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On some Nonclassical Two-Dimensional Models for Thermoelastic Plates with Variable Thickness

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ABSTRACT. In the present paper Green-Lindsay nonclassical three-dimensional model for thermoelastic plates with variable thickness is considered. Applying variational approach initial-boundary value problem corresponding to the three-dimensional model is investigated in suitable Sobolev spaces. The three-dimensional dynamical model for plate with variable thickness, when surface forces and heat flux are given along the upper and the lower faces of the plate, is reduced to a hierarchy of two-dimensional models. The initial-boundary value problems corresponding to the obtained two-dimensional models are investigated in suitable function spaces. Moreover, the convergence of the sequence of vector-functions of three space variables restored from the solutions of the reduced two-dimensional problems to the solution of the original three-dimensional problem is proved and under additional conditions the rate of convergence is estimated. *© 2010 Bull. Georg. Natl. Acad. Sci.*

Key words: *nonclassical models of thermoelastic solids, plates with variable thickness, Fourier-Legendre series, modeling error estimates.*

Nonclassical mathematical models for thermoelastic solids, which generalize classical models and depend on one or more relaxation time, were developed to eliminate shortcomings of the classical thermoelasticity, particularly, infinite velocity of thermoelastic disturbances. One of such type models was obtained by A. Green and K. Lindsay [1], which is characterized by a system of partial differential equations where, in comparison with the classical linear system of thermoelasticity, the constitutive relations for the stress tensor and the entropy are generalized by introducing two different relaxation times. For Green-Lindsay nonclassical model the problem of propagation of a thermoelastic wave was studied and domain of influence result was obtained in [2] in classical spaces of twice continuously differentiable functions. In the case of infinite and semi-infinite bodies initial-boundary value problems corresponding to Green-Lindsay model were investigated in [3, 4]. Applying method of potential and theory of integral equations the problems of stable and pseudo oscillations for Green-Lindsay nonclassical model were studied in [5]. Since numerical solution of three-dimensional initial-boundary value problems of nonclassical thermoelasticity requires complicated algorithms, it is important to construct two-dimensional or one-dimensional models of thermoelastic bodies. One of dimensional reduction methods for plates with variable thickness in the classical theory of elasticity was suggested by I. Vekua in [6]. To construct two-dimensional models of plate Vekua considered differential formulation of the three-dimensional initial-boundary value problem and approximating components of the displacement vector-function by partial sums of orthogonal Fourier-Legendre series with respect to the variable of plate thickness a hierarchy of initial-boundary value problems defined on two-dimensional space domain was obtained. Mathematical results on the relationship between the two-dimensional hierarchical models constructed in [6] and three-dimensional one in static case first were obtained in the spaces of classical regular functions in the paper [7], and the reduced two-dimensional models for thin shallow shells were investigated in Sobolev spaces in [8]. Later on, various hierarchical models were constructed and investigated applying Vekua's reduction method and its generalizations (see [9-11] and references given therein).

The present paper is devoted to the construction and investigation of a hierarchy of two-dimensional mathematical models for plates with variable thickness, when the stress-strain state of thermoelastic solid is described by Green-Lindsay nonclassical three-dimensional model. We consider initial-boundary value problem corresponding to Green-Lindsay three-dimensional model and applying variational approach we obtain the existence and uniqueness result in suitable Sobolev spaces. We construct hierarchical two-dimenisonal models for plate with variable thickness, when the density of surface forces and heat flux is given along the upper and the lower faces of the plate. We investigate initial-boundary value problems corresponding to the constructed two-dimensional models in suitable function spaces. Moreover, we prove that the sequence of vector-functions of three space variables restored from the solutions of the reduced two-dimensional problems converges to the solution of the three-dimensional problem and under additional regularity conditions we estimate the rate of convergence.

We denote by $W^{r,2}(D)=H^r(D)$, $r\geq 1$, $r\in\mathbf{R}$, the Sobolev space of order r based on the space $L^2(D)$ of square-integrable functions in $D\subset\mathbf{R}^p$, $p\in\mathbf{N}$, in Lebesgue sense, $\mathbf{H}^r(D)=[H^r(D)]^3$, $\mathbf{L}^2(D)=[L^2(D)]^3$ and $\mathbf{L}^s(\hat{\Gamma})=[L^s(\hat{\Gamma})]^3$, $s\geq 1$, $s\in\mathbf{R}$, where $\hat{\Gamma}$ is a Lipschitz surface. For any Banach space X, $C^0([0,T];X)$ denotes the space of continuous functions on $[0,T]$ with values in X, $L^2(0,T;X)$ is the space of such functions $g:(0,T)\to X$ that $\|g(t)\|_X\in L^2(0,T)$. We denote by $g'=dg/dt$ the generalized derivative of $g\in L^2(0,T;X)$.

Let us consider thermoelastic plate with variable thickness vanishing on a part of its boundary, i.e. plate that occupies three-dimensional Lipschitz domain Ω of the following form

$$\Omega=\{(x_1,x_2,x_3)\in\mathbf{R}^3;\ h^-(x_1,x_2)<x_3<h^+(x_1,x_2),\ (x_1,x_2)\in\omega\},$$

where $\omega\subset\mathbf{R}^2$ is a two-dimensional bounded Lipschitz domain with boundary $\partial\omega$, $h^\pm\in C^0(\overline{\omega})\cap C^{1,1}_{loc}(\omega\cup\tilde{\gamma})$ are Lipschitz continuous in the interior of the domain ω and on $\tilde{\gamma}\subset\partial\omega$ together with the first order derivatives, $h^+(x_1,x_2)>h^-(x_1,x_2)$, for $(x_1,x_2)\in\omega\cup\tilde{\gamma}$, $\tilde{\gamma}\subset\partial\omega$ is a Lipschitz curve, $h^+(x_1,x_2)=h^-(x_1,x_2)$, for $(x_1,x_2)\in\partial\omega\setminus\tilde{\gamma}$. The upper and the lower faces of Ω, defined by the equations $x_3=h^+(x_1,x_2)$ and $x_3=h^-(x_1,x_2)$, $(x_1,x_2)\in\omega$, we denote by Γ^+ and Γ^-, respectively, and the lateral face, where the thickness of Ω is positive, we denote by $\tilde{\Gamma}=\partial\Omega\setminus\overline{(\Gamma^+\cup\Gamma^-)}=\{(x_1,x_2,x_3)\in\mathbf{R}^3;h^-(x_1,x_2)<x_3<h^+(x_1,x_2),\ (x_1,x_2)\in\tilde{\gamma}\}$. We assume that plate consists of homogeneous, isotropic thermoelastic material. The applied body force density we denote by $\mathbf{f}=(f_i)\ :\Omega\times(0,T)\to\mathbf{R}^3$ and the density of heat sources we denote by $f^\theta:\Omega\times(0,T)\to\mathbf{R}$. Plate is clamped and the temperature θ vanishes along a part $\tilde{\Gamma}_0=\{(x_1,x_2,x_3)\in\mathbf{R}^3;h^-(x_1,x_2)<x_3<h^+(x_1,x_2),\ (x_1,x_2)\in\tilde{\gamma}_0\}$ of the lateral face $\tilde{\Gamma}$, $\tilde{\gamma}_0\subset\tilde{\gamma}$ is a Lipschitz curve, and on the remaining part $\Gamma_1=\Gamma\setminus\overline{\tilde{\Gamma}_0}$ of the boundary the surface forces with density $\mathbf{g}=(g_i):\Gamma_1\times(0,T)\to\mathbf{R}^3$ and heat flux with density $g^\theta:\Gamma_1\times(0,T)\to\mathbf{R}$ is given.

