Physics

Temperature Dependence of Hydrogen Mobility in Ordering Rare-Earth Dihydrides

Ioseb Ratishvilii*, Natela Namoradze**

* E. Andronikashvili Institute of Physics, Tbilisi ** Institute of Cybernetics, Tbilisi

(Presented by Academy Member T. Sanadze)

ABSTRACT. The rare-earth ordering dihydrides are considered. The influence of ordering processes on the mobility of interstitial H-atoms is investigated. It is shown that those H-H interactions which induce formation of ordered superstructures in the hydrogen sublattice, provide as well the increment of the dwell time of the moving particles. The latter effect can be interpreted as a "weighting" of mobile H-atoms. The influence of the "particle-weighting effect" on the temperature dependence of the dipole-dipole interaction induced nuclear spin-lattice relaxation time is examined. It is shown that although the corresponding changes in the $T_{Id}(T)$ curve are remarkable, their influence on the total $T_I(T)$ dependence, determined as well by interactions of nuclear spins with conduction electrons, is negligibly small. © 2009 Bull. Georg. Natl. Acad. Sci.

Key words: metal hydrides, ordering processes, hydrogen "weighting"; nuclear spin-lattice relaxation.

The model and general relations. It is well known that in a number of M-H compounds (metal-hydrogen interstitial alloys) the temperature lowering provides formation of ordered superstructures in the subsystem of hydrogen atoms (H-atoms). Inelastic neutron scattering experiments had established that in such ordering hydrides the frequencies of local vibrations of interstitial H-atoms significantly decrease below the disorderorder transition temperature [1]. The local vibration frequencies v(loc) are directly related to the time-parameter τ_0 describing the hydrogen "from site – to site" jumping processes. The same time-parameter τ_0 turns out to be an essential part of expressions describing the spin-lattice relaxation processes caused by the dipoledipole interactions of the spins of hydrogen and metal nuclei [2, 3]. In the latter case τ_0 is usually assumed to be constant in the whole temperature range under consideration. There arises therefore a natural question what type of modifications can be expected in the temperature dependence of the spin-lattice relaxation time $T_I(T)$ if we take into account the temperature variations of τ_0 parameter. We intend to discuss this problem in the given investigation consisting of two parts. In the first one we consider the interrelations between the hydrogen ordering processes and the temperature dependence of the time parameter τ_0 , while in the second part we shall estimate the degree of changes in the spin-lattice relaxation rates caused by the obtained $\tau_0(T)$ dependence.

As an example we consider the ordered rare-earth dihydrides MH_{2+c} ($0 < c < c_{max}$; $c_{max} \leq 1$), where *N* metal atoms form a *fcc* lattice, 2*N* hydrogen atoms fill the set of 2*N* tetrahedral interstitial positions (H_T -atoms), while the remaining *cN* H-atoms (H_0 -atoms) are distributed among N octahedral interstitial positions. Just the latter part of hydrogen atoms reveal a considerably high mobility (with respect to H_T -atoms) and form the ordered superstructures at temperatures lower than the disorder-order transition point T_{tr} . The ordered superstructures formed, particularly, in lanthanum dihydrides are described by two long-range-order parameters (η_1 and η_2), and reveal a step-wise ordering process involving disorder-order and order-order transition points, T_{tr1} and T_{tr2} . In case of the compound under consideration, LaH_{2.27}, $T_{tr1} = 365$ K and $T_{tr2} = 343$ K, respectively (see e.g. [4]). The development of the ordering process subdivides the total set of octahedral positions into three groups of sites, which differ by the occupation probabilities, or, in other words, by the "sites occupation numbers" - n_1 , n_2 and n_3 , related to the order parameters η_1 and η_2 [5]:

$$n_1 = c + \eta_1 g + 2\eta_2 g, n_2 = c + \eta_1 g - 2\eta_2 g, n_3 = c - \eta_1 g. (1)$$

where g = 0.25 is a normalizing factor which provides the occupation numbers to be located within the physically meaningful range $[0 \le n_1, n_2, n_3 \le 1]$. The differences between occupation numbers arise at temperatures below the transition points T_{tr1} and T_{tr2} (see Fig. 1).

