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Mathematics

On the Generalized Fast Convergent Sampling Series

Zurab A. Piranashvili

Vladimir Chavchanidze Institute of Cybernetics, Tbilisi

(Presented by Academy Member Elizbar Nadaraya)

ABSTRACT. The generalized fast convergent sampling series and estimates of the remainder term are given. These estimates enable to receive the mentioned representation for stochastic processes and fields. *© 2011 Bull. Georg. Natl. Acad. Sci.*

Key words: *entire function, stochastic process, stochastic field,* $L^{\alpha}(\Omega)$ *-process.*

Let $\xi(t),\ -\infty<t<\infty$ be a separable stochastic process with real or complex values and with separable derivatives, if they exist. For simplicity suppose that the expectation $E\xi(t)=0,\ -\infty<t<\infty.$

Theorem 1. *If the conditions of the Theorem* 6 *from* [1] *are fulfilled, then for almost all sampling functions of the process* $\xi(t)$ *the following formula is true*

$$\frac{1}{p!}\lim_{\zeta\to t}\frac{d^p}{d\zeta^p}\left(\frac{\xi(\zeta)}{(\zeta-c)^{N_0+1}\left(ae^{\delta\zeta}+be^{-\delta\zeta}\right)\sin^{N+1}(\alpha\zeta)}\left(\frac{\sin\beta(\zeta-t)}{\beta(\zeta-t)}\right)^q\right)=\sum_{k=-\infty}^{\infty}\left(\frac{(-1)^k}{\alpha}\right)^{N+1}\cdot$$

$$\cdot\left\{\sum_{\tau=0}^{N}\frac{1}{(N-\tau)!}\left[\sum_{\mu=0}^{\tau}\frac{\alpha^{\tau-\mu}}{(\tau-\mu)!}\cdot A_{\mu\tau N}\cdot\sum_{j=0}^{\mu}\frac{\xi^{(j)}\left(\frac{k\pi}{\alpha}\right)}{j!\left(t-\frac{k\pi}{\alpha}\right)}\cdot\sum_{r=0}^{\mu-j}\frac{(p+r)!(N_0+\mu-j-r)!(-1)^{p+\mu-j-r}}{r!N_0!p!(\mu-j-r)!\left(t-\frac{k\pi}{\alpha}\right)^r\left(\frac{k\pi}{\alpha}-c\right)^{N_0+\mu+1-j-r}}\right]\right\}. \qquad (1)$$

$$\cdot\frac{\varphi_{\tau N}(t;k,q,\alpha,\beta,\delta,a,b)}{\left(t-\frac{k\pi}{\alpha}\right)^p}+\sum_{\tau=0}^{N_0}\frac{\xi^{(\tau)}(c)}{p!(N_0-\tau)!(t-c)^{p+1}}\cdot\sum_{\mu=0}^{\tau}\frac{(p+\tau-\mu)!\psi_{\tau NN_0}(t;q,\alpha,\beta,\delta,a,b,c)}{\mu!(\tau-\mu)!(c-t)^{\tau-\mu}},$$

$t\neq\frac{\nu\pi}{\alpha},\quad \nu=0,\pm1,\pm2,\ldots$

for every $\alpha>\frac{\sigma}{N+1},\quad 0<\beta<\frac{(N+1)\alpha-\sigma}{q},\quad 0<\delta<(N+1)\alpha-\sigma-q\beta$*, where* N_0,N,p,q *are fixed nonnegative integers,* a,b,α,β,δ *are positive real numbers,* $c\neq0$ *is some fixed number and*

$$A_{\mu\tau N}=\lim_{x\to0}\frac{d^{\tau-\mu}}{dx^{\tau-\mu}}\left(\frac{x}{\sin x}\right)^{N+1}, \qquad (2)$$

