23)$	$\frac{\tau^2}{m_2}+\frac{\tau^2}{m_3}$

(2)

To the system represented in the form (2) we can apply the iteration process [4]

$$S = C^{-1}(M \cdot S) + C^{-1}(\delta^p). \tag{3}$$

The iteration process (3) has a clear mechanical meaning, while from the computational standpoint it reduces to a simple operation of multiplication of matrices by vectors.

Special emphasis should be placed on the following important fact: nonlinear effects associated with possible changes of the physical and mechanical properties of the springs and their damage are taken on by a separate unit which is the matrix C. This makes it easy to take into account these nonlinear effects when considering the oscillation process of the system of masses.

Now let us turn our attention to the conditions of convergence of the iteration process described by the expression (3). As is known [4], the iteration process (3) converges if the matrices C, ($C + M$) and ($C – M$) are positive definite. Below it is shown how these conditions can be formulated in our case.

The first row and the first column of the matrix D are multiplied by $\left(\sqrt{\Delta_{13}}\cdot\sqrt{\Delta_{23}}\right)$, the second row and the second column by $\left(\sqrt{\Delta_{12}}\cdot\sqrt{\Delta_{23}}\right)$, the third row and the third column by $\left(\sqrt{\Delta_{12}}\cdot\sqrt{\Delta_{13}}\right)$. This operation and further elementary transformations are shown below.

$\left(\frac{\tau^2}{m_1}+\frac{\tau^2}{m_2}+\Delta_{12}\right)x$ $x\sqrt{\Delta_{13}}x\sqrt{\Delta_{23}}x\sqrt{\Delta_{13}}x\sqrt{\Delta_{23}}$	$\frac{\tau^2}{m_1}\cos(13,12)\ x$ $x\sqrt{\Delta_{13}}x\sqrt{\Delta_{23}}x\sqrt{\Delta_{12}}x\sqrt{\Delta_{23}}$	$\frac{\tau^2}{m_2}\cos(23,12)x$ $x\sqrt{\Delta_{13}}x\sqrt{\Delta_{23}}x\sqrt{\Delta_{12}}x\sqrt{\Delta_{13}}$
$\frac{\tau^2}{m_1}\cos(12,13)x$ $x\sqrt{\Delta_{13}}x\sqrt{\Delta_{23}}x\sqrt{\Delta_{12}}x\sqrt{\Delta_{23}}$	$\left(\frac{\tau^2}{m_1}+\frac{\tau^2}{m_2}+\Delta_{13}\right)x$ $x\sqrt{\Delta_{12}}x\sqrt{\Delta_{23}}x\sqrt{\Delta_{12}}x\sqrt{\Delta_{23}}$	$\frac{\tau^2}{m_3}\cos(23,13)x$ $x\sqrt{\Delta_{12}}x\sqrt{\Delta_{23}}x\sqrt{\Delta_{12}}x\sqrt{\Delta_{13}}$
$\frac{\tau^2}{m_2}\cos(12,23)x$ $x\sqrt{\Delta_{13}}x\sqrt{\Delta_{23}}x\sqrt{\Delta_{12}}x\sqrt{\Delta_{13}}$	$\frac{\tau^2}{m_3}\cos(13,23)x$ $x\sqrt{\Delta_{12}}x\sqrt{\Delta_{23}}x\sqrt{\Delta_{12}}x\sqrt{\Delta_{13}}$	$\left(\frac{\tau^2}{m_1}+\frac{\tau^2}{m_2}+\Delta_{23}\right)x$ $x\sqrt{\Delta_{12}}x\sqrt{\Delta_{13}}x\sqrt{\Delta_{12}}x\sqrt{\Delta_{13}}$

=

=

$\left(\frac{\tau^2}{m_1}+\frac{\tau^2}{m_2}\right)x\Delta_{13}x\Delta_{23}+$ $+\Delta_{12}x\Delta_{13}x\Delta_{23}$	$\frac{\tau^2}{m_1}\cos(13,12)x$ $x\Delta_{23}x\sqrt{\Delta_{12}}x\sqrt{\Delta_{13}}$	$\frac{\tau^2}{m_2}\cos(23,12)x$ $x\Delta_{13}x\sqrt{\Delta_{12}}x\sqrt{\Delta_{23}}$
$\frac{\tau^2}{m_1}\cos(12,13)x$ $x\Delta_{23}x\sqrt{\Delta_{12}}x\sqrt{\Delta_{13}}$	$\left(\frac{\tau^2}{m_1}+\frac{\tau^2}{m_3}\right)x\Delta_{12}x\Delta_{23}+$ $+\Delta_{12}x\Delta_{13}x\Delta_{23}$	$\frac{\tau^2}{m_3}\cos(23,13)x$ $x\Delta_{12}x\sqrt{\Delta_{13}}x\sqrt{\Delta_{23}}$
$\frac{\tau^2}{m_2}\cos(12,23)x$ $x\Delta_{13}x\sqrt{\Delta_{12}}x\sqrt{\Delta_{23}}$	$\frac{\tau^2}{m_3}\cos(13,23)x$ $x\Delta_{12}x\sqrt{\Delta_{13}}x\sqrt{\Delta_{23}}$	$\left(\frac{\tau^2}{m_2}+\frac{\tau^2}{m_3}\right)x\Delta_{12}x\Delta_{13}+$ $+\Delta_{12}x\Delta_{13}x\Delta_{23}$

