საქართველოს მეცნიერებათა ეროვნული აკადემიის მოამბე, ტ. 4, №3, 2010
BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, vol. 4, no. 3, 2010

Mechanics

Study of the Influence of Clearances on the Dynamical Load of Machine Systems

Robert Adamia*, Svetlana Chagelishvili, Tengiz Nadiradze****

* *Academy Member, R. Dvali Institute of Machine Mechanics, Tbilisi*
** *R. Dvali Institute of Machine Mechanics, Tbilisi*

ABSTRACT. A generalized method of calculation of dynamic characteristics in transmissions of machine aggregates at different stages of machine starting on power transmissions of the main electrical drive with account of the co-existence of clearances is prepared. Analytical dependence between the sizes of gaps and additional forces (of shock character) caused by their influence is determined. *© 2010 Bull. Georg. Natl. Acad. Sci.*

Key words: *machine aggregates, clearance, impact forces, dynamical loads, partial frequency.*

The formation of clearances in main driving lines of mechanical systems is due to presence of spindles, gear and helical joints in transmissions of machine aggregates. Magnitudes of clearances are in direct proportional dependence to the life-time of these machines: clearances cause generation of percussive forces; which in their turn are sources of dynamical loads and they lead to accelerated fatigue of main units of machine aggregates (spindle, gear, cages, reducing gears and other details). Additional dynamical loads caused by clearances may exceed several times the static forces generated by the basic technologic regimes. Influence of clearances is especially significant for aggregates operating in reverse regimes, namely for rolling mills and especially for sheet rolling machines of high efficiency, where seizure of metal by rollers is realized at high speeds and always is characterized by significant percussive phenomena.

It is evident that new rolling machines must be designed considering additional impact forces caused by clearances. Unfortunately, reference material is not available in technical literature till today, which would enable us to establish analytical relationship between magnitudes of clearances and additional impact forces caused by them.

The present research aims to eradicate this shortcoming to a certain extent. Consider dynamical loads in transmissions with clearances at starting of a machine, for the case when reduced calculating scheme of a machine aggregate is a three-mass mechanical system (Fig. 1).

The following designations are adopted in the drawing: I_1 – moment of inertia of the electric motor rotor; I_2 – moment of inertia of intermediate masses (gear cage, reducing gear, flywheels, etc.) reduced to the electro motor shaft; I_3 – mass moment of inertia of the machine aggregate rollers, reduced to the electro motor shaft; ${}_1$, ${}_2$ – reduced radial clearances in corresponding sections. We assume that mass I_1 is acted on by driving moment M_1 $const$ and other masses – by corresponding moments of resistance M_2 and M_3.

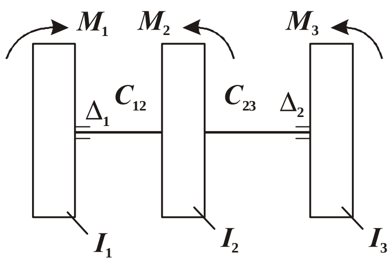

Fig. 1.

At the first stage, when the electromotor rotor starts motion, other masses of the system under consideration are immobile. In this case the differential equation of movement has the simplest form:

$$I_1 \frac{d^2\varphi}{dt^2} \quad M_1 . \tag{1}$$

At constant driving moment of the electromotor and zero initial conditions, expressions for the rotor angular velocity ω_1 and angular displacement φ_1 are found from equation (1):

$$\omega_1 \quad \varepsilon_0 t\ , \ \ \varphi_1 \quad \varepsilon_0 t^2 / 2\ , \tag{2}$$

where $\varepsilon_0 \quad \frac{M_1}{I_1}$ is the rotor angular acceleration at constant acceleration of the system. At passing the clearance of magnitude $\ _1$ in the time interval T_{01}, the electromotor is accelerated up to certain magnitude ω_{1init}.

If we assume that $t \quad T_0$ and $\varphi_1 \quad \ _1$ for equation (2), then we obtain an expression defining ω_{1init}:

$$\omega_{1init} \quad \sqrt{\frac{2M_1 \ _1}{I_1}} \quad \sqrt{2\varepsilon_0 \ _1}\ . \tag{3}$$

The process of passing the clearance is determined by elastic percussion at the end of the first stage, since mass I_2 was immobile.