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$$\varphi_{\tau N}\left(t;k,q,\alpha,\beta,\delta,a,b\right)=\lim_{\zeta\to\frac{k\pi}{\alpha}}\frac{d^{N-\tau}}{d\zeta^{N-\tau}}\left[\left(\frac{\sin\beta(\zeta-t)}{\beta(\zeta-t)}\right)^{q}\cdot\frac{1}{\left(ae^{\delta\zeta}+be^{-\delta\zeta}\right)}\right],\tag{3}$$

$$\psi_{\tau NN_0}\left(t;q,\alpha,\beta,\delta,a,b,c\right)=\lim_{\zeta\to c}\frac{d^{N_0-\tau}}{d\zeta^{N_0-\tau}}\left[\left(\frac{\sin\beta(\zeta-t)}{\beta(\zeta-t)}\right)^{q}\cdot\frac{1}{\left(ae^{\delta\zeta}+be^{-\delta\zeta}\right)\sin^{N+1}(\alpha\zeta)}\right].\tag{4}$$

The proof is the same as for Theorem 1 from [1], and is based on the following estimate: if $f(z)$ is an entire function, which satisfies the condition [1]

$$\left|f(z)\right|\le L_f\cdot\left(1+|z|^m\right)\cdot e^{\sigma|y|},\qquad z=x+iy,\tag{5}$$

for some nonnegative integer m and positive real numbers L_f,σ, then for arbitrary fixed z and for all sufficiently large positive integers n we have

$$R_n(z)=\left|\frac{1}{p!}\lim_{\zeta\to z}\frac{d^p}{d\zeta^p}\left(\frac{f(\zeta)}{(\zeta-c)^{N_0+1}\left(ae^{\delta\zeta}+be^{-\delta\zeta}\right)\sin^{N+1}(\alpha\zeta)}\left(\frac{\sin\beta(\zeta-z)}{\beta(\zeta-z)}\right)^q\right)-\sum_{k=-n}^{n}\left(\frac{(-1)^k}{\alpha}\right)^{N+1}\cdot\right.$$
$$\left\{\sum_{\tau=0}^{N}\frac{1}{(N-\tau)!}\left[\sum_{\mu=0}^{\tau}\frac{\alpha^{\tau-\mu}}{(\tau-\mu)!}A_{\mu\tau N}\sum_{j=0}^{\mu}\frac{f^{(j)}\left(\frac{k\pi}{\alpha}\right)}{j!\left(z-\frac{k\pi}{\alpha}\right)}\sum_{r=0}^{\mu-j}\frac{(p+r)!(N_0+\mu-j-r)!(-1)^{p+\mu-j-r}}{r!N_0!p!(\mu-j-r)!\left(z-\frac{k\pi}{\alpha}\right)^r\left(\frac{k\pi}{\alpha}-c\right)^{N_0+\mu+1-j-r}}\right]\right\}\cdot$$
$$\left.\cdot\frac{\varphi_{\tau N}(z;k,q,\alpha,\beta,\delta,a,b)}{\left(z-\frac{k\pi}{\alpha}\right)^p}-\sum_{\tau=0}^{N_0}\frac{f^{(\tau)}(c)}{p!(N_0-\tau)!(z-c)^{p+1}}\sum_{\mu=0}^{\tau}\frac{(p+\tau-\mu)!\psi_{\tau NN_0}(z;q,\alpha,\beta,\delta,a,b,c)}{\mu!(\tau-\mu)!(c-z)^{\tau-\mu}}\right|\le L_f\cdot Q_{p,q,N,N_0}(z)\cdot\tag{6}$$
$$\cdot\frac{e^{q\beta|y|}}{\beta^q\left[(N+1)\alpha-\sigma-q\beta-\delta\right]}e^{-\delta\left(n+\frac{1}{2}\right)\frac{\pi}{\alpha}}\cdot\left[\left(\frac{\alpha}{\pi\left(n+\frac{1}{2}\right)}\right)^{p+q+N_0+2}+\left(\frac{\alpha}{\pi\left(n+\frac{1}{2}\right)}\right)^{p+q+N_0+2-m}\right],$$