The internal energy of the ordering subsystem can be written in the following form [5]

$$E(c,T) = [N k_{B}/2][V(0)c^{2} +$$
$$+V(k_{1})(\eta_{1}(T) g)^{2} + 2V(k_{2})(\eta_{2}(T)g)^{2}], \qquad (2)$$

Here k_1 and k_2 are the wave vectors of the distribution function describing the equilibrium superstructure, while V(0), $V(\mathbf{k}_1)$ and $V(\mathbf{k}_2)$ are the H-H interaction potential Fourier-components (taken in temperature units). Numerical values of the energy parameters V(0), $V(\mathbf{k}_1)$ and $V(\mathbf{k}_2)$ are defined basing on experimental data. Particularly, it was established [4, 6] that for the superstoichiometric dihydrides of lanthanum the energy parameter $V(\mathbf{k}_1)$ can be estimated from the relation T_{tr1} $= -V(\mathbf{k}_1)c(1-c)$, which for c = 0.27 and $T_{tr1}(LaH_{2,27}) =$ 365 K, gives us $V(k_1) = -1853$ K [4], while for the ratio of energy constants $[V(\mathbf{k}_2)/V(\mathbf{k}_1)] \equiv p$ the value was determined: p = 0.7743 [6]. The energy parameter V(0)remains unknown, but as we are interested in the temperature variations of the ordering system with a fixed concentration, the latter parameter is less important and we shall assume that V(0) = 0.

Some words about the energy ascribed to the local vibrations of H₀-atoms. It was defined as $E_0(loc) = hv(loc)$ and determined from the diffuse inelastic neutron scattering data [7, 8]. For our numerical estimations a tentative value $E_0(loc) \approx 100$ meV was selected, or in the temperature units, $E_0(loc) = 1160$ K.

The dipole-dipole interaction of the mobile proton spins with the nuclear spins of surrounding metal atoms and those of the remaining H_0 and H_T atoms induces the spin lattice relaxation process characterized by the temperature-dependent relaxation time T_{1d} . The latter is usually described by an expression [2, 3] which can be schematically presented in the following form

$$T_{1d}^{-1} = \tau \left[J^{H}(T) \Sigma_{j}^{H} + J^{M}(T) \Sigma_{j}^{M} \right], \qquad (3)$$

where $J^{H}(T)$ and $J^{M}(T)$ are the temperature-and-timedependent factors, while Σ_{j}^{H} and Σ_{j}^{M} denote the sums of functions of the distances separating the resonating Hparticle from the surrounding hydrogen and metal atoms, respectively, while τ denotes the dwell time of jumping H₀-atoms (i.e. τ^{-1} is the number of the site-to-site jumps in a time unit).

The redistribution of H-atoms in the metal lattice occurs at temperatures well above the low-temperature range, where the quantum diffusion dominates. For this reason we describe the hydrogen motion within the metal lattice as an activation process characterised by an activation energy E_a , In these conditions the dwell-time τ of the moving particle can be presented as

$$\tau = \tau_0 \exp\left(E_a/k_B T\right),\tag{4}$$

where τ_0 is usually supposed to be constant in the whole temperature range under consideration.

The dwell-time τ was defined above as inversely proportional to the particle's jumping frequency v. A potential barrier overcome implies that $v = v_0 \exp[-E_a/k_BT]$, where v_0 is the jumping frequency at extremely high temperatures and is proportional, on the one hand, to the local oscillation frequency v(loc), and, on the other - to $v_0 = (\tau_0)^{-1}$. Thus we have

$$\tau_0 \sim v(loc)^{-1}.$$
 (5)

Let us consider the behavior of an H-atom local frequency v(loc) during the ordering process. For clarity, we shall consider a simplest example of a quantum one-dimensional oscillator which moves in the field given by the potential function $V(x) = Kx^2/2$. The quantum state of such a particle is described by the Schrödinger equation:

 $[d^2/dx^2]\psi(x) + [(2m\varepsilon/\hbar^2) - (m K x^2 /\hbar^2)] \psi(x) = 0.$ (6)

The eigenvalues of this equation are well known:

$$\varepsilon \equiv \varepsilon_0 = \hbar \omega_0 (n + 1/2), \text{ where } \omega_0 \equiv \sqrt{(K/m)}, \quad (7)$$

and *n* are integers. In this case $v(loc) = v(loc)_0 = \frac{1}{2\pi}\sqrt{(K/m)}$.