At the second stage, which begins after closing of the clearance in the first elastic section, differential equations of movement of the system will have the form (At this stage, mass I_2 and elastic section connected to it, begin movement):

$$\begin{aligned} & I_1 \frac{d^2\varphi_1}{dt^2} \quad C_{12}(\varphi_1 \quad \varphi_2) \quad M_1 \\ & I_2 \frac{d^2\varphi_2}{dt^2} \quad C_{12}(\varphi_1 \quad \varphi_2) \quad M_2 \end{aligned} \tag{4}$$

with the following initial conditions:

$$t_1 \quad 0; \quad \varphi_1 \quad \varphi_2 \quad 0\ ; \quad \frac{d\varphi_1}{dt_1} \quad \omega_{1init}; \quad \frac{d\varphi_2}{dt_1} \quad 0 \tag{5}$$

After solution of differential equations (4), initial conditions (5) we determine the magnitude of $\varphi_2, \omega_1, \omega_2$, $M_{12} \quad C_{12}(\varphi_1 \quad \varphi_2)$.

$$\begin{aligned} & \varphi_1 \quad \frac{M_a}{I_1 p^2}(1 \quad \cos pt_1) \quad \frac{I_2\omega_{1init}}{(I_1 \quad I_2)p}\sin pt_1 \quad \frac{I_2\omega_{1init}}{I_1 \quad I_2}t_1 \quad \varepsilon_{ar}^{(2)} \\ & \varphi_2 \quad \frac{M_a}{I_2 p^2}(1 \quad \cos pt_1) \quad \frac{I_1\omega_{1init}}{(I_1 \quad I_2)}\sin pt_1 \quad \frac{I_1\omega_{1init}}{I_1 \quad I_2}t_1 \quad \varepsilon_{ar}^{(2)}\frac{t_1^2}{2} \\ & \omega_1 \quad \frac{M_a}{I_1 p}\sin pt_1 \quad \frac{I_2\omega_{1init}}{I_1 \quad I_2}\cos pt_1 \quad \frac{I_1\omega_{1init}}{I_1 \quad I_2} \quad \varepsilon_{ar}^{(2)} t_1 \\ & \omega_2 \quad \frac{M_a}{\theta_2 p}\sin pt_1 \quad \frac{I_1\omega_{1init}}{I_1 \quad I_2}\cos pt_1 \quad \frac{I_1\omega_{1init}}{I_1 \quad I_2} \quad \varepsilon_{ar}^{(2)} t_1 \\ & M_{12} \quad M_a(1 \quad \cos pt_1) \quad \frac{\omega_{1init}C_{12}}{p}\sin pt_1 \end{aligned} \quad , \tag{6}$$

where $M_a \quad \frac{I_2 M_1 \quad I_1 M_2}{I_1 \quad I_2}$; $\varepsilon_{av}^{(2)} \quad \frac{M_1 \quad M_2}{I_1 \quad I_2}$ is average acceleration after the closing of the first clearance; p is partial frequency of proper oscillations, $p \quad \sqrt{\frac{C_{12}(I_1 \quad I_2)}{I_1 I_2}}$.

The above given expressions (6) enable us to determine the basic characteristics of the drive dynamical system after closing of the first clearance, when masses of the mechanical system perform complex rotary and vibratory movements. This latter is caused by the elasticity of the unit connecting masses I_1 and I_2.

The last expression in formulae (6) determines moment f elastic forces in the period of closing of the first clearance. The first term of this expression determines the moment of elastic forces caused by external forces acting on the system, and the second term – additional dynamical loads (of percussive character) generated by presence of the clearance. The second component of the moment of elastic forces can be analytically determined as follows:

$$M_{12} \quad \sqrt{2 M_1 I_1 C_{12} \quad \frac{I_2}{I_1 \quad I_2}} \sin p t_1 . \tag{7}$$

Determine the dynamic coefficient of the two-mass mechanical system considering the influence of the clearance. Its value is calculated by the ratio of maximum moment of elastic forces to moment M_a (moment M_a is a static load in the elastic unit in the case of lack of clearance):

$$K_d \quad 1 \quad \sqrt{1 \quad (\frac{\omega_{1init} c_{12}}{M_a p})^2} \quad 1 \quad \sqrt{1 \quad \frac{2 M_1 n_2 c_{12}}{M^2{}_a}} \quad {}_1 , \tag{8}$$

where $n_2 \quad \frac{I_2}{I_1 \quad I_2}$.