The nonclassical dynamical linear three-dimensional model of stress-strain state of thermoelastic body obtained by A. Green and K. Lindsay in differential form is given by

$$\rho\frac{\partial^2 u_i}{\partial t^2}-\sum_{j=1}^{3}\frac{\partial}{\partial x_j}\left(\lambda\sum_{p=1}^{3}e_{pp}(\mathbf{u})\delta_{ij}+2\mu e_{ij}(\mathbf{u})-\eta\theta\delta_{ij}-\eta\tau_1\frac{\partial\theta}{\partial t}\delta_{ij}\right)=f_i \text{ in } \Omega\times(0,T),\ i=1,2,3, \tag{1}$$

$$\chi\left(\frac{\partial\theta}{\partial t}+\tau_0\frac{\partial^2\theta}{\partial t^2}\right)-\kappa\sum_{j=1}^{3}\frac{\partial^2\theta}{\partial x_j^2}+\Theta_0\eta\frac{\partial}{\partial t}\sum_{p=1}^{3}e_{pp}(\mathbf{u})=f^\theta \text{ in } \Omega\times(0,T), \tag{2}$$

$$\sum_{j=1}^{3}\left(\lambda\sum_{p=1}^{3}e_{pp}(\mathbf{u})\delta_{ij}+2\mu e_{ij}(\mathbf{u})-\eta\theta\delta_{ij}-\eta\tau_1\frac{\partial\theta}{\partial t}\delta_{ij}\right)\nu_j=g_i \quad \text{on } \Gamma_1\times(0,T),\ \mathbf{u}(0)=\mathbf{0} \quad \text{on } \tilde{\Gamma}_0\times(0,T), \tag{3}$$

$$\kappa\sum_{j=1}^{3}\frac{\partial\theta}{\partial x_j}\nu_j=g^{\theta} \quad \text{on } \Gamma_1\times(0,T), \qquad \theta=0 \quad \text{on } \tilde{\Gamma}_0\times(0,T), \tag{4}$$

$$u_i(x,0)=u_{0i}(x),\ \frac{\partial u_i}{\partial t}(x,0)=u_{1i}(x),\ \theta(x,0)=\theta_0(x),\ \frac{\partial\theta}{\partial t}(x,0)=\theta_1(x) \quad \text{in } \Omega, i=1,2,3, \tag{5}$$

where δ_{ij} is the Kronecker delta, $\mathbf{u}=(u_i):\Omega\times(0,T)\to\mathbf{R}^3$ is the displacement vector-function of thermoelastic body, $\theta:\Omega\times(0,T)\to\mathbf{R}$ is the temperature distribution, λ, μ are Lamé constants, ρ is a mass density, $\kappa>0$ is a heat conductivity coefficient, $\chi>0$ is a thermal capacity, η is a thermoelastic constant, $\Theta_0>0$ is a temperature of the medium in the natural state and τ_0,τ_1 are relaxation times. Note that in the case of $\tau_0=\tau_1=0$ the nonclassical three-dimensional model (1)-(5) coincides with the classical linear three-dimensional model for thermoelastic bodies.

To investigate the existence and uniqueness of weak solution of the three-dimensional initial-boundary value problem (1)-(5) we consider the following variational formulation: Find $\mathbf{u}\in C^0([0,T];\mathbf{V}(\Omega))$, $\mathbf{u}'\in L^\infty(0,T;\mathbf{V}(\Omega))$, $\mathbf{u}''\in L^\infty(0,T;\mathbf{L}^2(\Omega))$, $\theta\in C^0([0,T];V^\theta(\Omega))$, $\theta'\in L^\infty(0,T;V^\theta(\Omega))$, $\theta''\in L^\infty(0,T;L^2(\Omega))$, which satisfies the following equations in the sense of distributions on $(0,T)$,

$$\frac{d}{dt}(\rho\mathbf{u}'(.),\mathbf{v})_{\mathbf{L}^2(\Omega)}+a(\mathbf{u}(.),\mathbf{v})-\eta(\theta,\sum_{p=1}^{3}\frac{\partial v_p}{\partial x_p})_{L^2(\Omega)}-\eta\tau_1(\theta',\sum_{p=1}^{3}\frac{\partial v_p}{\partial x_p})_{L^2(\Omega)}=(\mathbf{f},\mathbf{v})_{\mathbf{L}^2(\Omega)}+(\mathbf{g},\mathbf{v})_{\mathbf{L}^2(\Gamma_1)},\quad \forall\mathbf{v}\in\mathbf{V}(\Omega), \tag{6}$$

$$\frac{d}{dt}(\chi\tau_0\theta'(.),\varphi)_{L^2(\Omega)}+(\chi\theta'(.),\varphi)_{L^2(\Omega)}+a^\theta(\theta(.),\varphi)+\Theta_0\eta(\sum_{p=1}^{3}\frac{\partial u'_p}{\partial x_p},\varphi)_{L^2(\Omega)}=(f^\theta,\varphi)_{L^2(\Omega)}+(g^\theta,\varphi)_{L^2(\Gamma_1)},\ \forall\varphi\in V^\theta(\Omega), \tag{7}$$

together with the initial conditions

$$\mathbf{u}(0)=\mathbf{u}_0, \qquad \mathbf{u}'(0)=\mathbf{u}_1, \qquad \theta(0)=\theta_0, \qquad \theta'(0)=\theta_1, \tag{8}$$

where $\mathbf{u}_0$, $\mathbf{u}_1$ are the initial displacement and velocity vector-functions, θ_0, θ_1 are the initial distributions of the temperature and of its rate of change, $\mathbf{V}(\Omega)=\{\mathbf{v}\in\mathbf{H}^1(\Omega);\mathbf{tr}(\mathbf{v})=\mathbf{0}$ on $\tilde{\Gamma}_0\}$, $V^\theta(\Omega)=\{\varphi\in H^1(\Omega);tr(\varphi)=0$ on $\tilde{\Gamma}_0\}$, $\mathbf{tr}:\mathbf{H}^1(\Omega)\to\mathbf{H}^{1/2}(\Gamma)$ and $tr:H^1(\Omega)\to H^{1/2}(\Gamma)$ are the trace operators,