where $z\ne\frac{\nu\pi}{\alpha},\nu=0,\pm1,\pm2,...$ and function $Q_{p,q,N,N_0}(z)=\frac{2^{p+q+N_0+2}}{D_0(a,b)}\cdot\left(\frac{2}{1-e^{-\pi}}\right)^{N+1}$, $D_0(a,b)=D_0(b,a)=\min\{a,b,|a-b|\}$ is finite on the arbitrarily bounded domain of the variable z (when $\delta>0$, then we have “exponential” convergence). The estimate (6) can be obtained if we apply Cauchy’s residuals theorem for the integral

$$\frac{1}{2\pi i}\int_{C_n}\frac{f(\zeta)}{(\zeta-c)^{N_0+1}\left(ae^{\delta\zeta}+be^{-\delta\zeta}\right)\sin^{N+1}(\alpha\zeta)}\left(\frac{\sin\beta(\zeta-z)}{\beta(\zeta-z)}\right)^q\frac{d\zeta}{(\zeta-z)^{p+1}},\tag{7}$$

where C_n is circle $|\zeta|=\left(n+\frac{1}{2}\right)\frac{\pi}{\alpha}$. To obtain estimate (6) the following inequality is used

$$\left|\frac{\sin z}{z}\right| \le \frac{e^{|y|}}{|z|}, \qquad z = x + iy. \tag{8}$$

Using the inequality

$$\left|\frac{\sin z}{z}\right| \le e^{|y|}, \qquad z = x + iy. \tag{9}$$

instead of (6) we shall obtain the following estimate

$$R_n(z) \le L_f Q_{p,0,N,N_0(z)} \frac{e^{q\beta|y|}}{(N+1)\alpha - \sigma - q\beta - \delta} e^{-\delta\left(n+\frac{1}{2}\right)\frac{\pi}{\alpha}} \left[\left(\frac{\alpha}{\pi\left(n+\frac{1}{2}\right)}\right)^{p+N_0+2} + \left(\frac{\alpha}{\pi\left(n+\frac{1}{2}\right)}\right)^{p+N_0+2-m}\right]. \tag{10}$$

The estimate (10) is suitable for small values of β, even at $\beta = 0$, however, in this case the estimate (6) makes no sense. When $N_0 = -1,$ then subintegral function from (7) has no pole in a point $\zeta = c,$ $(c \ne 0)$. The remainder term is equal to zero, therefore an addend from the right-hand member of the formula (1), is equal to zero. In a corollary the formula (1) of [4], and also its other special cases are obtained [1-5]. In this case, if also $p = q = \delta = N = 0,$ then from (1) the well-known Whittaker-Kotelnikov-Shannon‘s interpolation formula is obtained [1]. When $p = 0,$ then from (1) we obtain the folowing representation

$$\xi(t) = (t-c)^{N_0+1}\left(ae^{\delta t} + be^{-\delta t}\right)\sum_{k=-\infty}^{\infty}\left(\frac{\sin\alpha\left(t-\frac{k\pi}{\alpha}\right)}{\alpha\left(t-\frac{k\pi}{\alpha}\right)}\right)^{N+1}\sum_{\tau=0}^{N}\frac{1}{(N-\tau)!}$$

$$\left[\sum_{\mu=0}^{\tau}\frac{\alpha^{\tau-\mu}}{(\tau-\mu)!}A_{\mu\tau N}\sum_{j=0}^{\mu}\frac{\xi^{(j)}\left(\frac{k\pi}{\alpha}\right)}{j!}\sum_{r=0}^{\mu-j}\frac{(N_0+\mu-j-r)!(-1)^{\mu-j-r}\left(t-\frac{k\pi}{\alpha}\right)^{N-r}}{N_0!(\mu-j-r)!\left(\frac{k\pi}{\alpha}-c\right)^{N_0+\mu+1-j-r}}\right]\varphi_{\tau N}(t;k,q,\alpha,\beta,\delta,a,b) + \tag{11}$$