Reduction of dynamical loads caused by percussions can be practically reached by qualitative fabrication and mounting of details of connecting units. Minimization of percussive loads caused by the influence of clearances can be realized also with the help of regulation of the electromotor starting and braking according to a certain optimal law.

Formula (8) determines maximum value of the dynamic coefficient of two-mass mechanical systems. After closing of the first clearance the second mass of the mechanical system performs complex movement (rotary and vibratory). After a certain interval of time T_{02}, the second mass will pass clearance $_2$ and close the driving system on the third mass. Time T_{02} of passing of the clearance is calculated by solution of the transcendental equation and substitution of magnitudes $\varphi_2 =\ _2$ and $t_1 = T_{02}$ in expression (6), taking into account the uniformly accelerated and vibratory movement of mass I_2:

$$_2 \quad \frac{M_a}{I_2 p^2}(1 \quad \cos p T_{02}) \quad \frac{I_1 \omega_{1init}}{(I_1 \quad I_2) p} \sin p T_{02} \quad \frac{I_1 \omega_{1init}}{I_1 \quad I_2} T_{02} \quad \varepsilon_{ar}{}^{(2)} \frac{T_{02}{}^2}{2} . \tag{9}$$

Namely, parameters defining inertness and stiffness of rolling mills are chosen in the way that in such system $p \quad 100 \frac{1}{\text{sec}}$. This enables us to neglect the influence of the first and second terms in equation (9). Then T_{02} is calculated by solving the following quadratic equation:

$$\varepsilon_{ar}{}^{(2)} T_{02}{}^2 \quad \frac{2 I_1 \omega_{1init}}{I_1 \quad I_2} T_{02} \quad 2 \ _2 \quad 0 , \tag{10}$$

which has one root:

$$T_{02} \quad \frac{I_1\omega_{1init}}{I_1 \quad I_2} \quad \sqrt{\frac{I_1{}^2\omega^2{}_{1init}}{(I_1 \quad I_2)^2} \quad 2 \quad {}_2\varepsilon_{ar}^{(2)}} \quad \frac{1}{\varepsilon_{ar}^{(2)}}. \tag{11}$$

Approximate formulae (10) and (11) for calculation of T_{02} can be used for solution of the exact transcendental equation (9), from which we find the true magnitude of T_{02}.

At the third stage, after closing the clearance $_2$, the differential equations of the system movement will have the form:

$$\begin{array}{l} I_1\dfrac{d^2\varphi_1}{dt_2{}^2} \quad C_{12}(\varphi_1 \quad \varphi_2) \quad M_1 \\ I_2\dfrac{d^2\varphi_2}{dt^2} \quad C_{12}(\varphi_1 \quad \varphi_2) \quad C_{23}(\varphi_2 \quad \varphi_3) \quad M_2 \\ I_3\dfrac{d^2\varphi_3}{dt^2} \quad C_{23}(\varphi_2 \quad \varphi_3) \quad M_3 \end{array} . \tag{12}$$

We present equations (12) in another form:

$$\begin{array}{l} I_1\ddot{\varphi}_1 \quad I_2\ddot{\varphi}_2 \quad I_3\ddot{\varphi}_3 \quad M_1 \quad M_2 \quad M_3; \\ \ddot{\varphi}_1 \quad \ddot{\varphi}_2 \quad \dfrac{C_{12}\ I_1 \quad I_2}{I_1 I_2}\ \varphi_1 \quad \varphi_2 \quad \dfrac{C_{23}}{I_2}\ \varphi_2 \quad \varphi_3 \quad \dfrac{M_1}{I_1} \quad \dfrac{M_2}{I_2}; \\ \ddot{\varphi}_2 \quad \ddot{\varphi}_3 \quad \dfrac{C_{23}\ I_2 \quad I_3}{I_2 I_3}\ \varphi_2 \quad \varphi_3 \quad \dfrac{C_{12}}{I_2}\ \varphi_1 \quad \varphi_2 \quad \dfrac{M_2}{I_2} \quad \dfrac{M_3}{I_3} \end{array} . \tag{13}$$

We use the rule of transformation of coordinates:

$$\begin{array}{l} \varphi_1 \quad \bar{\xi} \quad \dfrac{I_2 \quad I_3}{I_0}\theta_{12} \quad \dfrac{I_3}{I_0}\theta_{23} \\ \varphi_2 \quad \bar{\xi} \quad \dfrac{I_1}{I_0}\theta_{12} \quad \dfrac{I_3}{I_0}\theta_{23} \\ \varphi_3 \quad \bar{\xi} \quad \dfrac{I_1}{I_0} \quad \dfrac{I_1 \quad I_2}{I_0}\theta_{23} \\ (\dot{\xi} \quad U) \end{array} , \tag{14}$$

where $I_0 \quad I_1 \quad I_2 \quad I_3$.

