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Physics

Magnetic Field Dependence of $A \rightarrow B$ Phase Transition Temperature in Superfluid ^{3}He

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ABSTRACT. The magnetic field dependence of the $A\rightarrow B$ transition temperature T_{AB} in superfluid ^3He is considered in order to take into account the linear-in-field contribution. In the high field region, where the quadratic-in-field contribution prevails, the well-known answer is restored. On the other hand, it is pointed out that the Fermi liquid effects shift the observability of the linear-in-field region to rather low magnetic fields.

Key words: *superfluidity of liquid ^{3}He, magnetic field.*

Soon after the discovery of the superfluidity of liquid ^3He it was established that near the critical temperature $T_C(H)$ the phase diagram of superfluid state undergoes a profound modification under the action of even small external magnetic fields [1,2,3]. This modification shows up in the elimination of the direct normal to *B* phase transition over the entire phase diagram in favour of an anisotropic *A* phase.

The $A\rightarrow B$ transition temperature $T_{AB}<T_C$ can be established by equating the free energies $F_A(H)$ and $F_B(H)$. Near the zero-field transition temperature $T_C(P)$ the free energy can be found by minimizing the Ginzburg-Landau functional

$$F_S = 3\alpha_{\mu\nu} < \Delta_\mu \Delta_\nu^* > + F_S^{(4)}(\vec{\Delta}), \qquad (1)$$

where the fourth order contribution in the order-parameter $\vec{\Delta}(\hat{k})$ reads as

$$F_S^{(4)} = 9\ \{\beta_1 < \Delta_\mu \Delta_\mu >< \Delta_\nu^* \Delta_\nu^* > + \beta_2 < \left|\vec{\Delta}\right|^2 >^2 + \\ + \beta_3 < \Delta_\mu \Delta_\nu >< \Delta_\mu^* \Delta_\nu^* > + \beta_4 < \Delta_\mu \Delta_\nu^* >< \Delta_\mu^* \Delta_\nu > + \\ + \beta_5 < \Delta_\mu \Delta_\nu^* >< \Delta_\mu \Delta_\nu^* >\}. \qquad (2)$$

In Eqs. (1) and (2) the spin vector $\vec{\Delta}$ is defined according to the general expression for the order-parameter $A_{\mu i}$ of the spin triplet *P*-wave Cooper condensate: $\Delta_\mu(\hat{k}) = A_{\mu i}\hat{k}_i$, and the angular brackets <...> stand for an average over the momentum direction $\hat{k}$ on the Fermi surface. In the presence of a magnetic field $\vec{H}$ the tensor coefficient $\alpha_{\mu\nu}$ of the second order term in Eq.(1) reads as:

$$\alpha_{\mu\nu} = \alpha\delta_{\mu\nu} + ig_1\varepsilon_{\mu\nu\lambda}H_\lambda + g_2 H_\mu H_\nu, \qquad (3)$$

where:

$$\alpha = \frac{1}{3}N_F \ln\left(\frac{T}{T_C}\right), \qquad (4a)$$

$$g_1 H = -\frac{1}{3}N_F \eta h, \qquad (4b)$$

$$g_2 H^2 = \frac{1}{3}N_F \kappa h^2, \qquad (4c)$$

with $h = \hbar\gamma H/(2k_B T_C) = H/H_0$. The dimensionless coefficients η and κ could be considered as phenomenological quantities although their values can be estimated according to microscopic calculations. In the weak-coupling approximation [4]

$$\eta_{wc} = \frac{N_F^{'}}{N_F} k_B T_C \ \ln\left(\frac{2\gamma_E \ \hbar\omega_C}{\pi \ k_B T_C} \right), \qquad (5)$$

where $N_F^{'}$ stands for the derivative of the quasiparticle DOS with respect to the energy at the Fermi level. Detailed calculations which take into account the linear-in-field corrections to the Fermi liquid parameters are performed in Ref.[5].