$$+(t-c)^{N_0}\left(ae^{\delta t}+be^{-\delta t}\right)\sin^{N+1}(\alpha t)\cdot\sum_{\tau=0}^{N_0}\frac{\xi^{(\tau)}(c)}{(N_0-\tau)!}\sum_{\mu=0}^{\tau}\frac{\psi_{\tau N N_0}(t;q,\alpha,\beta,\delta,a,b,c)}{\mu!(c-t)^{\tau-\mu}}.$$

When $q = 0$, then from (11) we obtain

$$\xi(t) = (t-c)^{N_0+1}\cdot\left(ae^{\delta t} + be^{-\delta t}\right)\cdot\sum_{k=-\infty}^{\infty}\left(\frac{\sin\alpha\left(t-\frac{k\pi}{\alpha}\right)}{\alpha\left(t-\frac{k\pi}{\alpha}\right)}\right)^{N+1}\cdot\sum_{\tau=0}^{N}\frac{1}{(N-\tau)!}\cdot$$

$$\cdot\left[\sum_{\mu=0}^{\tau}\frac{\alpha^{\tau-\mu}}{(\tau-\mu)!}A_{\mu\tau N}\sum_{j=0}^{\mu}\frac{\xi^{(j)}\left(\frac{k\pi}{\alpha}\right)}{j!}\sum_{r=0}^{\mu-j}\frac{(N_0+\mu-j-r)!(-1)^{\mu-j-r}\left(t-\frac{k\pi}{\alpha}\right)^{N-r}}{N_0!(\mu-j-r)!\left(\frac{k\pi}{\alpha}-c\right)^{N_0+\mu+1-j-r}}\right]\varphi_{\tau N}(t;k,0,\alpha,\beta,\delta,a,b) + \tag{12}$$

$$+(t-c)^{N_0}\left(ae^{\delta t}+be^{-\delta t}\right)\sin^{N+1}(\alpha t)\cdot\sum_{\tau=0}^{N_0}\frac{\xi^{(\tau)}(c)}{(N_0-\tau)!}\sum_{\mu=0}^{\tau}\frac{\psi_{\tau N N_0}(t;0,\alpha,\beta,\delta,a,b,c)}{\mu!(c-t)^{\tau-\mu}},$$

where $\varphi_{\tau N}\left(t;k,0,\alpha,\beta,\delta,a,b\right)=\lim\limits_{\zeta\to\frac{k\pi}{\alpha}}\dfrac{d^{N-\tau}}{d\zeta^{N-\tau}}\left[\dfrac{1}{\left(ae^{\delta\zeta}+be^{-\delta\zeta}\right)}\right]$ and

$$\psi_{\tau NN_0}\left(t;0,\alpha,\beta,\delta,a,b,c\right)=\lim_{\zeta\to c}\frac{d^{N_0-\tau}}{d\zeta^{N_0-\tau}}\left[\frac{1}{\left(ae^{\delta\zeta}+be^{-\delta\zeta}\right)\sin^{N+1}\left(\alpha\zeta\right)}\right].$$

When $\delta=0$, then

$$\varphi_{\tau N}\left(t;k,0,\alpha,\beta,0,a,b\right)=\begin{cases}\dfrac{1}{a+b}, & when\quad \tau=N,\\ 0, & when\quad \tau=0,1,2,...,N-1,\end{cases}$$

$$\psi_{\tau NN_0}\left(t;0,\alpha,\beta,0,a,b,c\right)=\frac{1}{a+b}\cdot\lim_{\zeta\to c}\frac{d^{N_0-\tau}}{d\zeta^{N_0-\tau}}\left[\frac{1}{\sin^{N+1}\left(\alpha\zeta\right)}\right]$$