Those H-H interactions, which cause spatial redistribution of H-atoms in the ordering metal hydrides, can be considered to be responsible for the H-atom's local frequency changes. In order to account for this perturbation phenomenon let us introduce in (6) a corresponding potential V_{HH} and assume that V_{HH} is proportional to the (1/*cN*) part of the ordering energy of H₀-atoms E(c, T),

$$V_{HH} \sim (1/cN)E(c, T), \tag{8}$$

then (6) will take the form

 $[d^2/dx^2]\psi(x) + [(2m(\varepsilon - V_{_{HH}})/\hbar^2) - (mKx^2/\hbar^2)]\psi(x) = 0.(6')$

Taking into account that the additional term V_{HH} can be as large as the oscillator energy ε_0 , the traditional perturbation theory cannot be used, and we shall try to account for this perturbation by a corresponding renormalization of the mass of oscillating particles. The following obvious mathematical transformations give:

$$m(\varepsilon - V_{HH}) = m\varepsilon(1 - V_{HH}/\varepsilon) \equiv m^*\varepsilon; \quad m^* \equiv m (1 - V_{HH}/\varepsilon).(9a)$$
$$mK \equiv K[m^*/(1 - V_{HH}/\varepsilon)] = K^*m^*, \quad K^* = K(1 - V_{HH}/\varepsilon)^{-1}.(9b)$$

Using these new notations, equation (3') will take a shape coinciding formally with (3), and its eigenvalues will coincide outwardly with those given by (4). We shall have

 $[d^2/dx^2]\psi(x) + [(2m^*\varepsilon/\hbar^2) - (m^*K^*x^2/\hbar^2)]\psi(x) = 0. (10)$

$$\varepsilon = \hbar \,\omega_{\mu}(T)(n+1/2),\tag{11}$$

where

$$\omega_{H}(T) = \omega_{0} (1 - V_{HH} / \hbar \omega_{0})^{-1}.$$
(12)

Within the high-temperature disordered state the ordering energy of H₀-atoms E(c, T) is zero and the local mode frequency has to be presented by its traditional expression - $v(loc)_0 = (1/2\pi) \sqrt{(K/m)}$. When temperature is lowered below the disorder-order transition point, then E(c, T) becomes negative and following relations (8) and (12) it has to be concluded that v(loc) will become lower than $v(loc)_0$. Thus, it follows that formation of an ordered state provides temperature variations of the time parameter τ_0 - it increases monotonically at temperature lowering.

A detailed numerical analysis of the corresponding changes in the spin-lattice relaxation time performed for the ordering lanthanum dihydride $LaH_{2.27}$ will be given below. Here we present only intermediate numerical results characterizing the H-H interaction induced temperature variations of the τ_0 -factor in the selected M-H compound. Particularly, we present in Fig. 1 the ordered state formation process (by giving the temperature changes of occupation numbers n_1 , n_2 and n_3), in Fig. 2 - temperature dependence of the ordering energy E(c, T), and in Fig. 3 – the calculated τ_0 (T) dependence.

As a concluding remark we can formulate the following statement.

Formation of ordered superstructures in ordering metal-hydrogen systems is accompanied by the H-H

interaction induced hydrogen mobility reduction, which can be interpreted as some "effective weighting" of spatially ordered particles.

Application to spin-lattice relaxation processes in the rare-earth metal dihydrides. Numerical estimations of the spin-lattice relaxation time $T_1(T)$ are performed for the lanthanum dihydride LaH_{2.27}, where the subsystem of hydrogen atoms located in the octahedral interstitial positions (the subsystem of H_O-atoms) undergoes a two-step disorder-order type phase transformation and forms a sequence of ordered spatial configurations. For simplicity we assume that spatial redistribution of H_O-atoms do not influence neither on the metal *fcc* lattice, nor on the equilibrium states of the remaining part of H-atoms located in the tetrahedral interstitial positions (H_T-atoms).



Fig. 1. Formation of an ordered configuration in the subsystem of H_0 -atoms; $n_1(T)$, $n_2(T)$ and $n_3(T)$ are the sites occupation numbers (the sites occupation probabilities).



Fig. 2. The internal ordering energy E(c, T) of the system of H_0 -atoms.