$$a(\tilde{\mathbf{v}},\mathbf{v})=\int_\Omega\left(\lambda\sum_{p=1}^{3}e_{pp}(\tilde{\mathbf{v}})\sum_{q=1}^{3}e_{qq}(\mathbf{v})+2\mu\sum_{i,j=1}^{3}e_{ij}(\tilde{\mathbf{v}})e_{ij}(\mathbf{v})\right)dx, \qquad \forall\mathbf{v},\tilde{\mathbf{v}}\in\mathbf{V}(\Omega),$$

$$a^\theta(\tilde{\varphi},\varphi)=\kappa\int_\Omega\sum_{j=1}^{3}\frac{\partial\tilde{\varphi}}{\partial x_j}\frac{\partial\varphi}{\partial x_j}dx, \quad \forall\varphi,\tilde{\varphi}\in H_0^1(\Omega), \quad e_{ij}(\mathbf{v})=\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right),\ i,j=\overline{1,3},$$

$(.,.)_{\mathbf{L}^2(\Omega)}$, $(.,.)_{L^2(\Omega)}$, $(.,.)_{\mathbf{L}^2(\Gamma_1)}$ and $(.,.)_{L^2(\Gamma_1)}$ are scalar products in the spaces $\mathbf{L}^2(\Omega)$, $L^2(\Omega)$, $\mathbf{L}^2(\Gamma_1)$ and $L^2(\Gamma_1)$, respectively.

For Green-Lindsay nonclassical dynamical three-dimensional model for thermoelastic body (6)-(8) the following theorem is valid.

Theorem 1. *Suppose that* $\mathbf{u}_0 \in \mathbf{H}^2(\Omega)\cap \mathbf{V}(\Omega)$, $\mathbf{u}_1 \in \mathbf{V}(\Omega)$, $\theta_0 \in H^2(\Omega)\cap V^{\theta}(\Omega)$, $\theta_1 \in V^{\theta}(\Omega)$, $\mathbf{f},\mathbf{f}' \in L^2(0,T;\mathbf{L}^2(\Omega))$, $\mathbf{g},\mathbf{g}',\mathbf{g}'' \in L^2(0,T;\mathbf{L}^{4/3}(\Gamma_1))$, $f^{\theta}, f^{\theta'} \in L^2(0,T;L^2(\Omega))$, $g^{\theta}, g^{\theta'}, g^{\theta''} \in L^2(0,T;L^{4/3}(\Gamma_1))$, *and the following compatibility conditions*

$$g_i(0)=\sum_{j=1}^{3}\left(\lambda\sum_{p=1}^{3}e_{pp}(\mathbf{u}_0)\delta_{ij}+2\mu e_{ij}(\mathbf{u}_0)-\eta\theta_0\delta_{ij}-\eta\tau_1\theta_1\delta_{ij}\right)\nu_j,\ i=1,2,3,\quad g^{\theta}(0)=\sum_{j=1}^{3}\kappa\frac{\partial\theta_0}{\partial x_j}\nu_j .$$

If $\mu>0$, $3\lambda+2\mu>0$ *and* $0<\tau_0\le\tau_1$, *then the initial-boundary value problem (6)-(8) possesses a unique solution.*

In order to construct an algorithm of approximation of Green-Lindsay nonclassical three-dimensional model for thermoelastic plates with variable thickness by a sequence of two-dimensional models let us consider the subspaces $\mathbf{V}^2_{\mathbf{N}}(\Omega)$, $\mathbf{V}_{\mathbf{N}}(\Omega)$ and $\mathbf{H}_{\mathbf{N}}(\Omega)$ of $\mathbf{H}^2(\Omega)\cap\mathbf{V}(\Omega)$, $\mathbf{V}(\Omega)$ and $\mathbf{L}^2(\Omega)$, respectively, $\mathbf{N}=(N_1,N_2,N_3)$, consisting of vector-functions whose components are polynomials with respect to the variable of plate thickness x_3,

$$\mathbf{v}_{\mathbf{N}}=(v_{\mathbf{N}i}),\quad v_{\mathbf{N}i}=\sum_{r_i=0}^{N_i}\frac{1}{h}(r_i+\frac{1}{2})\overset{r_i}{v}_{\mathbf{N}i}P_{r_i}(z),\quad \overset{r_i}{v}_{\mathbf{N}i}\in L^2(\omega),\ 0\le r_i\le N_i,\ i=1,2,3,$$

where $z=\dfrac{x_3-\overline{h}}{h}$, $h=\dfrac{h^+-h^-}{2}$, $\overline{h}=\dfrac{h^++h^-}{2}$. In addition, we consider the subspaces $V^{\theta,2}_{N_\theta}(\Omega)$, $V^{\theta}_{N_\theta}(\Omega)$ and $H^{\theta}_{N_\theta}(\Omega)$ of $H^2(\Omega)\cap V^{\theta}(\Omega)$, $V^{\theta}(\Omega)$ and $L^2(\Omega)$, respectively, which consist of the following functions

$$\varphi_{N_\theta}=\sum_{r=0}^{N_\theta}\frac{1}{h}(r+\frac{1}{2})\overset{r}{\varphi}_{N_\theta}P_r(z),\quad \overset{r}{\varphi}_{N_\theta}\in L^2(\omega),\ 0\le r\le N_\theta .$$

