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Mechanics

On Dynamic Effect Caused by Moving Punches on Elastic Half-Plane with the Account of Friction Force

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ABSTRACT. The problem of agitation of orthotropic half-plane, caused by motion of concentrated force on the boundary has been considered. It is shown that stresses and area of wave distribution in anisotropic body have significantly greater values than in isotropic materials. *© 2010 Bull. Georg. Natl. Acad. Sci.*

Key words: *orthotropic half-plane, anisotropy, isotropy, stress, wave distribution.*

The aim of the present paper is to determine stressed state of the body, caused by pressure of n-punches moving with constant velocity V_1 on its boundaries.

Not limiting general outline of the question, we can assume that $n = 2$.

Suppose that excited on the boundary normal and tangent stresses are connected by Coulomb's law, then we have to solve the problem of stressed state of half-plane with the following boundary conditions:

$$\begin{aligned}
&\tau_{xy} = \kappa_1 \sigma_y, \quad \text{when} \quad a_1 \le x \le b_1; \\
&\frac{\partial v}{\partial x} = f_1'(x), \quad \text{when} \quad a_1 \le x \le b_1; \\
&\tau_{xy} = \kappa_2 \sigma_y, \quad \text{when} \quad a_2 \le x \le b_2; \\
&\frac{\partial v}{\partial x} = f_2'(x), \quad \text{when} \quad a_2 \le x \le b_2; \\
&\tau_{xy} = \sigma_y = 0, \quad \text{when} \quad x \in L,
\end{aligned} \tag{1}$$

where
$$L = \{x; \ -\infty < x < \infty\}\ ; \ L_2 = \{(a_1; b_1) \cup (a_2; b_2)\}\ ; \ L = L_1 - L_2. \tag{2}$$

Having transformed the variables $x - v_1 t = \xi, \quad y = \eta$ and introducing new variables $z_1 = \xi + i\beta_1\eta, \quad z_2 = \xi + i\beta_2\eta$, for the components of stresses and displacements we shall have:

$$\begin{aligned}
&\sigma_x = -2\,\mathrm{Re}\left[(\beta_1^2 + m)F_1''(z_1) + (\beta_2^2 + m)F_2''(z_2)\right], \\
&\sigma_y = 2n\,\mathrm{Re}\left[F_1''(z_1) + F_2''(z_2)\right], \\
&\tau_{xy} = \mathrm{Im}\left[aF_1''(z_1) + bF_2''(z_2)\right], \\
&u = -2\,\mathrm{Re}\left[rF_1'(z_1) + lF_2'(z_2)\right]\ , \\
&v = 2\,\mathrm{Im}\left[r_1F_1'(z_1) + l_1F_2'(z_2)\right]\ ,
\end{aligned}$$

where $F_1(z_1)$ and $F_2(z_2)$ are analytical functions in a lower half-plane (y<0), $\beta_1, \beta_2, m, n, a, b, r, l, l_1$ are constants, characterizing anisotropy of elastic body. Taking into account the last equalities, boundary condition (1) will be written in the following way:

$$\operatorname{Im}\left[aF_1''(z_1)+bF_2''(z_2)\right]_{y=0}=2k_1 n\operatorname{Re}\left[F_1''(z_1)+F_2''(z_2)\right]_{y=0},\quad a<\xi<b_1,$$

$$\operatorname{Im}\left[r_1F_1''(z_1)+l_1F_2''(z_2)\right]_{y=0}=\frac{1}{2}f_1'(\xi),\quad a_1<\xi<b_1, \tag{3}$$

$$\operatorname{Im}\left[aF_1''(z_1)+bF_2''(z_2)\right]_{y=0}=2k_2 n\operatorname{Re}\left[F_1''(z_1)+F_2''(z_2)\right]_{y=0},\quad a_2<\xi<b_2,$$

$$\operatorname{Im}\left[r_1F_1''(z_1)+l_1F_2''(z_2)\right]_{y=0}=\frac{1}{2}f_2'(\xi),\quad a_2<\xi<b_2,$$

$$2n\operatorname{Re}\left[F_1''(z_1)+F_2''(z_2)\right]_{y=0}=\operatorname{Im}\left[aF_1''(z_1)+bF_2''(z_2)\right]_{y=0},\quad \xi\in L.$$