=

=

1		
	1	
		1

$x(\Delta_{12}\Delta_{13}\Delta_{23})$ +

$+\tau^2 x$

$\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\Delta_{13}x\Delta_{23}$	$\frac{1}{m_1}\cos(13,12)x$ $x\Delta_{23}x\sqrt{\Delta_{12}}x\sqrt{\Delta_{13}}$	$\frac{1}{m_2}\cos(23,12)x$ $x\Delta_{13}x\sqrt{\Delta_{12}}x\sqrt{\Delta_{23}}$
$\frac{1}{m_1}\cos(12,13)x$ $x\Delta_{23}x\sqrt{\Delta_{12}}x\sqrt{\Delta_{13}}$	$\left(\frac{1}{m_1}+\frac{1}{m_3}\right)x\Delta_{12}x\Delta_{23}$	$\frac{1}{m_3}\cos(23,13)x$ $x\Delta_{12}x\sqrt{\Delta_{13}}x\sqrt{\Delta_{23}}$
$\frac{1}{m_2}\cos(12,23)x$ $x\Delta_{13}x\sqrt{\Delta_{12}}x\sqrt{\Delta_{23}}$	$\frac{1}{m_3}\cos(13,23)x$ $\Delta_{12}x\sqrt{\Delta_{13}}x\sqrt{\Delta_{23}}$	$\left(\frac{1}{m_2}+\frac{1}{m_3}\right)x\Delta_{12}x\Delta_{13}$

Thus the initial matrix D is transformed in the form $\overline{D}=I+\varepsilon\,\overline{M}$,

where $\varepsilon=\dfrac{\tau^2}{\Delta_{12}\cdot\Delta_{13}\cdot\Delta_{23}}$.

As is known [5], by an appropriate choice of a real number ε we can always obtain the positive definiteness of the matrix $\overline{D}$.

Thus we have shown that the iteration process described by the expression (3) converges provided that the small time interval t is appropriately chosen.

The proposed iteration algorithm (3) can be considered as the description of the process of motion of a system of particles in the medium in which the position and motion of particles obey non-stationary geometrical conditions, in particular with time delay.

მექანიკა

ძლიერი დინამიკური ზემოქმედებისას არათავისუფალი მატერიალური წერტილების სისტემის მოძრაობის არაწრფივი ამოცანის შესწავლის ერთი იტერაციული ალგორითმის შესახებ

გ. გაბრიჩიძე

აკადემიის წევრი, კ. ზავრიევის სამშენებლო მექანიკის და სეისმომედეგობის ინსტიტუტი, თბილისი

სათაურში აღნიშნული ამოცანის ამოხსნის რიცხვითი ალგორითმი მოითხოვს დროის ყოველ დისკრეტულ ბიჯზე ალგებრულ განტოლებათა სისტემის ხელახლა ფორმირებასა და ამოხსნას, რაც ძალზე ართულებს ამოცანის ამოხსნის რიცხვითი რეალიზაციის პროცესს. ნაშრომში შემოთავაზებულია სპეციალური ალგორითმი, რომელიც ნაცვლად მატრიცების ფორმირებისა და შებრუნებისა, შეიცავს შედარებით იოლ ოპერაციებს მატრიცებისა და ვექტორების გადამრავლებისას. ნაშრომში ჩამოყალიბებულია იტერაციული პროცესის კრებადობის პირობები. შემოთავაზებული იტერაციული ალგორითმი შეიძლება განვიხილოთ როგორც აღწერა მატერიალური წერტილების სისტემის მოძრაობისა გარემოში, რომელიც მატერიალური წერტილების მდებარეობასა და მოძრაობას ადებს გეომეტრიულ პირობებს არასტაციონარული ხასიათისას, კერძოდ — პირობები ჩნდებიან გარკვეული τ დროის დაგვიანებით.

REFERENCES

1. *G. Gabrichidze* (1991), Doctoral Thesis. Tbilisi (in Russian).
2. *G. Gabrichidze* (2008), Bull. Georg. Natl. Acad. Sci. New Series, vol. II, no.1: 54-56.
3. *G. Gabrichidze, D. Giorgadze, K. Chkhikvadze, N. Chlaidze* (2008), In: Materials of the First International Conference on Seismic Safety Problems of Caucasus Region Population, Cities and Settlements. September 8-11, 2008, Tbilisi, Georgia, 104-107.
4. *D.K. Fadeev, V.N. Fadeeva* (1963), Vychislitel'nye metody lineinoi algebry. M.-L. (in Russian).
5. *P. Lancaster* (1978), Teoriya matrits. M. (in Russian).

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