Since the functions h^+ and h^- are Lipschitz continuous together with their first order derivatives in the interior of the domain ω, from Rademacher's theorem [12] it follows that $h^{\pm}$ and $\partial_\alpha h^{\pm}$ are differentiable almost everywhere in ω^* and $\partial_\alpha h^{\pm},\partial_\alpha\partial_\beta h^{\pm}\in L^{\infty}(\omega^*)$ for all subdomains ω^*, $\overline{\omega^*}\subset\omega$, $\alpha,\beta=1,2$. Therefore, the positiveness of h in ω implies that for any vector-function $\mathbf{v}_{\mathbf{N}}=(v_{\mathbf{N}i})_{i=1}^3\in\mathbf{V}^2_{\mathbf{N}}(\Omega)$ the corresponding functions $\overset{r_i}{v}_{\mathbf{N}i}\in H^2(\omega^*)$ for all ω^*, $\overline{\omega^*}\subset\omega$, i.e. $\overset{r_i}{v}_{\mathbf{N}i}\in H^2_{loc}(\omega)$, $0\le r_i\le N_i$, $i=1,2,3$. Similarly, if $\mathbf{v}_{\mathbf{N}}=(v_{\mathbf{N}i})_{i=1}^3\in\mathbf{V}_{\mathbf{N}}(\Omega)$, then $\overset{r_i}{v}_{\mathbf{N}i}\in H^1(\omega^*)$ for all ω^*, $\overline{\omega^*}\subset\omega$, i.e. $\overset{r_i}{v}_{\mathbf{N}i}\in H^1_{loc}(\omega)$, $0\le r_i\le N_i$, $i=1,2,3$. For functions from the spaces $V^{\theta,2}_{N_\theta}(\Omega)$ and $V^{\theta}_{N_\theta}(\Omega)$ we also have $\overset{r}{\varphi}_{N_\theta}\in H^2_{loc}(\omega)$, if $\varphi_{N_\theta}\in V^{\theta,2}_{N_\theta}(\Omega)$ and $\overset{r}{\varphi}_{N_\theta}\in H^1_{loc}(\omega)$, if $\varphi_{N_\theta}\in V^{\theta}_{N_\theta}(\Omega)$, $0\le r\le N_\theta$. Moreover, the norms $\|.\|_{\mathbf{H}^2(\Omega)}$, $\|.\|_{\mathbf{H}^1(\Omega)}$ and $\|.\|_{H^2(\Omega)}$, $\|.\|_{H^1(\Omega)}$ in the spaces $\mathbf{H}^2(\Omega)$, $\mathbf{H}^1(\Omega)$ and $H^2(\Omega)$, $H^1(\Omega)$ define weighted norms $\|.\|_{**}$, $\|.\|_{*}$ and $\|.\|_{\theta**}$, $\|.\|_{\theta*}$ of vector-functions $\vec{v}_{\mathbf{N}}\in[H^2_{loc}(\omega)]^{N_{1,2,3}}$, $\vec{v}_{\mathbf{N}}\in[H^1_{loc}(\omega)]^{N_{1,2,3}}$, $N_{1,2,3}=N_1+N_2+N_3+3$, with components $\overset{r_i}{v}_{\mathbf{N}i}$, $\vec{v}_{\mathbf{N}}=(\overset{r_i}{v}_{\mathbf{N}i})$, and $\vec{\varphi}_{N_\theta}\in[H^2_{loc}(\omega)]^{N_\theta+1}$, $\vec{\varphi}_{N_\theta}\in[H^1_{loc}(\omega)]^{N_\theta+1}$, with components $\overset{r}{\varphi}_{N_\theta}$, $\vec{\varphi}_{N_\theta}=(\overset{r}{\varphi}_{N_\theta})$, such that $\|\vec{v}_{\mathbf{N}}\|_{**}=\|\mathbf{v}_{\mathbf{N}}\|_{\mathbf{H}^2(\Omega)}$, $\|\vec{v}_{\mathbf{N}}\|_{*}=\|\mathbf{v}_{\mathbf{N}}\|_{\mathbf{H}^1(\Omega)}$ and $\|\vec{\varphi}_{N_\theta}\|_{\theta**}=\|\varphi_{N_\theta}\|_{H^2(\Omega)}$, $\|\vec{\varphi}_{N_\theta}\|_{\theta*}=\|\varphi_{N_\theta}\|_{H^1(\Omega)}$. Using the properties of the Legendre polynomials, we can obtain explicit expressions for the norms $\|.\|_{**}$, $\|.\|_{*}$, $\|.\|_{\theta**}$ and $\|.\|_{\theta*}$, particularly, the norm $\|.\|_{*}$ is given by

$$\|\vec{v}_{\mathbf{N}}\|_*^2=\sum_{i=1}^{3}\sum_{r_i=0}^{N_i}(r_i+\frac{1}{2})\left[\left\|\sum_{s_i=r_i}^{N_i}(s_i+\frac{1}{2})(1-(-1)^{r_i+s_i})h^{-3/2}\overset{s_i}{v}_{\mathbf{N}i}\right\|^2_{L^2(\omega)}+\left\|h^{-1/2}\overset{r_i}{v}_{\mathbf{N}i}\right\|^2_{L^2(\omega)}+\right.$$

$$\left.+\sum_{\alpha=1}^{2}\left\|\sum_{s_i=r_i+1}^{N_i}(s_i+\frac{1}{2})(\partial_\alpha h^+-(-1)^{r_i+s_i}\partial_\alpha h^-)h^{-3/2}\overset{s_i}{v}_{\mathbf{N}i}-h^{-1/2}\partial_\alpha\overset{r_i}{v}_{\mathbf{N}i}+(r_i+1)h^{-3/2}\partial_\alpha h\overset{r_i}{v}_{\mathbf{N}i}\right\|^2_{L^2(\omega)}\right],$$

where we assume that the sum with the lower limit greater than the upper one equals zero.

For components $\overset{r_i}{v}_{\mathbf{N}i}$ and $\overset{r}{\varphi}_{N_\theta}$ of vector-functions $\vec{v}_{\mathbf{N}} \in [H^1_{loc}(\omega)]^{N_{1,2,3}}$ and $\vec{\varphi}_{N_\theta} \in [H^1_{loc}(\omega)]^{N_\theta+1}$, which satisfy the conditions $\|\vec{v}_{\mathbf{N}}\|_* < \infty$ and $\|\vec{\varphi}_{N_\theta}\|_{\theta*} < \infty$ we can define the traces on $\tilde{\gamma}$. Indeed, the corresponding vector-function $\mathbf{v_N} = (v_{\mathbf{N}i})_{i=1}^3$ and function φ_{N_θ} of three space variables belong to the space $\mathbf{V_N}(\Omega) \subset \mathbf{H}^1(\Omega)$ and $V^\theta_{N_\theta}(\Omega) \subset H^1(\Omega)$, respectively. Consequently, applying the trace operator on the space $H^1(\Omega)$ we define the traces on $\tilde{\gamma}$ for functions $\overset{r_i}{v}_{\mathbf{N}i}$ and $\overset{r}{\varphi}_{N_\theta}$,

$$tr_{\tilde{\gamma}}(\overset{r_i}{v}_{\mathbf{N}i}) = \int_{h^-}^{h^+} tr(v_{\mathbf{N}i})|_{\tilde{\Gamma}} P_{r_i}(z)dx_3, \quad tr_{\tilde{\gamma}}(\overset{r}{\varphi}_{N_\theta}) = \int_{h^-}^{h^+} tr(\varphi_{N_\theta})|_{\tilde{\Gamma}} P_r(z)dx_3, \quad r_i = \overline{0, N_i}, \; i = \overline{1,3}, \; r = \overline{0, N_\theta}.$$