After such transformation equation (14) assumes the form

$$\begin{aligned}
&\dot{U} \quad \varepsilon_{ar}^{(3)} \\
&\ddot{\theta}_{12} \quad \frac{C_{12}(I_1 \quad I_2)}{I_1 I_2}\theta_{12} \quad \frac{C_{23}}{I_2}\theta_{23} \quad M_a \frac{I_1 \quad I_2}{I_1 I_2} \\
&\ddot{\theta}_{23} \quad \frac{C_{23}(I_2 \quad I_3)}{I_2 I_3}\theta_{23} \quad \frac{C_{12}}{I_2}\theta_{12} \quad M_b \frac{I_1 \quad I_2}{I_1 I_2}
\end{aligned}, \tag{15}$$

where $\varepsilon_{av}^{(3)} \quad \frac{M_1 \quad M_2 \quad M_3}{I_0}$ is average acceleration of the system after closing of clearance $\quad_2$;

$$M_b \quad \frac{M_3 I_2 \quad M_2 I_3}{I_2 I_3}.$$

Initial conditions of differential equations (15) will be determined considering the fact that at initial moment of closing of the second clearance connecting section (unit) of the first and second masses is deformed:

$$\theta_{12}(0) \quad \theta_{12}(T_{02}) \quad \frac{M_a}{C_{12}}(1 \quad \cos pT_{02}) \quad \frac{\omega_{1init}}{p}\sin pT_{02} \tag{16}$$

and the connecting unit of the second and third masses is not deformed:

$$\theta_{23}\ 0 \quad \theta_{23}\ T_{02} \quad 0. \tag{17}$$

Angular velocities and displacements of masses I_1 and I_2 at initial moment of closing of the second clearance are determined by the following formulae:

$$\begin{aligned}
&\omega_1(T_{02}) \quad \frac{M_a}{I_1 p}\sin pT_{02} \quad \frac{I_2}{I_1 \quad I_2}\omega_{1init}\cos pT_{02} \quad \frac{I_1}{I_1 \quad I_2}\omega_{1init} \quad \varepsilon_{av}{}^{(2)}T_{02} \\
&\omega_2(T_{02}) \quad \frac{M_a}{I_2 p}\sin pT_{02} \quad \frac{I_1}{I_1 \quad I_2}\omega_{1init}\cos pT_{02} \quad \frac{I_1}{I_1 \quad I_2}\omega_{1init} \quad \varepsilon_{av}{}^{(2)}T_{02} \\
&\varphi_1(T_{02}) \quad \frac{M_a}{I_1 p^2}(1 \quad \cos pT_{02}) \quad \frac{I_2}{(I_1 \quad I_2)p}\omega_{1init}\sin pT_{02} \quad \frac{I_1}{I_1 \quad I_2}\omega_{1init}T_{02} \quad \varepsilon_{av}{}^{(2)}\frac{T_{02}}{2} \\
&\varphi_2(T_{02}) \quad \frac{M_a}{I_2 p^2}(1 \quad \cos pT_{02}) \quad \frac{I_2}{(I_1 \quad I_2)p}\omega_{1init}\sin pT_{02} \quad \frac{I_1}{I_1 \quad I_2}\omega_{1init}T_{02} \quad \varepsilon_{av}{}^{(2)}\frac{T_{02}}{2} \quad _2
\end{aligned}. \tag{18}$$