and from (12) we obtain the following representation

$$\xi\left(t\right)=\left(t-c\right)^{N_0+1}\cdot\sum_{k=-\infty}^{\infty}\left(\frac{\sin\alpha\left(t-\frac{k\pi}{\alpha}\right)}{\alpha\left(t-\frac{k\pi}{\alpha}\right)}\right)^{N+1}\cdot$$

$$\left[\sum_{\mu=0}^{N}\frac{\alpha^{N-\mu}}{\left(N-\mu\right)!}A_{\mu NN}\sum_{j=0}^{\mu}\frac{\xi^{(j)}\left(\frac{k\pi}{\alpha}\right)}{j!}\sum_{r=0}^{\mu-j}\frac{\left(N_0+\mu-j-r\right)!\left(-1\right)^{\mu-j-r}\left(t-\frac{k\pi}{\alpha}\right)^{N-r}}{N_0!\left(\mu-j-r\right)!\left(\frac{k\pi}{\alpha}-c\right)^{N_0+\mu+1-j-r}}\right]+$$

$$+\left(t-c\right)^{N_0}\sin^{N+1}\left(\alpha t\right)\cdot\sum_{\tau=0}^{N_0}\frac{\xi^{(\tau)}\left(c\right)}{\left(N_0-\tau\right)!}\sum_{\mu=0}^{\tau}\frac{\psi_{\tau NN_0}\left(\alpha,c\right)}{\mu!\left(c-t\right)^{\tau-\mu}},\qquad(13)$$

where $\psi_{\tau NN_0}\left(\alpha,c\right)=\lim\limits_{\zeta\to c}\dfrac{d^{N_0-\tau}}{d\zeta^{N_0-\tau}}\left(\dfrac{1}{\sin^{N+1}\left(\alpha\zeta\right)}\right)$.

By virtue of (6), for convergence of series (13) the condition $N_0+2-m>0$ should be satisfied.

When $N=0,$ then in the first term belonging to a right part of the formula (13), indexes of summation are equal to zero $\mu=j=r=0$, and from (13) we receive

$$\xi\left(t\right)=\left(t-c\right)^{N_0+1}\cdot\sum_{k=-\infty}^{\infty}\frac{\sin\alpha\left(t-\frac{k\pi}{\alpha}\right)}{\alpha\left(t-\frac{k\pi}{\alpha}\right)}\cdot\frac{\xi\left(\frac{k\pi}{\alpha}\right)}{\left(\frac{k\pi}{\alpha}-c\right)^{N_0+1}}+\left(t-c\right)^{N_0}\sin\left(\alpha t\right)\cdot\sum_{\tau=0}^{N_0}\frac{\xi^{(\tau)}\left(c\right)}{\left(N_0-\tau\right)!}\sum_{\mu=0}^{\tau}\frac{\tilde{\psi}_{\tau N_0}\left(\alpha,c\right)}{\mu!\left(c-t\right)^{\tau-\mu}},\qquad(14)$$

where $\tilde{\psi}_{\tau N_0}\left(\alpha,c\right)=\lim\limits_{\zeta\to c}\dfrac{d^{N_0-\tau}}{d\zeta^{N_0-\tau}}\left(\dfrac{1}{\sin\left(\alpha\zeta\right)}\right)$. When $N_0=0$, then in the second term belonging to the right part of the formula (14), indexes of summation are equal to zero $\mu=\tau=0,\ \ \tilde{\psi}_{00}\left(\alpha,c\right)=\dfrac{1}{\sin\left(\alpha c\right)}$ and from (14) we receive

$$\xi\left(t\right)=\left(t-c\right)\sum_{k=-\infty}^{\infty}\frac{\xi\left(\frac{k\pi}{\alpha}\right)}{\frac{k\pi}{\alpha}-c}\cdot\frac{\sin\alpha\left(t-\frac{k\pi}{\alpha}\right)}{\alpha\left(t-\frac{k\pi}{\alpha}\right)}+\frac{\sin\left(\alpha t\right)}{\sin\left(\alpha c\right)}\cdot\xi\left(c\right).\qquad(15)$$