Fig. 3. H-H interaction induced temperature variations of τ_0 timeparameter (τ_{00} denotes the unperturbed, high-temperature value).

In a pure (undoped) nonmagnetic metal hydride the experimentally determined spin-lattice relaxation rate $R_1 \equiv (1/T_1)$ is a summary of two relaxation processes – the first one, based on the dipole-dipole interactions of hydrogen and metal nuclear spins, is characterized by the dipolar relaxation time T_{1d} , and the second one, based on the contact interaction of the proton and electron spins (the so-called "Korringa-interaction"), is characterized by the relaxation time T_{1e} . Mathematical relations which describe the corresponding relaxation processes are well known and they were intensively applied to the analyses of experimentally investigated different metal hydrides (see e.g. [2-4, 9 11]). Particularly, the relaxation time T_{1e} caused by the contact interaction of the mobile H₀-atoms with conduction electrons, is described usually by the Korringa relation

$$T_{1e}(T) = K_{korr}/T, \quad K_{korr} = \text{const},$$
 (13)

while the dipole-dipole interactions of the mobile proton spins with the nuclear spins of surrounding metal atoms and those of the remaining H_0 and H_T atoms, are described by the set of following expressions [2-4, 9-11]:

$$\begin{split} & [1/T_{1d}] = (2/5)(C_{I}/\omega_{I}) \ y\{[(1/[1+y^{2}]) + (4/[1+(2y)^{2}])] \times \\ & \times [\Sigma_{i}^{H_{T}}(1/r_{i}^{H})^{6} + \Sigma_{i}^{H_{O}} \ n(i) \ (1/r_{i}^{H})^{6}] + (C_{s} / C_{I}) \ [(1/3) \times \\ & \times (1/[1+k_{1}y^{2}]) + (1/[1+y^{2}]) + (2/[1+k_{2} \ y^{2}])] \Sigma_{i}^{m}(1/r_{i}^{m})^{6}\}, (14) \end{split}$$
where

 $y \equiv \omega_I \tau$, $\tau = \tau_0 \exp[E_a / k_B T]$,

$$k_1 \equiv (1 - (\omega_s / \omega_l))^2, \quad k_2 \equiv (1 + (\omega_s / \omega_l))^2; \quad (15b)$$

(15a)

$$C_{I} = \gamma_{I}^{4} \hbar^{2} I(I+1) , \quad C_{S} = \gamma_{I}^{2} \gamma_{S}^{2} \hbar^{2} S(S+1) ;$$

$$\omega_{I} = \gamma_{I} H_{0} , \quad \omega_{S} = \gamma_{S} H_{0} ; \quad (15c)$$

In the relation (14) symbols Σ^m , Σ^{H_T} and Σ^{H_O} denote, respectively, the sums over the metal lattice sites, and the two types of interstitial positions occupied by Hatoms. It is assumed that the sums $\Sigma_i^m (1/r_i^m)^6$ and $\Sigma_i^{H_T} (1/r_i^H)^6$ are constant over the whole temperature range, while the sum $\Sigma_i^{H_O} n_i (1/r_i^H)^6$ contains the temperature-dependent factors $n_i \equiv n_i$ (T) denoting the occupation numbers of interstitial positions, which change significantly in the vicinity of the disorder-order transition points. (An example of the temperature dependence of occupation numbers in the ordering system is shown in Fig. 1). In expressions (15) numerical parameters $(\gamma_{I^{5}} \omega_{I})$ and $(\gamma_{S^{5}} \omega_{S^{5}})$ represent, respectively, the gyromagnetic ratios and Larmor frequencies of interstitial H-nuclei (with spin I) and those of metal M-nuclei (with spin S). In expressions (15*a*) τ is the dwell-time of the jumping H-atoms. (If the mobility of H-atoms is characterized by the number of site-to-site jumps v, then $\tau = v^{-1}$. E_a is the corresponding activation energy, and τ_0 is the "preexponential factor").

In summarizing the mathematical scheme of calculations we formulate:

$$R_1 = R_{1d} + R_{1e}, (16)$$

where $R_{1e} \equiv (1/T_{1e})$ is determined by the relations (13), and $R_{1d} \equiv (1/T_{1d})$ is given by the expressions (14)-(15).