Then on the basis of formula (18) we have:

$$\dot{\theta}_{12}(0) \quad \dot{\theta}_{12}(T_{02}) \quad \omega_1(T_{02}) \quad \omega_2(T_{02}) \quad \frac{M_a(I_1 \quad I_2)}{pI_1 I_2}\sin pT_{02} \quad \omega_{1init}\cos pT_{02}, \tag{19}$$

$$\dot{\theta}_{23}\ 0 \quad \dot{\theta}_{23}\ T_{02} \quad \omega_2\ T_{02}\ . \tag{20}$$

can be determined through φ_1, φ_2 and φ_3 from expressions (14)

$$\bar{\xi} \quad \frac{1}{I_0}\ I_1\varphi_1 \quad I_2\varphi_2 \quad I_3\varphi_3\ . \tag{21}$$

Considering that

$$U \quad \dot{\xi}, \tag{22}$$

we can write

$$U(T_{02}) \quad \dot{\xi}(T_{02}) \quad \frac{1}{I_0} \; I_1\omega_{1init} \quad I_1 \quad I_2 \; \varepsilon_{ar}^{(2)} T_{02} \;\; . \tag{23}$$

Thus, at the moment of closing of the second clearance t_2=0, initial conditions (17,18,20,21,23) are observed by the system of differential equations.

By integration of the first equation of the system (15) we have

$$U(t_2) \quad \varepsilon_{av}{}^{(3)} t_2 \quad U(T_{02}) \, . \tag{24}$$

Omitting intermediate transformations and considering the initial conditions (17,18,20,21) we write the solution of the second and third differential equations of system (15):

$$\begin{aligned} M_{12}(t_2) \quad C_{12}\theta_{12}(t_2) \quad \frac{1}{p_2{}^2 \quad p_1{}^2} \quad C_{12}\theta_{12}(T_{02})(p_2{}^2 \quad p_{12}{}^2) \quad p_{12}{}^2 M_a \quad p_2{}^2 \frac{I_2 \quad I_3}{I_0} m_{12} \;\; \cos p_1 t_2 \\ C_{12}\theta_{12}(T_{02})(p_1{}^2 \quad p_{12}{}^2) \quad p_{12}{}^2 M_a \quad p_1{}^2 \frac{I_2 \quad I_3}{I_0} m_{12} \;\; \cos p_2 t_2 \;\; \frac{1}{p_1} \;\; C_{12}\dot{\theta}_{12}(T_{02})(p_2{}^2 \quad p_{12}{}^2) \\ \frac{C_{12}C_{23}}{I_2}\dot{\theta}_{23}(T_{02}) \;\; \sin p_1 t_2 \quad \frac{1}{p_2} \;\; C_{12}\dot{\theta}_{12}(T_{02})(p_1{}^2 \quad p_{12}{}^2) \quad \frac{C_{12}C_{23}}{I_2}\dot{\theta}_{23}(T_{02}) \quad \sin p_2 t_2 \quad \frac{I_2 \quad I_3}{I_0} m_{12} \, , \end{aligned} \tag{25}$$

$$\begin{aligned} M_{23}(t_2) \quad C_{23}\theta_{23}(t_2) \quad \frac{1}{p_1{}^2 \quad p_{12}{}^2} \quad C_{12}\theta_{12}(T_{02}) \quad p_{23}{}^2 \frac{I_1 \quad I_2}{I_0} m_{23} \;\; \cos p_1 t_2 \\ \frac{C_{12}C_{23}}{I_2}\theta_{12}(T_{02}) \quad p_{23}{}^2 M_b \quad p_1{}^2 \frac{I_1 \quad I_2}{I_0} m_{23} \;\; \cos p_2 t_2 \quad \frac{1}{p_1} C_{23}\dot{\theta}_{23}(T_{02})(p_2{}^2 \quad p_{23}{}^2) \\ \frac{C_{12}C_{23}}{I_2}\dot{\theta}_{12}(T_{02}) \sin p_1 t_2 \quad \frac{1}{p_2} \;\; (p_1{}^2 \quad p_{23}{}^2)\dot{\theta}_{23}(T_{02}) \quad \frac{C_{12}C_{23}}{I_2}\dot{\theta}_{12}(T_{02}) \quad \sin p_2 t_2 \quad \frac{I_1 \quad I_2}{I_0} m_{23} \, , \end{aligned} \tag{26}$$

