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Hydrology

Theoretical Basis for the Computer Simulation of River Floods

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ABSTRACT. High waters and floods are considered as an unsteady motion of water in a river, as described by the non-linear differential equations of Saint Venant. A new method is proposed for the numerical solution of these equations based on stable difference schemes and the method of matrix "runs", which permits broader possibilities of coverage under complex initial and boundary conditions that unavoidably occur in practice. This method is proposed for the computer simulation of river floods. *© 2011 Bull. Georg. Natl. Acad. Sci.*

***Key words**: floods, numerical methods.*

A river basin is the space where river runoff is formed. It is characterized by a hydrograph a curve representing the changes of discharge of level in time. Hydrograph of the runoff in the upper river is formed by a number of water rises of different water lift. However, for practical purposes the hydrograph has to be made at sites lying downstream, thus being a major problem of hydrology, which calls for calculation of the velocity and change of the form of each flood wave along its movement towards the mouth. But the basic difficulties in solving this task are due to the unsteady character of the movement of the flood [1-3].

To describe the transformation of a flood wave two trends should be identified. The first is a mathematical description of this phenomenon with the aid of a corresponding system of differential equations and the second, approximate engineering methods [1,2,4,5].

To describe an unsteady movement of water arising at the passage of a freshet through a system of rivers and hydrotechnical structures, a full system [1,6] of differential equations of the dynamics and continuing (equations of Saint-Venan) must be used:

$$\left.\begin{aligned} &2c\frac{\partial c}{\partial x}+\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=g\left(i-\frac{u^2}{E^2R}\right)\\ &c\frac{\partial u}{\partial x}+2u\frac{\partial c}{\partial x}+2\frac{\partial c}{\partial t}=0\end{aligned}\right\}, \qquad (1)$$

where $c=\sqrt{gF/B}, F(x,t)$ - cross sectional area of flow, $B(x,t)$ – cross-sectional water surface width, u - rate of flow, g - is the acceleration of gravity, i - inclination of bottom, E - Chezy coefficient, R - hydraulic radius, x – distance, t - time.

It does not appear feasible to obtain an analytical solution of the system (1) owing to the nonlinearity of these equations and other reasons. Therefore, to solve various equations of unsteady water flow numerical (finite-difference) methods are used to the present day [1,6-10].

It should be noted that application of numerical methods has its specific features, namely the choice of the difference scheme, initial and boundary conditions, building the algorithm of the course of the process, programming, adjustment on the computer, etc. All this should ensure a steady step-by-step calculation and the possibility to introduce various changes both in the algorithm of the process and in the programme for the computer [7-10].

We write down the system (1) in the characteristic form. To this end, we in turns sum and subtract the first and second equations of this system, as a result we obtain

$$\left.\begin{aligned} (u+c)\frac{\partial u}{\partial x}+2(u+c)\frac{\partial c}{\partial x}+\frac{\partial u}{\partial t}+2\frac{\partial c}{\partial t}=g\left(i-\frac{u^2}{E^2R}\right) \\ (u-c)\frac{\partial u}{\partial x}+2(c-u)\frac{\partial c}{\partial x}+\frac{\partial u}{\partial t}-2\frac{\partial c}{\partial t}=g\left(i-\frac{u^2}{E^2R}\right) \end{aligned}\right\}. \quad (2)$$

The first equation corresponds to direct characteristics, the second to reverse. We approximate the derivatives with the aid of the scheme with central difference

$$\left.\begin{aligned} \frac{\partial u}{\partial x}=\frac{u_{n+1}^{k+1}-u_{n-1}^{k+1}}{2\Delta}, \quad \frac{\partial c}{\partial x}=\frac{c_{n+1}^{k+1}-c_{n-1}^{k+1}}{2\Delta} \\ \frac{\partial u}{\partial t}=\frac{u_n^{k+1}-u_n^{k}}{\tau}, \quad \frac{\partial c}{\partial t}=\frac{c_n^{k+1}-c_n^{k}}{\tau} \end{aligned}\right\}, \quad (3)$$

where k is the number of layers over time, n is number of point at each layer of time, Δ is step over distance, τ is step over time.