As to the κ coefficient, it stems from the free energy part

$$\delta F_H^{(2)} = \frac{1}{2}\delta\chi_s H^2, \ \ \delta\chi_s = \chi_s - \chi_N , \qquad (6)$$

where $\chi_s(\chi_N)$ stands for the magnetic susceptibility of the superfluid (normal) phase. For the *B* phase near T_C, $\delta F_H^{(2)} = g_2 H^2 \Delta^2$ where $g_2 H^2$ is given according to Eq. (4c) with

$$\kappa = \frac{7\varsigma(3)}{4\pi^2} \ \frac{1}{(1+F_0^a)^2}. \qquad (7)$$

Here the Fermi liquid parameter $F_0^a \cong -3/4$ and is weakly pressure dependent.

As is well known, the linear-in-field term (4b) is the origin of a tiny splitting of the *A* phase (due to a small asymmetry of the density of quasiparticle states at the Fermi level). The quadratic-in-field contribution (4c) gives rise to the magnetic field suppression of the $\Delta_{\uparrow\downarrow}$ component of the energy gap of the *B* phase.

Based on the argument that the term (4b) is rather small, in the majority of considerations of $T_{AB}=T_{AB}(H)$ this term is usually discarded, and in a standard way one starts from the following expressions for the equilibrium free energies of the *A* and *B* phases:

$$F_A = -\frac{1}{4\beta_{245}}\alpha^2, \qquad (8a)$$

$$F_B = -\frac{1}{2(3\beta_{12}+\beta_{345})} \times \left[\frac{3}{2}\alpha^2 + g_2 H^2 \alpha + \frac{2\beta_{12}+\beta_{345}}{2\beta_{345}}(g_2H^2)^2\right]. \qquad (8b)$$

Since here the linear-in-field contribution (proportional to g_1) is dropped, the action of the magnetic field appears only in F_B.

Equating (8a) and (8b), it is found that $T_{AB}(H)$ is to be obtained from the equation:

$$\varphi = \alpha^2 + a\alpha + b = 0 \qquad (9)$$

with

$$a = 2P_1 g_2 H^2, \qquad (10)$$
$$b = P_1^2(1-q_1)(g_2H^2)^2,$$

where the coefficients P_1 and q_1 are defined in the Appendix. From Eq. (9) follows the answer for $\tau_{AB} = 1 - T_{AB}/T_C$:

$$\tau_{AB} = P_1(1+\sqrt{q_1}\ )\kappa h^2 \qquad (11)$$

which reproduces a well-known result obtained in Ref.[6].

Now we turn to the role of the linear-in-field contribution to $T_{AB}(H)$. This question was first posed in Ref. [7]. The starting point is the construction of the expressions for $F_{A2}(H)$ and $F_B(H)$, the contribution (4b) being taken into account. In a standard way it is established that

$$F_{A2} = -\frac{1}{4\beta_{245}}\left[\alpha^2 - \frac{\beta_{245}}{\beta_5} g_1^2 H^2\right], \qquad (12)$$

$$F_B = -\frac{1}{2(3\beta_{12}+\beta_{345})} \times \left\{\frac{3}{2}\alpha^2 + g_2H^2\alpha + \frac{3\beta_{12}+\beta_{345}}{\beta_4-(3\beta_1+\beta_{35})} g_1^2H^2 + \frac{2\beta_{12}+\beta_{345}}{2\beta_{345}}(g_2H^2)^2 + \frac{(3\beta_{12}+\beta_{345})^2}{(3\beta_1+\beta_{35}-\beta_4)^2} \times \left[\frac{\beta_1}{\beta_{345}}\frac{g_1^2 g_2 H^4}{\alpha} + \frac{\beta_1(3\beta_{12}+\beta_{345})}{4(3\beta_1+\beta_{35}-\beta_4)^2}\frac{g_1^4H^4}{\alpha^2}\right]\right\}. \qquad (13)$$

Comparison of $F_{A2}(H)$ and $F_B(H)$ gives an equation for $T_{AB}(H)$

$$\varphi\left(\frac{T_{AB}}{T_C}\right) = \alpha^4 + a\alpha^3 + b\alpha^2 + c\alpha + d = 0, \qquad (14)$$

where now

$$a = 2P_1 g_2 H^2,$$
$$b = -P_1P_2P_3 g_1^2 H^2 + P_1^2(1-q_1)(g_2H^2)^2, \qquad (15)$$
$$c = -P_1P_2^2P_4 g_1^2 g_2 H^4,$$
$$d = -P_1P_2^3P_5(g_1H)^4 .$$

The definition of all coefficients P_a is given in the Appendix.