Since the vector-functions $\mathbf{v_N} = (v_{\mathbf{N}i})$ from the subspaces $\mathbf{V_N}(\Omega)$ and $\mathbf{H_N}(\Omega)$, and the functions φ_{N_θ} from $V^\theta_{N_\theta}(\Omega)$ and $H^\theta_{N_\theta}(\Omega)$ are uniquely defined by functions $\overset{r_i}{v}_{\mathbf{N}i}$ and $\overset{r}{\varphi}_{N_\theta}$ of two space variables, therefore considering the original three-dimensional problem (6)-(8) on these subspaces, we obtain the following hierarchy of two-dimensional initial-boundary value problems: Find $\vec{w}_{\mathbf{N}} \in C^0([0,T]; \vec{V}_{\mathbf{N}}(\omega))$, $\vec{w}'_{\mathbf{N}} \in L^\infty(0,T; \vec{V}_{\mathbf{N}}(\omega))$, $\vec{w}''_{\mathbf{N}} \in L^\infty(0,T; \vec{H}_{\mathbf{N}}(\omega))$, $\vec{\zeta}_{N_\theta} \in C^0([0,T]; \vec{V}^\theta_{N_\theta}(\omega))$, $\vec{\zeta}'_{N_\theta} \in L^\infty(0,T; \vec{V}^\theta_{N_\theta}(\omega))$, $\vec{\zeta}''_{N_\theta} \in L^\infty(0,T; \vec{H}^\theta_{N_\theta}(\omega))$, which satisfy the following equations in the sense of distributions on $(0,T)$,

$$\frac{d}{dt} R_{\mathbf{N}}(\vec{w}'_{\mathbf{N}}, \vec{v}_{\mathbf{N}}) + a_{\mathbf{N}}(\vec{w}_{\mathbf{N}}, \vec{v}_{\mathbf{N}}) - b_{\mathbf{N}N_\theta}(\vec{\zeta}_{N_\theta} + \tau_1 \vec{\zeta}'_{N_\theta}, \vec{v}_{\mathbf{N}}) = L_{\mathbf{N}}(\vec{v}_{\mathbf{N}}), \quad \forall \vec{v}_{\mathbf{N}} \in \vec{V}_{\mathbf{N}}(\omega), \tag{9}$$

$$\frac{d}{dt} R^\theta_{N_\theta}(\vec{\zeta}_{N_\theta} + \tau_0 \vec{\zeta}'_{N_\theta}, \vec{\varphi}_{N_\theta}) + a^\theta_{N_\theta}(\vec{\zeta}_{N_\theta}, \vec{\varphi}_{N_\theta}) + \Theta_0 b^\theta_{\mathbf{N}N_\theta}(\vec{w}'_{\mathbf{N}}, \vec{\varphi}_{N_\theta}) = L^\theta_{N_\theta}(\vec{\varphi}_{N_\theta}), \quad \forall \vec{\varphi}_{N_\theta} \in \vec{V}^\theta_{N_\theta}(\omega), \tag{10}$$

together with the initial conditions

$$\vec{w}_{\mathbf{N}}(0) = \vec{w}_{\mathbf{N}0}, \; \vec{w}'_{\mathbf{N}}(0) = \vec{w}_{\mathbf{N}1}, \; \vec{\zeta}_{N_\theta}(0) = \vec{\zeta}_{N_\theta 0}, \; \vec{\zeta}'_{N_\theta}(0) = \vec{\zeta}_{N_\theta 1}, \tag{11}$$

where $\vec{V}_{\mathbf{N}}(\omega) = \{\vec{v}_{\mathbf{N}} = (\overset{r_i}{v}_{\mathbf{N}i}) \in [H^1_{loc}(\omega)]^{N_{1,2,3}};\ \|\vec{v}_{\mathbf{N}}\|_* < \infty,\ \ tr_{\tilde{\gamma}}(\overset{r_i}{v}_{\mathbf{N}i}) = 0 \text{ on } \tilde{\gamma}_0,\ r_i = \overline{0,N_i},\ i = \overline{1,3}\}$,

$\vec{V}^2_{\mathbf{N}}(\omega) = \{\vec{v}_{\mathbf{N}} = (\overset{r_i}{v}_{\mathbf{N}i}) \in [H^2_{loc}(\omega)]^{N_{1,2,3}} \cap \vec{V}_{\mathbf{N}}(\omega); \|\vec{v}_{\mathbf{N}}\|_{**} < \infty\}$, $\vec{H}_{\mathbf{N}}(\omega) = \{\vec{v}_{\mathbf{N}} = (\overset{r_i}{v}_{\mathbf{N}i}) \in [L^2(\omega)]^{N_{1,2,3}};$

$\|\vec{v}_{\mathbf{N}}\|^2_{\vec{H}_{\mathbf{N}}(\omega)} = \sum_{i=1}^3 \sum_{r_i=0}^{N_i} \left\| h^{-1/2} \overset{r_i}{v}_{\mathbf{N}i} \right\|^2_{L^2(\omega)} < \infty\}$, $\vec{V}^\theta_{N_\theta}(\omega) = \{\vec{\varphi}_{N_\theta} = (\overset{r}{\varphi}_{N_\theta}) \in [H^1_{loc}(\omega)]^{N_\theta+1}; \|\vec{\varphi}_{N_\theta}\|_{\theta*} < \infty,\ \ tr_{\tilde{\gamma}}(\overset{r}{\varphi}_{N_\theta}) = 0$ on $\tilde{\gamma}_0, r = \overline{0, N_\theta}\}$, $\vec{V}^{\theta,2}_{N_\theta}(\omega) = \{\vec{\varphi}_{N_\theta} = (\overset{r}{\varphi}_{N_\theta}) \in [H^2_{loc}(\omega)]^{N_\theta+1} \cap \vec{V}^\theta_{N_\theta}(\omega); \|\vec{\varphi}_{N_\theta}\|_{\theta**} < \infty\}$,

$\vec{H}^\theta_{N_\theta}(\omega) = \{\vec{\varphi}_{N_\theta} = (\overset{r}{\varphi}_{N_\theta}) \in [L^2(\omega)]^{N_\theta+1};\ \|\vec{\varphi}_{N_\theta}\|^2_{\vec{H}^\theta_{N_\theta}(\omega)} = \sum_{r=0}^{N_\theta} \left\| h^{-1/2} \overset{r}{\varphi}_{N_\theta} \right\|^2_{L^2(\omega)} < \infty\}$, the bilinear forms $R_{\mathbf{N}}$, $R^\theta_{N_\theta}$, $a_{\mathbf{N}}$, $a^\theta_{N_\theta}$, $b_{\mathbf{N}N_\theta}$, $b^\theta_{\mathbf{N}N_\theta}$ are defined by the corresponding forms in the left-hand sides of the equations (9), (10) and by taking account of the properties of Legendre polynomials, we obtain the following explicit expressions

$$R_{\mathbf{N}}(\vec{y}_{\mathbf{N}}, \vec{v}_{\mathbf{N}}) = \sum_{i=1}^3 \sum_{r_i=0}^{N_i} \left(r_i + \frac{1}{2}\right) \rho \int_\omega \frac{1}{h} \overset{r_i}{y}_{\mathbf{N}i} \overset{r_i}{v}_{\mathbf{N}i}\, d\omega, \quad R^\theta_{N_\theta}(\vec{\psi}_{N_\theta}, \vec{\varphi}_{N_\theta}) = \sum_{r=0}^{N_\theta} \left(r + \frac{1}{2}\right) \chi \int_\omega \frac{1}{h} \overset{r}{\psi}_{N_\theta} \overset{r}{\varphi}_{N_\theta}\, d\omega,$$