where

$$\begin{aligned} & p_{12}{}^2 \quad \frac{1}{2}(p_{12}{}^2 \quad p_{23}{}^2) \quad \sqrt{\frac{1}{4}(p_{12}{}^2 \quad p_{23}{}^2)^2 \quad \frac{C_{12}C_{23}I_0}{I_1 I_2 I_3}}; \\ & p_{12}{}^2 \quad \frac{C_{12}(I_1 \quad I_2)}{I_1 I_2}; \qquad p_{23}{}^2 \quad \frac{C_{23}(I_2 \quad I_3)}{I_2 I_3}; \qquad\qquad . \\ & m_{12} \quad \frac{I_1 \quad I_2}{I_2} M_a \quad \frac{I_1}{I_2} M_b; \qquad m_{23} \quad \frac{I_3}{I_2} M_a \quad \frac{I_2 \quad I_3}{I_2} M_b \end{aligned} \tag{27}$$

Constant components of expressions (25) and (26) are elastic moments of units caused by influence of static forces. Their values can be determined on the basis of solution of the system of algebraic equations, which in its turn is obtained from expressions (16) by supposing θ_{12}=0 and θ_{23}=0.

Then

$$\begin{array}{l} M_{12} p_{12}^2 \quad \dfrac{C_{12}}{I_2} M_{23} \quad M_a p_{12}^2 ; \\ M_{12} \dfrac{C_{23}}{I_2} \quad M_{23} p_{23}^2 \quad M_b p_{23}^2 . \end{array} , \tag{28}$$

where $M_{12} = C_{12}\theta_{12}$, $M_{23} = C_{23}\theta_{23.}$

Maximum dynamic coefficient in elastic units after closing of the second clearance $_2$ is determined as follows

$$K_d^{(1)} \ _{\max} \quad \frac{\max M_{12} \ t_2}{m_{12}} \quad \frac{I_0}{I_2 \quad I_3} \ ; \qquad K_d^{(2)} \ _{\max} \quad \frac{\max M_{23} \ t_2}{m_{23}} \quad \frac{I_0}{I_1 \quad I_2} \ . \tag{29}$$

Maximum values of $M_{12}\,(t)$ and $M_{23}\,(t)$ can be determined by plotting graphs of variation of these moments with time on the basis of application of formulae (25) and (26).

It is necessary to know the character of variation of angular velocities of masses in the course of study of the mechanical system, then we have to apply expressions of transformation of coordinates (14), (25) and (26). These expressions should be divided at first by C_{12} and C_{23}, and then differentiated with respect to time. Then we will have

$$\begin{array}{l} \omega_1(t_2) \quad \dfrac{d\varphi}{dt_2} \quad \varepsilon_{ar}^{(3)} t_2 \quad \dot{U}(T_{02}) \quad \dfrac{p_1}{I_0(p_2^2 \quad p_1^2)} \quad \theta_{12}(T_{02}) \ \ I_2 \quad I_3 \ (p_2^2 \quad p_{12}^2) \ \dfrac{I_3}{I_2} C_{12} \\ \dfrac{m_{12}(I_2 \quad I_3)}{I_0} \ \dfrac{I_0}{I_1} \ \dfrac{I_2 \quad I_3}{C_{12}} p_1^2 \quad \dfrac{I_3(I_1 \quad I_2)}{I_0 C_{23}} p_2^2 m_{23} \quad \sin p_1 t \\ \dfrac{p_2}{I_0(p_2^2 \quad p_1^2)} \quad \theta_{12}(T_{02}) \ \ I_2 \quad I_3 \ (p_1^2 \quad p_{12}^2) \ \dfrac{I_3}{I_2} C_{12} \qquad \dfrac{m_{12}(I_2 \quad I_3)}{I_0} \ \dfrac{I_0}{I_1} \ \dfrac{I_2 \quad I_3}{C_{12}} p_1^2 \\ \dfrac{I_3(I_1 \quad I_2)}{I_0 C_{23}} p_2^2 m_{12} \quad \sin p_2 t \quad \dfrac{1}{I_0(p_2^2 \quad p_1^2)} \quad \dot{\theta}_{12}(T_{02}) \ \ I_2 \quad I_3 \ (p_2^2 \quad p_{12}^2) \ \dfrac{I_3}{I_2} C_{12} \\ \dot{\theta}_{23}(T_{02}) \ \dfrac{C_{23}(_I 2 \quad I_3)}{I_2} \quad I_3(p_2 \quad p_{23}^2) \quad \cos p_1 t \\ \dfrac{1}{I_0(p_2^2 \quad p_1^2)} \quad \dot{\theta}_{12}(T_{02}) \quad I_2 \quad I_3 \ (p_2^2 \quad p_{12}^2) \ \dfrac{I_3}{I_2} C_{12} \quad \dot{\theta}_{23} \ T_{02} \ \dfrac{C_{23}(I_2 \quad I_3}{I_2} \quad I_3(p_2^2 \quad p_{23}^2) \end{array} . \tag{30}$$

Similarly $\omega_2(t_2)$, $\omega_3(t_2)$ can be determined.