On the boundary for y = 0, $z_1 = z_2 = \xi$ with account of boundary conditions for functions $F_1''(\xi)$ and $F_2''(\xi)$ we get:

$$F_1''(\xi)=\frac{b}{2\pi in(b-a)}\int_{-\infty}^{+\infty}\frac{(\sigma_y)_{y=0}d\xi}{\xi-\xi_0}-\frac{b}{n(b-a)}(\tau_{xy})_{y=0}-\frac{1}{\pi(b-a)}\int_{-\infty}^{+\infty}\frac{(\tau_{xy})_{y=0}d\xi}{\xi-\xi_0}-\frac{2i}{b-a}(\tau_{xy})_{y=0},$$

$$F_2''(\xi)=\frac{a}{2\pi in(b-a)}\int_{-\infty}^{+\infty}\frac{(\sigma_y)_{y=0}d\xi}{\xi-\xi_0}-\frac{a}{n(b-a)}(\sigma_y)_{y=0}-\frac{1}{\pi(b-a)}\int_{-\infty}^{+\infty}\frac{(\tau_{xy})_{y=0}d\xi}{\xi-\xi_0}+\frac{2i}{b-a}(\tau_{xy})_{y=0}. \tag{4}$$

Taking into account (4) boundary conditions (3) will have the form

$$\frac{al_1-br_1}{\pi n(b-a)}\int_{a_1}^{b_1}\frac{(\sigma_y)_{y=0}d\xi}{\xi-\xi_0}+\frac{2(r_1-l_1)}{b-a}(\tau_{xy})_{y=0}=\frac{1}{2}f_1'(\xi),\quad a_1\le\xi\le b_1,$$

$$\frac{al_1-br_1}{\pi n(b-a)}\int_{a_2}^{b_2}\frac{(\sigma_y)_{y=0}d\xi}{\xi-\xi_0}+\frac{2(r_1-l_1)}{b-a}(\tau_{xy})_{y=0}=\frac{1}{2}f_2'(\xi),\quad a_2\le b_1, \tag{5}$$

The remaining part of the boundary is free from loading.

If in the equalities (5) we consider that $\tau_{xy}=k_j\cdot\sigma_y$ $(j=1,2)$, we obtain:

$$\frac{al_1-br_1}{\pi n(b-a)}\int_{a_j}^{b_j}\frac{(\sigma_y)_{y=0}d\xi}{\xi-\xi_0}+\frac{2(r_1-l_1)k_j}{b-a}(\sigma_y)_{y=0}=\frac{1}{2}f_1'(\xi),\ \xi\in L_2, \tag{6}$$

$$f_1(\xi)=\begin{cases}f_1(\xi), & \text{when}\quad \xi\in(a_1b_1)\\ f_2(\xi), & \text{when}\quad \xi\in(a_2b_2)\end{cases}$$

As we see our task reduces to the solution of integral euation (6). Let us introduce the function

$$\Omega(z)=U-iV=\int_{L_2}\frac{(\sigma_y)_{y=0}d\xi}{\xi-z}. \tag{7}$$

If in the last equality we can come to the boundary when $z \to \xi$ we get:

$$\Omega(\xi) = \left[U - iV_1\right]_{y=0} = \int_{L_2} \frac{(\sigma_y)_{y=0} d\xi}{\xi - \omega_0} - i\pi(\sigma_y)_{y=0}$$

or

$$U_1 = \int_{L_2} \frac{(\sigma_y)_{y=0} d\xi}{\xi - \xi_0}; \qquad V = \pi(\sigma_y)_{y=0}. \tag{8}$$

Taking into consideration that the remaining part of the boundary is free from loading, we get

$$V_1 = 0 \text{ when } \xi \in L,$$

$$\frac{al_1 - br_1}{\pi n(b-a)} U_1 + \frac{2(r_1 - l_1)k_j}{(b-a)\pi} V_1 = \frac{1}{2} f_1'(\xi), \quad \xi \in L_2, \tag{9}$$

We shall write equality (9) as:

$$V_1 = 0 \text{ when } \xi \in L,$$

$$U - \frac{2n(l_1 - r_1)k_j}{al_1 - br_1} V_1 = \frac{\pi n(b-a)}{2(al_1 - br_1)} f_1'(\xi), \; \xi \in L_2, \tag{10}$$