Following equation (14), the development of an ordering process in M-H compounds should be reflected, first of all, in the temperature dependence of the sum $\Sigma_i^{H_T} \Sigma_i^{H_O} n_i(T)(1/r_i^{H_0^6})$. In [4] we had analyzed the experimental data [10] obtained on the ordering lanthanum dihydride LaH_{2.27}, and had established that the given mathematical refinement of the corresponding calculations provides a negligibly small influence on the numerical values of the summary dependence $T_1(T)$.

Now we examine the influence of the above established $\tau_0(T)$ dependence on the dipole-dipole interaction induced part of the spin-lattice relaxation time $T_{1d}(T)$ and on the total relaxation time $T_1(T)$. Estimations of tentative temperature changes of the τ_0 parameter values were shown in the corresponding figure (see Fig. 3) and they turned out to be sufficiently significant, but there remains a question – in what degree the calculated values of the spin-lattice relaxation time are influenced by taking into account the temperature variations of τ_0 parameter

It is obvious that the increment of the time spent by each of the moving particles at the visited sites (the increment of the dwell-time) will cause an increment of the corresponding $T_{1d}(T)$ values. This natural conclusion is justified by the results of numerical estimations presented in Fig. 4. The next question is - how can the resulting changes in the general temperature dependence of the dipole-dipole relaxation time $T_{1d}(T)$ be classified. From Fig. 5 it follows that these changes are remarkable, but not essential. Very significant is the fact that they are revealed in the low temperature range, where the total rate of the nuclear spin relaxation process is determined mainly by interactions of nuclear spins with conduction electrons. The latter statement is illustrated in Fig. 6. Here both calculated dependences $T_{1e}(T)$ and $T_{1d}(T)$ are denoted by corresponding thin lines, while the thick line presents the calculated resulting $T_1(T)$ dependence.

From the above short consideration two qualitative conclusions may be deduced.

1) In the case of ordering lanthanum dihydrides,



Fig. 4. Temperature dependence of the difference between $T_{1d}(T)$ values calculated in different conditions: on assuming a constant. τ_0 -factor, $T_{1d}(0)$, and on taking into account H-H interaction induced $\tau_0(T)$ variations, $T_{1d}(H-H)$.



Fig. 6. Temperature dependences of the Korringa relation $T_{1e}(T)$, of dipole-dipole relaxation $T_{1d}(T)$, and of the summary relaxation time $T_1(T)$.

even in conditions of taking into account the ordering process and the "particle weighting" effect related to it, the calculated general $T_1(T)$ dependence remains practically unchanged. It follows that the set of previously selected adjusting parameters, such as activation energy E_a , time-parameter τ_0 and Korringa constant K_{Korr} , may be left unchanged. Particularly, for the compound LaH_{2.27} successful results in the description of experimental $T_1(T)$ dependence can be obtained on using the previously determined values of mentioned parameters [10]:

$$K_{\text{Korr}} = 410 \text{ sec K}, \qquad E_a = 0.35 \text{ eV/atom},$$

 $\tau_0^{-1} = 7.4 \times 10^{11} \text{ sec}^{-1}.$ (17)

In Fig. 7, we have presented experimental results [10] (the sequence of triangles) and the corresponding theoretical curve calculated basing on the given values of mentioned parameters and on taking into account simultaneously the ordering and "weighting" effects. (The calculated curve practically repeats the previously obtained theoretical dependences published in [4] and [10]).



Fig. 5. Absolute values of dipole-dipole relaxation times $T_{1d}(T)$ calculated in different conditions: on taking into account the additional H-H interactions, T_{1d} (H-H), and without the latter refinement $T_{1d}(0)$.



Fig. 7. Description of the sequence of experimental points by the calculated $T_1(T)$ curve.

2) It was suggested (see e.g. [11]) that in the Ce-H system, as well as in La-H system, continuous increment of the hydrogen concentration provides a smooth depleting of the conduction band. It follows that the "hydrogen nuclear spin - conduction electron spin" interaction strength will be continuously reduced as well. The band-depleting process is terminated in LaH₃ and CeH₃ compounds, where conduction bands turned out to be practically empty and compounds reveal the insulator (or semiconductor) properties [10]. At some lower concentration values both compounds are non-metallic in the high-temperature disordered states and reveal the ordinary metal properties in the ordered low-temperature states (below the disorder-order transition temperature). It seems natural to expect that the ordering related properties of H₀-atoms, and, particularly, the "particle weighting" effects, can be registered in the maximally proper, "unmasked" situation just in the corresponding concentration range.