$$a_{\mathbf{N}}(\vec{y}_{\mathbf{N}}, \vec{v}_{\mathbf{N}}) = \sum_{r=0}^{N_{\max}} \left(r + \frac{1}{2}\right) \int_\omega \frac{1}{h} \left(\lambda \sum_{p=1}^3 \overset{r}{e}_{pp}(\vec{y}_{\mathbf{N}}) \sum_{q=1}^3 \overset{r}{e}_{qq}(\vec{v}_{\mathbf{N}}) + 2\mu \sum_{i,j=1}^3 \overset{r}{e}_{ij}(\vec{y}_{\mathbf{N}}) \overset{r}{e}_{ij}(\vec{v}_{\mathbf{N}}) \right) d\omega,$$

$$a^\theta_{N_\theta}(\vec{\psi}_{N_\theta}, \vec{\varphi}_{N_\theta}) = \sum_{r=0}^{N_\theta} \left(r + \frac{1}{2}\right) \kappa \int_\omega \left[\frac{1}{h^3} \left(\sum_{s=r}^{N_\theta} (s + \frac{1}{2}) \overset{s}{\psi}_{N_\theta} (1 - (-1)^{r+s}) \right) \left(\sum_{\hat{s}=r}^{N_\theta} (\hat{s} + \frac{1}{2}) \overset{\hat{s}}{\varphi}_{N_\theta} (1 - (-1)^{r+\hat{s}}) \right) + \right.$$

$$+\sum_{\alpha=1}^{2}\frac{1}{h}\left(\partial_\alpha \overset{r}{\psi}_{N_\theta}-(r+1)\frac{\partial_\alpha h}{h}\overset{r}{\psi}_{N_\theta}-\sum_{s=r+1}^{N_\theta}\frac{\overset{s}{\psi}_{N_\theta}}{h}\left(s+\frac{1}{2}\right)\left(\partial_\alpha h^+-(-1)^{r+s}\partial_\alpha h^-\right)\right)\times$$

$$\times\left(\partial_\alpha \overset{r}{\varphi}_{N_\theta}-(r+1)\frac{\partial_\alpha h}{h}\overset{r}{\varphi}_{N_\theta}-\sum_{\hat{s}=r+1}^{N_\theta}\frac{\overset{\hat{s}}{\varphi}_{N_\theta}}{h}\left(\hat{s}+\frac{1}{2}\right)\left(\partial_\alpha h^+-(-1)^{r+\hat{s}}\partial_\alpha h^-\right)\right)\Bigg]d\omega,$$

$$b_{\mathbf{N}N_\theta}(\vec{\varphi}_{N_\theta},\vec{v}_{\mathbf{N}})=b^\theta_{\mathbf{N}N_\theta}(\vec{v}_{\mathbf{N}},\vec{\varphi}_{N_\theta})=\eta\sum_{r=0}^{N_\theta}\left(r+\frac{1}{2}\right)\int_\omega\left[\frac{1}{h^2}\left(\sum_{s=r}^{N_3}(s+\frac{1}{2})\overset{s}{v}_{\mathbf{N}3}(1-(-1)^{r+s})\right)+\right.$$

$$\left.+\sum_{\alpha=1}^{2}\frac{1}{h}\left(\partial_\alpha \overset{r}{v}_{\mathbf{N}\alpha}-(r+1)\frac{\partial_\alpha h}{h}\overset{r}{v}_{\mathbf{N}\alpha}-\sum_{s=r+1}^{N_\alpha}\frac{\overset{s}{v}_{\mathbf{N}\alpha}}{h}\left(s+\frac{1}{2}\right)\left(\partial_\alpha h^+-(-1)^{r+s}\partial_\alpha h^-\right)\right)\right]\overset{r}{\varphi}_{N_\theta}\,d\omega,$$

where $N_{\max}=\max\{N_1,N_2,N_3\}$, $\overset{r}{e}_{ij}(\vec{v}_{\mathbf{N}})=\frac{1}{2}\left(\partial_i(\overset{r}{v}_{\mathbf{N}j})+\partial_j(\overset{r}{v}_{\mathbf{N}i})+\overset{r}{\tilde{e}}_{ij}(\vec{v}_{\mathbf{N}})\right)$, $i,j=1,2,3$,

$$\overset{r}{\tilde{e}}_{ij}(\vec{v}_{\mathbf{N}})=-\frac{r+1}{h}\left(\partial_i h\,\overset{r}{v}_{\mathbf{N}j}+\partial_j h\,\overset{r}{v}_{\mathbf{N}i}\right)-\sum_{s=r+1}^{N_{\max}}\frac{1}{h}\left(s+\frac{1}{2}\right)\left(\overset{s}{v}_{\mathbf{N}j}(\partial_i h^+-(-1)^{r+s}\partial_i h^-)+\right.$$

$$\left.+\overset{s}{v}_{\mathbf{N}i}(\partial_j h^+-(-1)^{r+s}\partial_j h^-)\right)+\sum_{s=r}^{N_{\max}}\frac{1}{h}\left(s+\frac{1}{2}\right)(1-(-1)^{r+s})\left(\frac{(i-1)(i-2)}{2}\overset{s}{v}_{\mathbf{N}j}+\frac{(j-1)(j-2)}{2}\overset{s}{v}_{\mathbf{N}i}\right).$$

The linear forms $L_{\mathbf{N}}$, $L^\theta_{N_\theta}$ are defined by the right-hand sides of the equations (6), (7) and are given by

$$L_{\mathbf{N}}(\vec{v}_{\mathbf{N}})=\sum_{i=1}^{3}\sum_{r_i=0}^{N_i}\left(r_i+\frac{1}{2}\right)\left[\int_\omega\frac{1}{h}\overset{r_i}{v}_{\mathbf{N}i}\left(\overset{r_i}{f}_i+g^+_{\mathbf{N}i}\lambda_++g^-_{\mathbf{N}i}\lambda_-(-1)^{r_i}\right)d\omega+\int_{\gamma_1}\frac{1}{h}\overset{r_i}{v}_{\mathbf{N}i}\,\overset{r_i}{g}_{\mathbf{N}i}\,d\gamma_1\right],$$

$$L^\theta_{N_\theta}(\vec{\varphi}_{N_\theta})=\sum_{r=0}^{N_\theta}\left(r+\frac{1}{2}\right)\left[\int_\omega\frac{1}{h}\overset{r}{\varphi}_{N_\theta}\left(\overset{r}{f}{}^\theta+g^{\theta+}_{N_\theta}\lambda_++g^{\theta-}_{N_\theta}\lambda_-(-1)^r\right)d\omega+\int_{\gamma_1}\frac{1}{h}\overset{r}{\varphi}_{N_\theta}\,\overset{r}{g}{}^\theta_{N_\theta}\,d\gamma_1\right],$$

