Magnetic Field Dependence of $A \rightarrow B$ Phase Transition Temperature in Superfluid $^3$He

Giorgi Baramidze*, Giorgi Kharadze**

* E. Andronikashvili Institute of Physics, Tbilisi  
** Academy Member, E. Andronikashvili Institute of Physics, Tbilisi

ABSTRACT. The magnetic field dependence of the $A \rightarrow B$ transition temperature $T_{AB}$ in superfluid $^3$He is considered in order to take into account the linear-in-field contribution. In the high field region, where the quadratic-in-field contribution prevails, the well-known answer is restored. On the other hand, it is pointed out that the Fermi liquid effects shift the observability of the linear-in-field region to rather low magnetic fields.

Key words: superfluidity of liquid $^3$He, magnetic field.

Soon after the discovery of the superfluidity of liquid $^3$He it was established that near the critical temperature $T_c(P)$ the phase diagram of superfluid state undergoes a profound modification under the action of even small external magnetic fields [1,2,3]. This modification shows up in the elimination of the direct normal to $B$ phase transition over the entire phase diagram in favour of an anisotropic $A$ phase.

The $A \rightarrow B$ transition temperature $T_{AB} < T_c$ can be established by equating the free energies $F_A(H)$ and $F_B(H)$. Near the zero-field transition temperature $T_c(P)$ the free energy can be found by minimizing the Ginzburg-Landau functional

$$F_S = 3a_{\mu\nu} < \Delta_\mu \Delta_\nu > + F_S^{(4)}(\tilde{\Delta}), \quad (1)$$

where the fourth order contribution in the order-parameter $\tilde{\Delta}(\tilde{k})$ reads as

$$F_S^{(4)} = \frac{9}{4} \left[ \beta_1 < \Delta_\mu \Delta_\mu > < \Delta_\nu^* \Delta_\nu > + \beta_2 (|\tilde{\Delta}|^2)^2 + \beta_3 < \Delta_\mu \Delta_\nu > < \Delta_\mu^* \Delta_\nu > + \beta_4 < \Delta_\mu \Delta_\nu^* > < \Delta_\mu^* \Delta_\nu > + \beta_5 < \Delta_\mu \Delta_\nu^* > < \Delta_\mu^* \Delta_\nu^* > \right]. \quad (2)$$

In Eqs. (1) and (2) the spin vector $\tilde{\Delta}$ is defined according to the general expression for the order-parameter $A_\mu$ of the spin triplet $P$-wave Cooper condensate: $\Delta_\mu(\tilde{k}) = A_\mu \hat{k}$, and the angular brackets $<...>$ stand for an average over the momentum direction $\hat{k}$ on the Fermi surface. In the presence of a magnetic field $\tilde{H}$ the tensor coefficient $a_{\mu\nu}$ of the second order term in Eq.(1) reads as:

$$a_{\mu\nu} = a_0 \delta_{\mu\nu} + i g_1 e_{\mu\nu\lambda} H_\lambda + g_2 H_\mu H_\nu, \quad (3)$$

where:

$$\alpha = \frac{1}{3} N_F \ln \left( \frac{T}{T_C} \right), \quad (4a)$$

$$g_1 H = - \frac{1}{3} N_F \eta h, \quad (4b)$$

$$\eta = \frac{1}{3} \kappa h^2, \quad (4c)$$

with $h = h\gamma H/(2k_BT_C) = H/H_0$. The dimensionless coefficients $\eta$ and $\kappa$ could be considered as phenomeno-
logical quantities although their values can be estimated according to microscopic calculations. In the weak-coupling approximation [4]

$$ \eta_{nc} = \frac{N_F^r}{N_F} k_B T_C \ln \left( \frac{2 \gamma_F \hbar \omega_C}{\pi k_B T_C} \right), $$

where $N_F^r$ stands for the derivative of the quasiparticle DOS with respect to the energy at the Fermi level. Detailed calculations which take into account the linear-in-field corrections to the Fermi liquid parameters are performed in Ref.[5].

As to the $\kappa$ coefficient, it stems from the free energy part

$$ \delta F_{H}^{(2)} = \frac{1}{2} \delta \chi_s H^2, \quad \delta \chi_s = \chi_s - \chi_N, $$

where $\chi_s(\chi_N)$ stands for the magnetic susceptibility of the superfluid (normal) phase. For the $B$ phase near $T_C$, \( \delta F_{H}^{(2)} = g_{2} H^2 \Delta^2 \) where $g_{2} H^2$ is given according to Eq. (4c) with

$$ \kappa = \frac{7 \gamma(3)}{4 \pi^2} \frac{1}{(1 + F_0^a)^2}. $$

Here the Fermi liquid parameter $F_0^a \equiv -3/4$ and is weakly pressure dependent.

