Astronomy

On the Regularization of the Fragments' Orbits Created by Explosion of Geostationary Satellites

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ABSTRACT. The phenomenon of regularization of geostationary satellite's fragments created after its explosion has been investigated. The formula for calculation of the moments of minimal dispersion of the fragments' orbital poles positions has been received. *© 2010 Bull. Georg. Natl. Acad. Sci.*

Key words: *satellite, explosion, fragments, orbits, regularization.*

Introduction. The fragments created by explosion of geostationary satellites (GS) scatter in space in every direction. Their space orientation at the initial moment is subject to a certain regularity but in the process of evolution it gradually assumes random character.

For 12 GS the evolution of the behavior of ensembles consisting of 32 fragments ejected by explosion at velocity of 75 m/s has been studied on the basis of their motion theory [1–6].

Computer modeling of orbital evolution shows that many years after explosion the orientations of the orbits of fragments are ordered anew. This phenomenon was called regularization of the orbits of fragments [7–9].

We shall discuss a simplified variant of regularization: we assume that before explosion the GS moved on circular orbit; we shall introduce other simplifications later.

Initial equations. To detrermine the geometric sense of regularization let us discuss Fig. 1 [8, 9], which represents a projection of part of the celestial sphere on tangent plane in the vicinity of the North pole of Laplace plane.

After explosion every fragment gains its own Laplace plane pole. These poles are disposed on a great circle of the sky passing celestial poles and the initial Laplace plane pole, which GS had before explosion.

The fragments analogously gain their own poles of orbital plane in the vicinity of the initial orbital pole.

Just after the moment of explosion, the poles of the fragments' orbits begin precession around their own Laplace plane's poles. Later we shall see that the speeds of precession for different fragments slightly differ from one another.

The inclination of Laplace plane to the equator is described by formula [10]:

$$\mathrm{tg}2\beta = \frac{\kappa \sin 2\varepsilon}{4\pi J_{20}\left(\frac{a_E}{a}\right)^2 n^2 + \kappa \cos 2\varepsilon}, \qquad (1)$$

where

$$\kappa = \frac{n_L^2 m_L}{m_E + m_L} + \frac{n_S^2 m_S}{m_E + m_L + m_S},$$

m_E, m_L and m_S consequently denote the masses of the Earth, the Moon and the Sun, a and a_E are the radii of GS orbit and the Earth and n, n_L, n_S - diurnal motions of GS, the Moon and the Sun, expressed in radians.

For GS

$$\beta = 7.34°. \qquad (2)$$

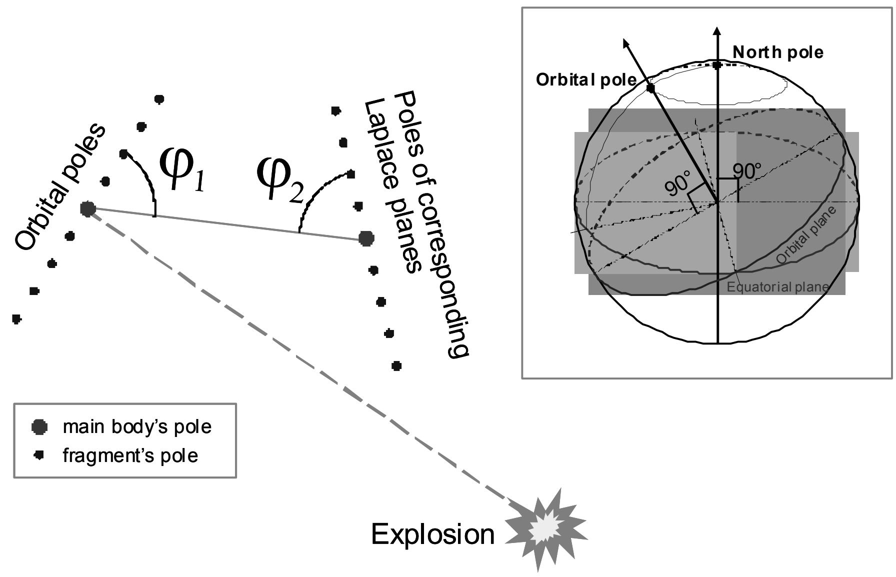

Fig. 1. The dispposition of poles of GS fragments orbits and Laplace planes at the moment of explosion.

The differentiation of (1) with respect to variable a, and substitution in the result of the typical parameters for geostationary orbit (a = 6.63 a_E) gives:

$$\frac{d\beta}{da} = 0.0695 = 3.98°. \qquad (3)$$

To determine the variation of the orbital elements of fragments because of explosion let us use the energy integral of two-body problem:

$$V^2 = k^2 m_E \left(\frac{2}{r} - \frac{1}{a}\right), \qquad (4)$$

where k denotes the gravitational constant and r the distance between the Earth and GS.

Because GS was moving on the circular orbit before the explosion, at that time $r = a$.

At the moment of explosion only the quantities V and a vary in (4). Hence, the differentiation of (4) with respect to variable a brings about the replacement of the semi-major axis of fragment's orbit with

$$\Delta a = \frac{2a}{V_c} \Delta V_T, \qquad (5)$$

where V_c and ΔV_T respectively denote the circular velocity of the GS and the tangent component of the speed of fragment's ejection ΔV.

On the other hand, the inclination of the fragment's orbit to the initial orbit of GS is equal to:

$$\Delta i = \frac{\Delta V_p}{V_c}, \qquad (6)$$

where ΔV_p denotes the polar component of ΔV velocity.