In Ref.[7] the second and third lines in Eq.(13), containing terms on the order H^4, were neglected. This has the following influence on the answer for $T_{AB}(H)$: i) neglecting of the contribution proportional to $(g_2H^2)^2$

makes it impossible to reproduce the correct answer given by Eq. (11) (the term $\sqrt{q_1}$ will be lost), ii) neglecting of the contribution collected in the square brackets of the third line of Eq.(13) changes the answer for the linear-in-field contribution to $T_{AB}(H)$. To avoid these drawbacks we address Eq.(14) and, as a first step, perform the variable transformation $\alpha \rightarrow x - \frac{1}{4}a$, after which an equation for $x(T_{AB}/T_C)$ is obtained:

$$\varphi\left(\frac{T_{AB}}{T_C}\right) = x^4 + Ax^2 + Bx + C = 0 \qquad (16)$$

with the coefficients

$$A = -\left[P_1P_2P_3(g_1H)^2 + \frac{1}{2}P_1^2(1+2q_1)(g_2H^2)^2\right],$$

$$B = q_1P_1^3(g_2H^2)^3 + q_2P_1P_2g_1^2g_2H^4, \qquad (17)$$

$$C = -P_1P_2^3P_5(g_1H)^4 + \frac{1}{16}P_1^4(1-4q_1)(g_2H^2)^4 + \frac{1}{4}P_1^2P_2(P_1P_3 - 2q_2)(g_1g_2H^3)^2.$$

It is to be noted that as a result of the variable transformation used, in Eq.(16) the cubic term is absent. At the same time the coefficient of the linear term B is zero in the weak-coupling approximation.

In order to solve Eq.(16) we use the decomposition $x = x_0 - \sqrt{q_1}P_1g_2H^2$ which, generates the decomposition $\varphi = \varphi_0 + \delta\varphi$ with φ_0 being the solution of the equation

$$\varphi_0 = x_0^4 + A_0x_0^2 + C_0 = 0, \qquad (18)$$

where A_0 and C_0 are the coefficients A and C taken at $q_1 = q_2 = 0$

From Eq. (18) it is found that

$$x_0\left(\frac{T_{AB}}{T_C}\right) = -\sqrt{P_0^2(g_1H)^2 + \frac{1}{4}P_1^2(g_2H^2)^2}, \qquad (19)$$

where

$$P_0^2 = \frac{1}{2}P_1P_2P_3\left[1 + \sqrt{1 + 4\frac{P_2P_5}{P_1P_3^2}}\right]. \qquad (20)$$

Direct inspection shows that $\delta\varphi$ is the sum of terms proportional to the powers of q and the minimal power in H contained in $\delta\varphi$ is H^5. For this reason we can use an approximation with $\delta\varphi$ disregarded, and as a result it is found that

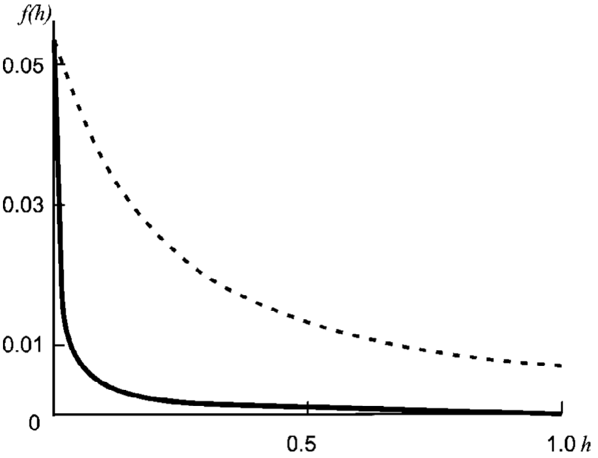

Fig. The bold curve corresponds to the Landau parameter $F_0^a = -3/4$ and the dashed curve is constructed at $F_0^a = 0$.

$$\alpha\left(\frac{T_{AB}}{T_C}\right) = x - \frac{1}{4}a = x_0 - \sqrt{q_1}P_1g_2H^2 - \frac{1}{2}P_1g_2H^2 =$$
$$= -P_1\left(\frac{1}{2} + \sqrt{q_1}\right)g_2H^2 - \sqrt{P_0^2(g_1H)^2 + \frac{1}{4}P_1^2(g_2H^2)^2}. \qquad (21)$$