We determine the moments of elastic forces $M_{12}\,(t)$ and $M_{23}\,(t)$ for the case when there is no clearance in the second elastic unit. In this case differential equations (13), after closing of clearance $_1$ in the first unit, must satisfy the following initial conditions:

$$\varphi_1 \ t_1 \quad \varphi_2 \ t_1 \quad \varphi_3 \ t_1 \ ; \ \ \dot{\varphi}_1 \ t_1 \quad \omega_{1init}; \ \ \dot{\varphi}_2 \ t_1 \quad \dot{\varphi}_3 \ t_1 \quad 0. \tag{31}$$

After solution of differential equations (13) with such initial conditions we find:

$$\begin{array}{l} M_{12} \ t_1 \quad \dfrac{1}{p_2^2 \quad p_1^2} \quad M_a p_{12}^2 \quad p_2^2 \dfrac{I_1 \quad I_3}{I_0} m_{12} \ \cos \ p_1 t_1 \quad M_a p_{12}^2 \quad p_1^2 \dfrac{I_2 \quad I_3}{I_0} m_{12} \ \cos \ p_2 t_1 \\ \dfrac{C_{12}\,\omega_{1init}}{p_1} \ p_2^2 \quad p_{12}^2 \ \sin \ p_1 t_1 \quad \dfrac{C_{12}\,\omega_{1init}}{p_2} \ p_1^2 \quad p_{12}^2 \ \sin \ p_2 t_1 \quad \dfrac{I_2 \quad I_3}{I_0} m_{12}. \end{array} , \tag{32}$$

$$M_{23}\ t_1 \quad \frac{1}{p_2^2\ \ p_1^2}\quad M_b\, p_{23}^2\quad p_2^2\frac{I_1\quad I_2}{I_0}m_{23}\ \cos\ p_1 t_1\quad M_b\, p_{23}^2\quad p_1^2\frac{I_1\quad I_2}{I_0}m_{23}\ \cos\ p_2 t_1$$
$$\frac{C_{12}\,C_{23}\,\omega_{1init}}{\theta_2 p_1}\sin\ p_1 t_1\quad \frac{C_{12}\,C_{23}\,\omega_{1init}}{\theta_2 p_2}\sin\ p_2 t_1\quad \frac{I_1\quad I_2}{I_0}m_{23}. \qquad (33)$$

Comparison of expressions (25), (26) and (32), (33) shows that these latter can be obtained from formulae (25) and (26), assuming that $\theta_{12}(T_{02})\quad 0, \dot{\theta}_{12}(T_{02})\quad \omega_1 init, \theta_{23}$ which corresponds to meeting the initial conditions (31) at moment t_1.

Consider calculation of dynamical loads with the example of the primary mill of the Krivoi-Rog Metallurgical Plant, which is characterized by the following parameters: $I_1\quad 9.8\text{ton}\ \ \text{m}\ \ \text{sec}^2$ – moment of inertia of the electromotor rotor; $I_2\quad 0.56\text{ton}\ \ \text{m}\ \ \text{sec}^2$ – moment of inertia of rollers of the gear cage; $I_3\quad 0.50\text{ton}\ \ \text{m}\ \ \text{sec}^2$ – moment of inertia of rollers reduced on the electromotor shaft; $C_{12}\quad 2\ \ 10^4\,\text{ton}\ \ \text{m}$ – stiffness of the electromotor shaft; $C_{23}\quad 1.1\ \ 10^4\,\text{ton}\ \ \text{m}$ – reduced rigidity of spindles; $_1$ = 0.01 rad – value of clearance in the first section.