The simplification of (5) and (6) is possible because of smallness of the speed of fragments' ejection relative to the circular velocity on the orbit (V_c = 3.07 km/s). From expressions (5) and (6) it is clear that Δa and Δi are independent quantities but their values are restricted by the inequality:

$$\Delta V^2 \geq \Delta V_T^2 + \Delta V_p^2, \qquad (7)$$

Substituting in (7) expressions (3), (5) and (6) one can receive:

$$L = 1.32°, \ F = 1.43°,$$

where L and F respectively denote the maximal values of $\Delta\beta$ and Δi.

Further simplification of the problem is possible using Lagrange equations [6, 9, 11]:

$$\frac{dz}{dt} = \dot{z} + \frac{m \cos 2i \cos z}{\sin i}, \quad \frac{di}{dt} = m \cos i \sin z, \qquad (8)$$

where

$$z = \Omega - \Omega_L, \ \ m = 6.18 \cdot 10^{-6}, \ \ \dot{\Omega} = -3.2 \cdot 10^{-4},$$
$$\dot{\Omega}_L = -9.2 \cdot 10^{-4}, \ \ \dot{z} = (9.2 - 3.2 \cos i) \cdot 10^{-4}. \qquad (9)$$

Ω and Ω_L respectively denote the longitudes of these nodes of GS and lunar orbits, i is the inclination of the GS orbit to the Laplace plane and $\dot{z}$, $\dot{\Omega}$ and $\dot{\Omega}_L$ denote the middle values of corresponding variables' derivatives with respect to time.

So, the smallness of m allows us to propose that because of precession, the inclination i and the velocity $\dot{z}$ (as well as $\dot{\Omega}$) are constants.

The evolution of the poles of fragments' orbital planes. In the case of the above-mentioned simplifications the evolution of fragments' orbital planes assumes the following form.

Let us fit the origin of co-ordinates with the pole of the initial Laplace plane and the *X*-axis direct to the orbital pole of GS, whose *X*-coordinate we denote by the *D*(0) symbol.

Let us note that

$$D(0) = tgi, \quad \varphi_1 = u, \quad \varphi_2 = \Omega, \qquad (10)$$

where the argument of the latitude for the point of explosion *u* and the longitude of the GS orbital node is counted relative to GS Laplace plane.

In such co-ordinate system the co-ordinates of the Laplace plane pole for some fragment can be described as follows:

$$x_0(\psi) = L\sin\psi\cos\Omega, \quad y_0(\psi) = L\sin\psi\sin u. \qquad (11)$$

where ψ denotes a parameter proportional to the value of ΔV_T.

Let us respectively denote the length of the segment between this pole and the GS orbital pole and the angle with X-axis and segment by $D(\psi)$ and $\varepsilon(\psi)$ symbols. Then

$$D(\psi) = \sqrt{tg^2 i - 2Ltgi\sin\psi\cos\Omega + L^2\sin^2\psi},$$
$$D(\psi)\sin\varepsilon(\psi) = y_0(\psi), \quad D(\psi)\cos\varepsilon(\psi) = tgi - x_0(\psi). \qquad (12)$$

From inequality (7) we have that the orbital pole of this fragment will be deployed on the straight segment between the points with co-ordinates:

$$x_{1,2}(0) = tgi \mp F\cos\psi\cos\Omega,$$
$$y_{1,2}(0) = \pm F\cos\psi\sin u. \qquad (13)$$

As a result of the precession every such segment will turn around its own center (having $x_0(\psi)$ and $y_0(\psi)$ coordinates) by the same angle:

$$\theta = \dot{\Omega}t, \qquad (14)$$

where *t* denotes the time after the explosion.

As a result of turning the coordinates of the segment's ends will become:

$$x_{1,2}(\theta) = tgi\cos\theta + L\sin\psi[\cos\Omega - \cos(\Omega-\theta)] \mp$$
$$\mp F\cos\psi\cos(u+\theta),$$
$$y_{1,2}(\theta) = -tgi\sin\theta + L\sin\psi[\sin\Omega - \sin(\Omega-\theta)] \pm \qquad (15)$$
$$\pm F\cos\psi\sin(u+\theta).$$

When ψ varies between 0 and 2π, the poles of Laplace planes pass all the possible positions and the contour consisting of the geometric places of points $x_{1,2}(\theta)$, $y_{1,2}(\theta)$ restricts the area occupied by the fragments' orbital poles in *x*, *y* phase plane.

The system (15) is an equation of ellipse in parametric form. In the case when its semi-minor axis becomes zero, the ellipse degenerates into a segment and we shall obtain the regularization of the fragments' orbits.

To solve this problem let us write the equation for the radius of the ellipse based on system (15):

$$R^2(\psi) = 2L^2\sin^2\psi(1-\cos\theta) + F^2\cos^2\psi +$$
$$+LF\sin 2\psi[\cos(u+\Omega) - \cos(u+\Omega-\theta)]. \qquad (16)$$

Equation to zero of the partial derivative of (16) with respect to θ-parameter gives us the value of ψ_e, corresponding to the extreme of the ellipse's radius:

$$tg\psi_e = -\frac{F\sin(u+\Omega+\theta_e)}{L\sin\theta_e}. \qquad (17)$$

Substituting expression (17) in (16) and equating its left hand side to zero, after simplification we shall obtain:

$$\cos(2u + 2\Omega + \theta_e) = -1. \qquad (18)$$

hence

$$t_e = \frac{(2n+1)\pi - u - \Omega}{\dot{\Omega}}, \qquad (19)$$

where *n* is a whole number.

In conditions of our simplifications the equality (19) is the necessary and enough condition for the regularization of GS fragments' orbits. Taking the ignored terms into consideration is followed by the bending of ellipse (15) and as a result the dispersion of fragments' orbital poles grows in the course of time and can never become zero.

ასტრონომია

გეოსტაციონარული თანამგზავრის აფეთქებისას წარმოქმნილ ფრაგმენტთა ორბიტების რეგულარიზაციის შესახებ

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