As is well known, the linear-in-field term (4b) is the origin of a tiny splitting of the $A$ phase (due to a small asymmetry of the density of quasiparticle states at the Fermi level). The quadratic-in-field contribution (4c) gives rise to the magnetic field suppression of the $\Delta^\uparrow \downarrow$ component of the energy gap of the $B$ phase.

Based on the argument that the term (4b) is rather small, in the majority of considerations of $T_{AB}=T_{AB}(H)$ this term is usually discarded, and in a standard way one starts from the following expressions for the equilibrium free energies of the $A$ and $B$ phases:

$$ F_A = -\frac{1}{4 \beta_{245}} \alpha^2, \quad \frac{1}{2(3 \beta_{12} + \beta_{345})} \times \left[ \frac{3}{2} \alpha^2 + g_{2} H^2 \alpha + \frac{2 \beta_{12} + \beta_{345}}{2 \beta_{345}} (g_{2} H^2)^2 \right]. $$

Since here the linear-in-field contribution (proportional to $g_{2}$) is dropped, the action of the magnetic field appears only in $F_C$.

Equating (8a) and (8b), it is found that $T_{AB}(H)$ is to be obtained from the equation:

$$ \varphi = a^2 + a \alpha + b = 0 $$

with

$$ a = 2 P_1 g_{2} H^2, $$

$$ b = P_1^2 (1 - q_1) (g_{2} H^2)^2, $$

where the coefficients $P_1$ and $q_1$ are defined in the Appendix. From Eq. (9) follows the answer for

$$ \tau_{AB} = 1 - T_{AB} / T_C : $$

$$ \tau_{AB} = P_1 (1 + \sqrt{q_1}) \kappa h^2 $$

which reproduces a well-known result obtained in Ref.[6].

Now we turn to the role of the linear-in-field contribution to $T_{AB}(H)$. This question was first posed in Ref. [7]. The starting point is the construction of the expressions for $F_{A}(H)$ and $F_{B}(H)$, the contribution (4b) being taken into account. In a standard way it is established that

$$ F_{A} = -\frac{1}{4 \beta_{245}} \left[ \alpha^2 - \frac{\beta_{345}}{\beta_5} g_{2} H^2 \right], $$

$$ F_{B} = -\frac{1}{2(3 \beta_{12} + \beta_{345})} \times \left[ \frac{3}{2} \alpha^2 + g_{2} H^2 \alpha + \frac{2 \beta_{12} + \beta_{345}}{2 \beta_{345}} (g_{2} H^2)^2 \right] \times \left[ \frac{3}{2} \alpha^2 + g_{2} H^2 \alpha + \frac{2 \beta_{12} + \beta_{345}}{2 \beta_{345}} (g_{2} H^2)^2 \right]. $$

Comparison of $F_{A}(H)$ and $F_{B}(H)$ gives an equation for $T_{AB}(H)$

$$ \varphi \left( \frac{T_{AB}}{T_C} \right) = a^4 + a \alpha^3 + b \alpha^2 + c \alpha + d = 0, $$

where now

$$ a = 2 P_1 g_{2} H^2, $$

$$ b = P_1^2 (1 - q_1) (g_{2} H^2)^2, $$

$$ c = -P_1^2 P_3 g_{2}^2 H^4, $$

$$ d = -P_1^2 P_5 g_{2} H^4. $$

The definition of all coefficients $P_a$ is given in the Appendix.

In Ref.[7] the second and third lines in Eq.(13), containing terms on the order $H^4$, were neglected. This has the following influence on the answer for $T_{AB}(H)$: i) neglecting of the contribution proportional to $(g_{2} H^2)^2$
Magnetic Field Dependence of $A \rightarrow B$ Phase Transition Temperature in Superfluid $^3$He

makes it impossible to reproduce the correct answer given by Eq. (11) (the term $\sqrt{q_1}$ will be lost), ii) neglecting of the contribution collected in the square brackets of the third line of Eq.(13) changes the answer for the linear-in-field contribution to $T_{ab}(H)$. To avoid these drawbacks we address Eq.(14) and, as a first step, perform the variable transformation $\alpha \rightarrow x - \frac{1}{4} a$, after which an equation for $x(T_{ab} / T_C)$ is obtained:

$$\phi\left(\frac{T_{ab}}{T_C}\right) = x^4 + Ax^2 + Bx + C = 0 \quad (16)$$

with the coefficients

$$A = -\left[ P_1 P_2 P_3 (g_1 H)^2 + \frac{1}{2} P_1^2 (1 + 2q_1)(g_2 H^2)^2 \right],$$

$$B = q_1 P_2 (g_2 H^2)^2 + q_2 P_1 P_3 g_2 H^4,$$

$$C = -P_1 P_2^2 P_3 (g_1 H)^4 + \frac{1}{16} P_1 (1 - 4q_1)(g_2 H^2)^4 +$$

$$+ \frac{1}{4} P_3^2 P_2 (P_1 - 2q_2)(g_2 H^2)^2.$$  (17)

It is to be noted that as a result of the variable transformation used, in Eq.(16) the cubic term is absent. At the same time the coefficient of the linear term $B$ is zero in the weak-coupling approximation.

In order to solve Eq.(16) we use the decomposition $x = x_0 - \sqrt{q_1} P_1 g_2 H^2$ which, generates the decomposition $\phi = \phi_0 + \delta \phi$ with $\phi_0$ being the solution of the equation

$$\phi_0 = x_0^4 + A_0 x_0^2 + C_0 = 0 , \quad (18)$$

where $A_0$ and $C_0$ are the coefficients $A$ and $C$ taken at $q_1 = q_2 = 0$.

From Eq. (18) it is found that

$$x_0 \left(\frac{T_{ab}}{T_C}\right) = -\sqrt{P_0^2 (g_1 H)^2 + \frac{1}{4} P_1^2 (g_2 H^2)^2}, \quad (19)$$

where

$$P_0 = \frac{1}{2} P_1 P_2 P_3 \left[ \frac{1 + 4 + \frac{2 P_2}{P_1^2}}{P_1 P_3} \right]. \quad (20)$$

Direct inspection shows that $\delta \phi$ is the sum of terms proportional to the powers of $q$ and the minimal power in $H$ contained in $\delta \phi$ is $H^5$. For this reason we can use an approximation with $\delta \phi$ disregarded, and as a result it is found that

$$f(h) = \frac{\delta T_{ab}(h)}{h} = P_0 \eta \left[ \frac{\sqrt{h}}{h_n} - \frac{h}{h_n} \right], \quad (25)$$

where the scaling value $h_n = 2(P_0 / P_1)(\eta / \kappa)$. In the Figure $f(h)$ is plotted for the pressure $H_0 = 2(P_0 / P_1)(\eta / \kappa)$.
It is evident that the Fermi liquid effect shifts the linear-in-field contribution to rather small values of $H << H_0 = h v \approx 420 \ G$.

Appendix

The coefficients $P_\alpha (\alpha = 1, 2, 3, 4, 5)$ and $q_\alpha (\alpha = 1, 2)$, introduced in the main text, are defined as follows:

$$P_1 = \frac{\beta_{35} + \beta_{34}}{2 \beta_{34} - 3 \beta_3}, \quad P_2 = \frac{2 \beta_{12} + \beta_{34}}{\beta_4 - 3 \beta_4 - \beta_{35}},$$

$$P_3 = \frac{\beta_{35} - 3 \beta_4 - \beta_3}{\beta_3 - \beta_3}, \quad P_4 = \frac{-2 \beta_3}{\beta_{34}},$$

$$P_5 = \frac{-\beta_3}{2 (2 \beta_4 - 3 \beta_4 - \beta_{35})}.$$

The coefficients $q_1$ and $q_2$ contain only the strong-coupling corrections $\delta \beta_1$ defined according to the decomposition

$$\beta_1 = -\beta_0 + \delta \beta_1, \quad \beta_2 = 2 \beta_0 + \delta \beta_2, \quad \beta_3 = 2 \beta_0 + \delta \beta_3,$$

$$\beta_4 = 2 \beta_0 + \delta \beta_4, \quad \beta_5 = -2 \beta_0 + \delta \beta_5,$$

$$\beta_0 = \frac{7 \zeta(3)}{240 (\pi k_B T_C)^2}.$$

Acknowledgements

The authors highly appreciate the valuable discussions with the late Dr. Shota Nikolaishvili.

REFERENCES


Received August, 2009