Inasmuch as we take the coefficients and free members of the system (2) at the upper layer (k+1), a corresponding system of algebraic equations for the determination of u_n^{k+1}, c_n^{k+1} (n=1,2,3 … N) proves nonlinear. To overcome these difficulties, various techniques have been proposed: iteration, recalculation and linearization. As our investigations have shown, each of these techniques are fairly effective.

The difference equations, obtained after substitution of approximations (3) in (2), may be represented in the following form.

For intermediate points

$$\left.\begin{aligned} a_{11}u_{n-1}^{k+1}+a_{12}c_{a-1}^{k+1}+b_{11}u_n^{k+1}+b_{12}c_a^{k+1}+c_{11}u_{n+1}^{k+1}+c_{12}\cdot c_{a+1}^{k+1}=e_1 \\ a_{21}u_{n-1}^{k+1}+a_{22}c_{a-1}^{k+1}+b_{21}u_n^{r+1}+b_{22}c_n^{k+1}+c_{21}u_{n+1}^{k+1}+c_{22}\cdot c_{n+1}^{k+1}=e_2 \end{aligned}\right\}. \quad (4)$$

For the left boundary

$$\left.\begin{aligned} a_{11}u_0^{k+1}+a_{12}c_0^{k+1}+b_{11}u_1^{k+1}+b_{12}c_1^{k+1}=e_1 \\ a_{21}u_0^{k+1}+a_{22}c_0^{k+1}+b_{21}u_1^{k+1}+b_{22}c_1^{k+1}=e_2 \end{aligned}\right\}. \quad (5)$$

For the right boundary

$$\left.\begin{aligned} b_{11}u_{n-1}^{k+1}+b_{12}c_{a-1}^{k+1}+c_{11}u_N^{k+1}+c_{12}\cdot c_N^{k+1}=e_1 \\ b_{21}u_{n-1}^{k+1}+b_{22}c_{N-1}^{k+1}+c_{21}u_N^{r+1}+c_{22}\cdot c_N^{k+1}=e_2 \end{aligned}\right\}. \quad (6)$$

The coefficients a,b,c in the equations (4)-(6) are determined on the basis of comparing these equations with corresponding difference equations.

The equations (4) - (6) may be appropriately written down in matrix form:

$$\left.\begin{aligned} A_n^k\psi_{n-1}^{k+1}+B_n^k\psi_n^{k+1}+C_n^k\psi_{n+1}^{k+1}=E_n^k \\ A_0^k\psi_0^{k+1}+B_0^k\psi_1^{k+1}=E_0^k \\ B_N^k\psi_{N-1}^{k+1}+C_N^k\psi_N^{k+1}=E_N^k \end{aligned}\right\}, \quad (7)$$

where $\psi=\psi\left|{}^{u}_{c}\right|$ - is the sought vector, A^k, B^k, C^k are matrices of four elements, and E^k vectors of two elements: the upper index corresponds to the number of the layers of time, and the lower index to the number of points at each time layer, N is the maximal number of points, constant at each time layer and corresponding to the right boundary. The first equation of the system (7) corresponds to the inner points (n=1,2,3,…N-1), the second equation to the left boundary (n=0), and the third equation to the right boundary (n=N).

The system (7) at each time layer is solved by the method of matrix "run" [8]. This method is proposed for the computer simulation of river floods.

Numerical example of flood computer imitation will be considered separately in the following publication.

ჰიდროლოგია

მდინარეებში წყალდიდობის გავრცობის კომპიუტერული იმიტაციის თეორიული საფუძვლები

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წყალდიდობა მდინარეებში განიხილება როგორც წყლის დაუმყარებელი მოძრაობა, რომელიც აისახება სენ ვენანის არაწრფივ დიფერენციალურ განტოლებათა სისტემით. წარმოდგენილია ამ განტოლებათა ამოხსნის ახალი რიცხვითი მეთოდი მდგრადი სასრულ-სხვაობიანი სქემების და მატრიცული გატარების მეთოდის გამოყენებით, რომელიც გამოირჩევა გაზრდილი შესაძლებლობებით ბუნებაში არსებული რთული საწყისი და სასაზღვრო პირობების დროს. ეს მეთოდი გამოყენებადია მდინარეში წყალდიდობის კომპიუტერული იმიტაციის მისაღწევად.

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