where $\gamma_1=\tilde{\gamma}\setminus\tilde{\gamma}_0$, $\lambda_\pm=\sqrt{1+(\partial_1 h^\pm)^2+(\partial_2 h^\pm)^2}$, $\overset{r}{v}=\int_{h^-}^{h^+}vP_r(z)dx_3$, for all functions $v\in L^2(\Omega)$, $r\in\mathbf{N}\cup\{0\}$, $g^+_{\mathbf{N}i}$, $g^{\theta+}_{N_\theta}$ and $g^-_{\mathbf{N}i}$, $g^{\theta-}_{N_\theta}$ are restrictions of

$$g_{\mathbf{N}i}(t)=g_i(t)+\sum_{j=1}^{3}\left(\lambda\sum_{p=1}^{3}e_{pp}(\mathbf{w}_{\mathbf{N}0})\delta_{ij}+2\mu e_{ij}(\mathbf{w}_{\mathbf{N}0})-\eta\zeta_{N_\theta 0}\delta_{ij}-\eta\tau_1\zeta_{N_\theta 1}\delta_{ij}\right)\nu_j\Bigg|_{\Gamma_1}-g_i(0),\ i=\overline{1,3},$$

$$g^\theta_{N_\theta}(t)=g^\theta(t)+\sum_{j=1}^{3}\kappa\frac{\partial\zeta_{N_\theta 0}}{\partial x_j}\nu_j\Bigg|_{\Gamma_1}-g^\theta(0),$$

on the upper Γ^+ and the lower Γ^- faces of the shell, respectively, $\mathbf{w}_{\mathbf{N}0}\in\mathbf{V}^2_{\mathbf{N}}(\Omega)$, $\zeta_{N_\theta 0}\in V^{\theta,2}_{N_\theta}(\Omega)$, $\zeta_{N_\theta 1}\in V^\theta_{N_\theta}(\Omega)$ correspond to the initial data $\vec{w}_{\mathbf{N}0}$, $\vec{\zeta}_{N_\theta 0}$, $\vec{\zeta}_{N_\theta 1}$ of the two-dimensional problem.

For the constructed two-dimensional initial-boundary value problems (9)-(11) the following existence and uniqueness theorem is proved.

Theorem 2. *If two-dimensional domain ω and functions h^+, h^- are such that $\Omega\subset\mathbf{R}^3$ is a Lipschitz domain, $\mu>0$, $3\lambda+2\mu>0$, $0<\tau_0\le\tau_1$, the functions $\overset{r_i}{f}_i$, $\overset{r_i}{g}_{\mathbf{N}i}$, $g^\pm_{\mathbf{N}i}$ $(r_i=\overline{0,N_i},i=\overline{1,3})$, $\overset{r}{f}{}^\theta$, $\overset{r}{g}{}^\theta_{N_\theta}$ $(r=\overline{0,N_\theta})$, $g^{\theta\pm}_{N_\theta}$ satisfy the following conditions*

$$h^{-1/2}\overset{r_i}{f}_i, h^{-1/2}(\overset{r_i}{f}_i)' \in L^2(0,T;L^2(\omega)),\quad \lambda_\pm^{3/4} g^\pm_{\mathbf{N}i}, \lambda_\pm^{3/4}(g^\pm_{\mathbf{N}i})', \lambda_\pm^{3/4}(g^\pm_{\mathbf{N}i})'' \in L^2(0,T;L^{4/3}(\omega)),$$

$$h^{-1/4}\overset{r_i}{g}_{\mathbf{N}i}, h^{-1/4}(\overset{r_i}{g}_{\mathbf{N}i})', h^{-1/4}(\overset{r_i}{g}_{\mathbf{N}i})'' \in L^2(0,T;L^{4/3}(\gamma_1)),\ r_i=\overline{0,N_i},\quad i=\overline{1,3},$$

$$h^{-1/2}\overset{r}{f}{}^\theta, h^{-1/2}(\overset{r}{f}{}^\theta)' \in L^2(0,T;L^2(\omega)),\ \lambda_\pm^{3/4} g^{\theta\pm}_{N_\theta}, \lambda_\pm^{3/4}(g^{\theta\pm}_{N_\theta})', \lambda_\pm^{3/4}(g^{\theta\pm}_{N_\theta})'' \in L^2(0,T;L^{4/3}(\omega)),$$

$$h^{-1/4}\overset{r}{g}{}^\theta_{N_\theta}, h^{-1/4}(\overset{r}{g}{}^\theta_{N_\theta})', h^{-1/4}(\overset{r}{g}{}^\theta_{N_\theta})'' \in L^2(0,T;L^{4/3}(\gamma_1)),\ r=\overline{0,N_\theta},$$

and $\vec{w}_{\mathbf{N}0} \in \vec{V}^2_{\mathbf{N}}(\omega)$, $\vec{w}_{\mathbf{N}1} \in \vec{V}_{\mathbf{N}}(\omega)$, $\vec{\zeta}_{N_\theta 0} \in \vec{V}^{\theta,2}_{N_\theta}(\omega)$, $\vec{\zeta}_{N_\theta 1} \in \vec{V}^{\theta}_{N_\theta}(\omega)$, *then the dynamical two-dimensional problem (9)-(11) possesses a unique solution.*

Thus, we have constructed a two-dimensional hierarchical model of thermoelastic plate with variable thickness on the basis of Green-Lindsay nonclassical three-dimensional model for thermoelastic bodies. In the following theorem we present the results on the relationship between the obtained two-dimensional and original three-dimensional models, but in order to formulate the theorem let us define the following anisotropic weighted Sobolev spaces

$$H^{0,0,s}_{h^\pm}(\Omega)=\{v; h^k\partial_3^k v\in L^2(\Omega),\ 0\le k\le s\},\ s\in\mathbf{N},$$

$$H^{1,1,s}_{h^\pm}(\Omega)=\{v; h^{k-1}\partial_3^{k-1}\partial_i^r v\in L^2(\Omega), h^{k-1}\partial_\alpha h^\pm \partial_3^k v\in L^2(\Omega), 1\le k\le s, r=0,1, i=1,2,3, \alpha=1,2\},$$

$$\tilde{H}^{1,1,s+1}_{h^\pm}(\Omega)=\{v; h^{k-1}\partial_3^{k-1}\partial_i^r\partial_j^{\tilde r} v\in L^2(\Omega), h^{k-1}\partial_\alpha h^\pm\partial_3^k\partial_i^r v\in L^2(\Omega), h^{k-1}\partial_\alpha\partial_\beta h^\pm\partial_3^k v\in L^2(\Omega),$$

$$1\le k\le s, h^{\tilde k-2}\partial_\alpha h^\pm\partial_\beta h^\pm\partial_3^{\tilde k} v\in L^2(\Omega), 1\le \tilde k\le s+1, \alpha,\beta=1,2, r,\tilde r=0,1,\ 1\le i,j\le 3\},$$

which are Hilbert spaces with respect to the corresponding norms.