Exploitation of rolling mills shows that clearances in spindle joints increase gradually as the result of wear of the washers surfaces. In this connection it is interesting to know how dynamical loads vary depending on the increase of clearances in main drive of the rolling mill. For this purpose we assume that the first clearance is constant ($_1$=const) and the second clearance varies in the range $_2$= (0.02 – 005) rad; the starting moment of the electromotor M_1=40ton·m, and moments of technological load (rolling moment), applied to the second and third masses, respectively are M_2=2ton·m; M_3= 4ton·m.

1. Determine the angular acceleration and velocity of the first mass (rotor) after closing of the first clearance

$$\varepsilon_0\quad \frac{M_1}{I_1}\quad 4.08\ {}^{1}\!/\!_{\text{sec}^2};\ \omega_{1init}\quad \sqrt{2\varepsilon_0\ \ _1}\quad 0.286\ {}^{1}\!/\!_{\text{sec}^2};\ T_{01}\quad \sqrt{2\ \ _1/\varepsilon_0}\quad 0.07\,\text{sec}$$

2. Frequency of proper oscillations p of the two-mass system elastic moment M_a and average acceleration after closing of the first clearance

$$p\quad \sqrt{\frac{C_{12}(I_1\quad I_2)}{I_1 I_2}}\quad 194.3\ {}^{1}\!/\!_{\text{sec}};\ M_a\quad \frac{M_1 I_2\quad M_2 I_1}{I_1\quad I_2}\quad 4.05\text{ton}\ \ \text{m};\quad \varepsilon_{av}^{(2)}\quad \frac{M_1\quad M_2}{I_1\quad I_2}\quad 3.67\ {}^{1}\!/\!_{\text{sec}^2}$$

3. Determine the time T_{02} necessary for closing of the second clearance with the use of equation (9)

$$\frac{4.05}{0.56\ \ 194.3^2}\ 1\quad \cos 194.3\,T_{02}\quad \frac{9.8\ \ 0.286}{10.36\ \ 194.3}\sin 194.3\,T_{02}\quad \frac{9.8\ \ 0.286}{10.36}T_{02}\quad 1.835T_{02}^2\quad \begin{matrix}0.02\\0.03\\0.05\end{matrix} \qquad (34)$$

At first we determine approximate values of T_{02} with the use of formula (11):

$$T_{02}\big|\ _{0.02}\quad 0.054;\quad T_{02}\big|\ _{0.03}\quad 0.073;\quad T_{02}\big|\ _{0.05}\quad 0.105. \qquad (35)$$

We find more exact values with the use of more precise transcendental equation (9):

$$T_{02}\big|\ _{0.02}\quad 0.0526;\quad T_{02}\big|\ _{0.03}\quad 0.0724;\quad T_{02}\big|\ _{0.05}\quad 0.1065.$$

Determine the values of $\theta_{12}(T_{02}), \theta_{23}(T_{02}), \theta(T_{02}),\ \theta_{23}(T_{02})$ with the use of expressions (16,17,19,20) and then using formulae (25), (26) we obtain the following expressions for calculation of moments of elastic forces $M_{12}(t)$ and $M_{23}(t)$:

მექანიკა

ღრეჩოების ზეგავლენის გამოკვლევა სამანქანო სისტემების დინამიკურ დატვირთვებზე

რ. ადამია*, ს. ჩაგელიშვილი, თ. ნადირაძე****

** აკადემიკოსი, რ. დვალის მანქანათა მექანიკის ინსტიტუტი, თბილისი*
*** რ. დვალის მანქანათა მექანიკის ინსტიტუტი, თბილისი*

შემოთავაზებულია სამანქანო აგრეგატების ტრანსმისიებში დინამიკური მახასიათებლების გაანგარიშების განზოგადოებული მეთოდიკა მანქანის გაშვების სხვადასხვა ეტაპზე მთავარი ელექტროამძრავის ძალოვან გადაცემებში ღრეჩოების თანაარსებობის გათვალისწიებით. დადგენილია ანალიზური დამოკიდებულება ღრეჩოების სიდიდეებსა და მათი გავლენით გამოწვეულ დამატებით (დარტყმითი ხასიათის) ძალვებს შორის.

REFERENCES:

1. *R. Adamia* (1978), Optimizatsyia dinamicheskikh nagruzok prokatnykh stanov, M., 232 (in Russian).
2. *R. Adamia, V. Loboda,* Osnovy ratsional'nogo proektirovaniya metalurgicheskikh mashin, M., 146 (in Russian).

Received June, 2010