Using this result, we finally have the following simple answer for τ_{AB}:

$$\tau_{AB} = P_1\left(\frac{1}{2} + \sqrt{q_1}\right)\kappa h^2 + \sqrt{P_0^2(\eta h)^2 + \frac{1}{4}P_1^2(\kappa h^2)^2}. \qquad (22)$$

By introducing a characteristic magnetic field $H_* = 2(P_0/P_1)(\eta/\kappa)H_0$ the asymptotic regions where the quadratic-in-field $(H >> H_*)$ and the linear-in-field $(H << H_*)$ contributions to T_{AB} prevail are found:

$$\tau_{AB} = \begin{cases} P_1(1+\sqrt{q_1})\kappa h^2, & H >> H_* \\ P_0\eta h, & H << H_* \end{cases} \qquad (23)$$

For $H >> H_*$ the well-known result [6,8] is reproduced.

In order to isolate the role of the linear-in-field contribution to T_{AB} it is convenient to consider

$$\delta\tau_{AB}(h) = \tau_{AB}(h) - (1+\sqrt{q_1})P_1\kappa h^2, \qquad (24)$$

and construct graphically

$$f(h) = \frac{\delta\tau_{AB}(h)}{h} = P_0\eta\left(\sqrt{1 + \left(\frac{h}{h_*}\right)^2} - \frac{h}{h_*}\right), \qquad (25)$$

where the scaling value $h_* = 2(P_0/P_1)(\eta/\kappa)$.

In the Figure $f(h)$ is plotted for the pressure

$P = 10\ bar\ (P_0\eta \approx 5,4\cdot 10^{-2}, h_* \approx 1,6\cdot 10^{-2})$. It is evident that the Fermi liquid effect shifts the linear-in field contribution to rather small values of $H << H_* = h_* H_0 \approx 420\ G$.

Appendix

The coefficients $P_a (a = 1,2,3,4,5)$ and $q_a (a = 1,2)$, introduced in the main text, are defined as follows:

$$P_1 = \frac{\beta_{245}}{2\beta_{345} - 3\beta_{13}}, \quad P_2 = \frac{3\beta_{12} + \beta_{345}}{\beta_4 - 3\beta_1 - \beta_{35}},$$

$$P_3 = \frac{\beta_{45} - 3\beta_1 - \beta_3}{-\beta_5}, \quad P_4 = \frac{-2\beta_1}{\beta_{345}},$$

$$P_5 = \frac{-\beta_1}{2(\beta_4 - 3\beta_1 - \beta_{35})}.$$

$$q_1 = \frac{(3\beta_{12} + \beta_{345})(2\beta_{13} - \beta_{345})}{\beta_{245}\beta_{345}}, \quad q_2 = P_1 P_3 - P_2 P_4,$$

where $\beta_{ij...} = \beta_i + \beta_j + ...$

The coefficients q_1 and q_2 contain only the strong-coupling corrections $\delta\beta_i$ defined according to the decomposition

$$\beta_1 = -\beta_0 + \delta\beta_1, \quad \beta_2 = 2\beta_0 + \delta\beta_2, \quad \beta_3 = 2\beta_0 + \delta\beta_3,$$

$$\beta_4 = 2\beta_0 + \delta\beta_4, \quad \beta_5 = -2\beta_0 + \delta\beta_5,$$

$$\beta_0 = \frac{7\varsigma(3)}{240} \frac{N_F}{(\pi k_B T_C)^2}.$$

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ფიზიკა

ზედენად ^{3}He-ში $A \rightarrow B$ ფაზური გადასვლის ტემპერატურის დამოკიდებულება მაგნიტურ ველზე

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განხილულია ზედენად ^{3}He-ში $A \rightarrow B$ ფაზური გადასვლის T_{AB} ტემპერატურის მაგნიტურ ველზე დამოკიდებულების თავისებურებანი. ნაჩვენებია, რომ ხსენებული დამოკიდებულების წრფივი მდგენელის ექსპერიმენტული დამზერა შესაძლებელია მხოლოდ საკმარისად მცირე მაგნიტური ველების პირობებში.

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