Theorem 3. If $\mu>0$, $3\lambda+2\mu>0$, $0<\tau_0\le\tau_1$, $\mathbf{u}_0\in\mathbf{H}^2(\Omega)\cap\mathbf{V}(\Omega)$, $\mathbf{u}_1\in\mathbf{V}(\Omega)$, $\theta_0\in H^2(\Omega)\cap V^\theta(\Omega)$, $\theta_1\in V^\theta(\Omega)$, $\mathbf{f},\mathbf{f}'\in L^2(0,T;\mathbf{L}^2(\Omega))$, $\mathbf{g},\mathbf{g}',\mathbf{g}''\in L^2(0,T;\mathbf{L}^{4/3}(\Gamma_1))$, $f^\theta, f^{\theta'}\in L^2(0,T;L^2(\Omega))$, $g^\theta, g^{\theta'}, g^{\theta''}\in L^2(0,T;L^{4/3}(\Gamma_1))$ *and vector-functions of three space variables* $\mathbf{w}_{\mathbf{N}0}\in\mathbf{V}^2_{\mathbf{N}}(\Omega)$, $\mathbf{w}_{\mathbf{N}1}\in\mathbf{V}_{\mathbf{N}}(\Omega)$, $\zeta_{N_\theta 0}\in V^{\theta,2}_{N_\theta}(\Omega)$, $\zeta_{N_\theta 1}\in V^{\theta}_{N_\theta}(\Omega)$, *corresponding to the initial conditions* $\vec{w}_{\mathbf{N}0}\in\vec{V}^2_{\mathbf{N}}(\omega)$, $\vec{w}_{\mathbf{N}1}\in\vec{V}_{\mathbf{N}}(\omega)$, $\vec{\zeta}_{N_\theta 0}\in\vec{V}^{\theta,2}_{N_\theta}(\omega)$, $\vec{\zeta}_{N_\theta 1}\in\vec{V}^{\theta}_{N_\theta}(\omega)$ *of the two-dimensional problems, tend to* $\mathbf{u}_0$, $\mathbf{u}_1$, θ_0 *and* θ_1 *in the spaces* $\mathbf{H}^2(\Omega)$, $\mathbf{L}^2(\Omega)$, $H^2(\Omega)$ *and* $H^1(\Omega)$, *respectively, as* $N_{\min}=\min\limits_{1\le i\le 3}\{N_i,N_\theta\}\to\infty$, *then the sequence of vector-functions* $\{\mathbf{w}_{\mathbf{N}}\}$ *and functions* $\{\zeta_{N_\theta}(t)\}$ *restored from the solutions* $\vec{w}_{\mathbf{N}}$ *and* $\vec{\zeta}_{N_\theta}$ *of the reduced two-dimensional problem (9)-(11), tend to the solution of the original three-dimensional problem (6)-(8),*

$$\left.\begin{array}{ll}\mathbf{w}_{\mathbf{N}}(t)\to\mathbf{u}(t) & \text{in } \mathbf{H}^1(\Omega),\\ \mathbf{w}'_{\mathbf{N}}(t)\to\mathbf{u}'(t) & \text{in } \mathbf{L}^2(\Omega),\\ \zeta_{N_\theta}(t)\to\theta(t) & \text{in } H^1(\Omega),\\ \zeta'_{N_\theta}(t)\to\theta'(t) & \text{in } L^2(\Omega),\end{array}\right.\qquad \textit{for all } t\in[0,T],\ \textit{as}\ N_{\min}\to\infty.$$

In addition, if $d^r\mathbf{u}/dt^r\in L^2(0,T;\,(H^{1,1,s_r}_{h^\pm}(\Omega))^3)$, $r=0,1$, $\mathbf{u}''\in L^2(0,T;\,(H^{0,0,s_2}_{h^\pm}(\Omega))^3)$, $d^r\theta/dt^r\in L^2(0,T;\,H^{1,1,s_r^\theta}_{h^\pm}(\Omega))$, $r=0,1$, $\theta''\in L^2(0,T;\,H^{0,0,s_2^\theta}_{h^\pm}(\Omega))$, $s_k, s_k^\theta\in\mathbf{N}$, $k=\overline{0,2}$, $s_0,s_0^\theta,s_1,s_1^\theta\ge 2$, *and* $\mathbf{u}_0\in(\tilde{H}^{1,1,\tilde s_0+1}_{h^\pm}(\Omega))^3$, $\mathbf{u}_1\in(H^{1,1,2}_{h^\pm}(\Omega))^3$, $\theta_0\in\tilde{H}^{1,1,\tilde s_0^\theta+1}_{h^\pm}(\Omega)$, $\theta_1\in H^{1,1,\tilde s_1^\theta}_{h^\pm}(\Omega)$, $\tilde s_0,\tilde s_0^\theta,\tilde s_1^\theta\in\mathbf{N}$, $\tilde s_0,\tilde s_0^\theta\ge 2$, *then for appropriate initial condition* $\vec{w}_{\mathbf{N}0}$, $\vec{w}_{\mathbf{N}1}$, $\vec{\zeta}_{N_\theta 0}$, $\vec{\zeta}_{N_\theta 1}$ *the following estimate is valid*

$$\|\mathbf{u}'-\mathbf{w}'_{\mathbf{N}}\|_{C^0([0,T];\mathbf{L}^2(\Omega))}+\|\mathbf{u}-\mathbf{w}_{\mathbf{N}}\|_{C^0([0,T];\mathbf{H}^1(\Omega))}+\|\theta'-\zeta'_{N_\theta}\|_{C^0([0,T];L^2(\Omega))}+$$

$$+\|\theta-\zeta_{N_\theta}\|_{C^0([0,T];H^1(\Omega))}\le\frac{1}{(N_{\min})^s}\,o(T,\Omega,\tilde\Gamma_0,h^\pm,\mathbf{N},N_\theta),$$

where $s=\min\{s_0-1,s_1-1,s_2,s_0^\theta-1,s_1^\theta-1,s_2^\theta,\tilde s_0-3/2,\tilde s_0^\theta-3/2,\tilde s_1^\theta-1\}$ *and* $o(T,\Omega,\tilde\Gamma_0,h^\pm,\mathbf{N},\ N_\theta)\to 0$, *as* $N_{\min}\to\infty$.

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ცვალებადი სისქის ფირფიტების ზოგიერთი არაკლასიკური ორგანზომილებიანი მოდელის შესახებ

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(წარმოდგენილია აკადემიის წევრის რ. ბანცურის მიერ)

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