Investigations were performed within the frames of the STCU Project N 3867.

ფიზიკა

წყალბადის ძვრადობის ტემპერატურული დამოკიდებულება იშვიათ მიწათა მოწესრიგებად წყალბადნაერთებში

ი. რატიშვილი*, ნ. ნამორაძე**

* ე. ანდრონიკაშვილის ფიზიკის ინსტიტუტი, თბილისი ** კიბერნეტიკის ინსტიტუტი, თბილისი

(წარმოდგენილია აკადემიკოს თ. სანაძის მიერ)

შესწავლილია მოწესრიგების პროცესის გავლენა ლითონის მესერში ჩანერგილი წყალბადის ატომების ძვრადობაზე იშვიათ მიწათა მოწესრიგებად წყალბადნაერთებში. ნაჩვენებია, რომ წყალბად-წყალბად ურთიერთქმედება, რომელიც უზრუნველყოფს მოწესრიგებული ზესტრუქტურის ჩამოყალიბებას წყალბადის ქვემესერში, ამავე დროს განაპირობებს წყალბადის ატომების კგანძთაშორის პოზიციებში დაყოვნების დროის გაზრდას, რაც შეიძლება აღიწეროს როგორც მოძრავი ნაწილაკის "დამძიმება". რიცზვითი შეფასებები შესრულებულია ლანთანის დიპიდრიდისათვის LaH_{2.27} და მიღებულია აქტივაციური პროცესის დროითი პარამეტრის ტემპერატურული დამოკიდებულება. განხილულია "ნაწილაკთა დამძიმების ეფექტის" გავლენა დიპოლ-დიპოლური ურთიერთქმედებით განპირობებული ბირთვული სპინ-მესერული რელაქსაციის დროის ტემპერატურულ დამოკიდებულებაზე. ნაჩვენებია, რომ თუმცა შესაბამი ცვლილებები $T_{1d}(T)$ მრუდზე საკმაოდ მკვეთრია, მაგრამ ჯამურ $T_1(T)$ დამოკიდებულებაზე, რომელიც ასევე შეიცავს ბირთვული სპინების გამტარებლობის ელექტრონებთან ურთიერთქმედების წევრსაც, ისინი პრაქტიკულად არ აისახება. ამიტომ შესაძლებელია უცვლელად იქნას დატოვებული წინათ განსაზღვრული იმ რიცზიებით პარამეტრების ერთობლიობა, რომელნიც უზრუნველყოფდნენ ექსპერიმენტული და თეორიული მონაცემების კარგ თანზვედრას.

REFERENCES

- 1. J.M. Rowe (1972), Sol. St. Comm., 11: 1299-1302.
- 2. R.M. Cotts (1978), In: Hydrogen in Metals (eds. G.Alefeld and J.Völkl), vol. 1: 227-288, Springer-Verlag.
- 3. Y. Fukai, S. Kazama (1977), Acta Metallurgica, 25: 59-77.
- 4. *N.Z. Namoradze, I.G. Ratishvili* (2007), In: Hydrogen Materials Science and Chemistry of Carbon Nanomaterials (eds. N. Veziroglu et al.). Springer: 87-94.
- 5. I.G. Ratishvili, P. Vajda (1997), Journ. of Alloys and Compounds, 253-254: 171-174.
- 6. I.G. Ratishvili, P. Vajda (1996), Phys. Rev. B 53 (2): 581 587.
- 7. T.J. Udovic, J.J. Rush, I.S. Anderson (1995), J. Phys. Cond. Matter, 7: 7005-7014.
- 8. P. Vorderwisch, S. Hautecler, J. Eckert (1997), Journ. All. Comp., 253-254: 252-254.
- 9. D. Zamir and R.M. Cotts (1964), Phys. Rev., 134, 3A: A671-A678.
- 10. T.-T. Phua, B.J. Beaudry, et al. (1983), Phys. Rev. B 28 (11): 6227-6250.
- 11. D. Zamir, R.G. Barnes, et al. (1984), Phys. Rev. B, 29 (1): 61-70.

Received July, 2009

Bull. Georg. Natl. Acad. Sci., vol. 3, no. 3, 2009