When $c = 0$ and $k = 0$, then (15) loses sense. Let us rewrite (15) in the following form

$$\xi(t) = (t-c)\sum_{k=-\infty,k\neq 0}^{\infty} \frac{\xi\left(\frac{k\pi}{\alpha}\right)}{\frac{k\pi}{\alpha}-c}\cdot\frac{\sin\alpha\left(t-\frac{k\pi}{\alpha}\right)}{\alpha\left(t-\frac{k\pi}{\alpha}\right)} + \frac{(c-t)\xi(0)\sin(\alpha t)}{\alpha ct} + \frac{\sin(\alpha t)}{\sin(\alpha c)}\cdot\xi(c). \tag{16}$$

For the entire function $f(z)$ we can write

$$\lim_{c\to 0}\left[\frac{(c-t)f(0)\sin(\alpha t)}{\alpha ct} + \frac{\sin(\alpha t)}{\sin(\alpha c)}\cdot f(c)\right] = \lim_{c\to 0}(c-t)\sin(\alpha t)\left[\frac{f(0)}{\alpha ct} + \frac{f(c)}{(c-t)\sin(\alpha c)}\right] = \sin(\alpha t)\left[\frac{f'(0)}{\alpha} + \frac{f(0)}{\alpha t}\right]. \tag{17}$$

By virtue of (17) formula (16) for process $\xi(t)$ will have the following form

$$\xi(t) = t\sum_{k=-\infty,k\neq 0}^{\infty} \frac{\xi\left(\frac{k\pi}{\alpha}\right)}{\frac{k\pi}{\alpha}}\cdot\frac{\sin\alpha\left(t-\frac{k\pi}{\alpha}\right)}{\alpha\left(t-\frac{k\pi}{\alpha}\right)} + \sin(\alpha t)\left[\frac{\xi'(0)}{\alpha} + \frac{\xi(0)}{\alpha t}\right]. \tag{18}$$

When $N_0 = 0$, $N = 0$, $\delta = 0$, $c = 0$, then $\zeta = 0$ is the second order pole of intergrand function in (7) and if we apply Cauchy's residuals theorem, the formula (18) directly turns out for the entire function $f(z)$. After that, the formula (18) is proved for random process $\xi(t)$ similarly [1]. For entire functions the formula (18) is given in [6: 197].

Let us note that under certain conditions and in a certain sense the formula (1) is fair as well for stochastic $L^{\alpha}(\Omega)$ –processes (about these processes see [7]).

For simplicity we shall reduce the generalization of the formula (1) only for a two-dimensional random field.

Theorem 2. *If the two-dimensional stochastic field* $\xi(t_1,t_2)$, $-\infty < t_1, t_2 < \infty$ *satisfies the conditions of the theorem* (2) *from* [3], *then for almost all sampling functions of the stochastic field* $\xi(t_1,t_2)$ *the following formula is true*

$$\frac{1}{p_1!p_2!}\lim_{\substack{\zeta_1\to t_1\\ \zeta_2\to t_2}}\frac{\partial^{p_1+p_2}}{\partial\zeta_1^{p_1}\partial\zeta_2^{p_2}}\left(\xi(\zeta_1,\zeta_2)\prod_{i=1}^{2}\frac{\left(\frac{\sin\beta_i(\zeta_i-t_i)}{\beta_i(\zeta_i-t_i)}\right)^{q_i}}{(\zeta_i-c_i)^{N_{0i}+1}\left(\left(a_i e^{\delta_i\zeta_i}+b_i e^{-\delta_i\zeta_i}\right)\sin^{N_i+1}(\alpha_i\zeta_i)\right)}\right) =$$