Expression (10) is the equation of Riemann-Hilbert. Hence, it is necessary to define analytically in the lower half-plane function $\Omega(z) = U_1 - iV_1$, satisfying the conditions on the boundary

$$a(\xi)U_1 + b(\xi)V_1 = f(\xi).$$

In our case

$$\begin{aligned} &a(\xi) = 1, \\ &b(\xi) = \frac{\pi n(r_1 - l)k_j}{2(al_1 - br_1)}, \\ &f(\xi) = \frac{\pi n(b-a)}{2(al_1 - br_1)} f_1'(\xi), \; \xi \in L_2, \quad j = 1,2; \\ &a = 0, \, b = 1; \, f(\xi) = 0, \; \xi \in L. \end{aligned} \tag{11}$$

Introduce the designations

$$\frac{2n(r_1 - l_2)k_j}{al_1 - br_1} = q_j \;, \quad j = 1,2, \tag{12}$$

$$\frac{\pi n(b-a)}{2(al_1 - br_1)} = r^*.$$

Considering (12) in (10) we get:

$$V_1 = 0 \text{ when } \xi \in L,$$

$$U_1 - qV_1 = r^* f_1(\xi), \text{ when } \xi \in L_2, \; j = 1,2.$$

At the solution of the obtained expression we assume that:

1. $\Omega(z)$ on the axis ξ has the peculiarity as z^{-a} where $0<a<1$, which is conditioned by impossible existence under the punch of concentrated force;

2. In the infinity $\Omega(z) \to \frac{P_j}{z}$ where P_j is the force acting on the punch. Then we shall have:

$$\Omega(z) = \frac{\mu^*}{\sqrt{1+(k_1 q_1)^2}} \frac{1}{(z-a_1)^{\theta_1}(z-b_1)^{1-\theta_1}(z-b_1)^{\theta_2}(z-b_2)^{1-\theta_2}} \times$$
$$\times \int_{L_2} \frac{f_j'(\xi)d\xi}{(\xi-a_1)^{\theta_1}(\xi-b_1)^{\theta-1}(z-a_2)^{\theta_2}(\xi-b_2)^{\theta-1}} + \frac{C_1 z + C_0}{(z-a_1)^{\theta_1}(z-a_2)^{\theta_2}(z-b_1)^{1-\theta_1}(z-b_2)^{1-\theta_2}}, \tag{13}$$

where $\theta_j = \frac{1}{\pi} arctg \frac{1}{k_j q_j}$.

In the formula (13) $x(z) = \frac{1}{\left[(z-a_1)(z-b_1)\right]^{1-\theta_1}\left[(z-a_2)^{\theta_2}(\xi-b_2)\right]^{1-\theta_2}}$ is partial solution of the equatilon (10), when near the points a_1 and a_2 there is the breakage of the degree ½.

In order to limit the solution in points a_1, b_1, a_2, b_2, instead of $x(z)$ it is necessary to use the function

$$x(z) = (z-a_1)(z-a_2)(z-a_1)^{\theta_1}(z-b)^{1-\theta_1}(z-a_2)^{\theta_2}(z-b_2)^{1-\theta_2}$$

If two rectangular punches press on elastic surface, then $f_1'(\xi) = 0$ and

$$x(z) = \frac{1}{\left[(z-a_1)(z-b_1)\right]^{\theta_1}\left[(z-a_2)(z-b_2)\right]^{\theta_2}}.$$

In contact area pressure distribution is defined by the expression

$$P(\xi) = -\operatorname{Im}\left[\Omega(z)\right]_{y=0}. \tag{14}$$

In the formula (13) real contacts C_1 and C_0 must be determined from the conditions

$$\int_{a_1}^{b_1} (\sigma_y)_{y=0} d\xi = P_1; \quad \int_{a_2}^{b_2} (\sigma_y)_{y=0} d\xi = P_2. \tag{15}$$

It should be noted that, if the module of the elastic body is large, the influence of punch motion speed is insignificant. If we ignore the module of the punch then in that case the distribution speed of longitudinal and transverse waves in the body also is not large and the influence of the punch motion even at small speeds (10-20 m/sec) is quite significant.

The case when motion speed of the punch is lower than distribution speed in the body of longitudinal and transverse waves is considered above. At that it is clear from the practical point of view, the motion with supersonic speed is of no interest.

მექანიკა

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REFERENCES

1. *R. Bantsuri* (1990), Proc. A. Razmadze Math. Inst., **100**.
2. *A.A. Galin* (1953), Kontaktnye zadachi teorii uprugosti, M. (in Russian).
3. *S.G. Lekhnitskii* (1947), Anizotropnye plastinki. L. (in Russian).
4. *N.I. Muskhelishvili* (1968), Singulyarnye integral'nye uravneniya. M. (in Russian).

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