$$= \sum_{k_1,k_2=-\infty}^{\infty}\prod_{i=1}^{2}\left[\frac{(-1)^{k_i}}{\alpha_i}\right]^{N_i+1}\left\{\sum_{\tau_1=0}^{N_1}\sum_{\tau_2=0}^{N_2}\frac{1}{(N_1-\tau_1)!(N_2-\tau_2)!}\sum_{\mu_1=0}^{\tau_1}\sum_{\mu_2=0}^{\tau_2}\prod_{i=1}^{2}\frac{\alpha_i^{\tau_i-\mu_i}}{(\tau_i-\mu_i)!}\cdot A_{\mu_i\tau_i N_i}\cdot\right.$$

$$\cdot\sum_{j_1=0}^{\mu_1}\sum_{j_2=0}^{\mu_2}\left[\frac{\partial^{j_1+j_2}\xi(t_1,t_2)}{\partial t_1^{j_1}\partial t_2^{j_2}}\right]_{\substack{t_1=\frac{k_1\pi}{\alpha_1}\\ t_2=\frac{k_2\pi}{\alpha_2}}}\cdot\frac{1}{j_1!j_2!\left(t_1-\frac{k_1\pi}{\alpha_1}\right)\left(t_2-\frac{k_2\pi}{\alpha_2}\right)}\cdot$$

$$\cdot\sum_{r_1=0}^{\mu_1-j_1}\sum_{r_2=0}^{\mu_2-j_2}\frac{(p_1+r_1)!(p_2+r_2)!(N_{01}+\mu_1-j_1-r_1)!(N_{02}+\mu_2-j_2-r_2)!(-1)^{p_1+p_2+\mu_1+\mu_2-j_1-r_1-j_2-r_2}}{r_1!r_2!N_{01}!N_{02}!p_1!p_2!(\mu_1-j_1-r_1)!(\mu_2-j_2-r_2)!\left(t_1-\frac{k_1\pi}{\alpha_1}\right)^{r_1}\left(t_2-\frac{k_2\pi}{\alpha_2}\right)^{r_2}}\cdot \tag{19}$$

$$\left.\cdot\frac{1}{\left(\frac{k_1\pi}{\alpha_1}-c_1\right)^{N_{01}+\mu_1+1-j_1-r_1}\left(\frac{k_2\pi}{\alpha_2}-c_2\right)^{N_{02}+\mu_2+1-j_2-r_2}}\right\}\cdot\frac{\varphi_{\tau_i N_i}(t_i;k_i,q_i,\alpha_i,\beta_i,a_i,b_i,\delta_i)}{\left(t_i-\frac{k_i\pi}{\alpha_i}\right)^{p_i}}+$$

$$+\sum_{\tau_1=0}^{N_{01}}\sum_{\tau_2=0}^{N_{02}}\left[\frac{\partial^{\tau_1+\tau_2}\xi(t_1,t_2)}{\partial t_1^{\tau_1}\partial t_2^{\tau_2}}\right]_{\substack{t_1=c_1\\ t_2=c_2}}\cdot\prod_{i=1}^{2}\sum_{\mu_i=0}^{\tau_i}\frac{(p_i+\tau_i-\mu_i)!\psi_{\tau_i N_i N_{0i}}(t_i;q_i,\alpha_i,\beta_i,\delta_i,a_i,b_i,c_i)}{\mu_i!(\tau_i-\mu_i)!(c_i-t_i)^{\tau_i-\mu_i}},$$

$$t_i \neq \frac{\nu_i \pi}{\alpha_i}, \quad \nu_i = 0, \pm 1, \pm 2, \ldots, \quad i = 1,2, \textit{ for every } \alpha_i > \frac{\sigma_i}{N_i + 1}, \quad 0 < \beta_i < \frac{(N_i + 1)\alpha_i - \sigma_i}{q_i}, \quad 0 < \delta_i < (N_i + 1)\alpha_i - \sigma_i - q_i \beta_i,$$

where $a_i, b_i, \alpha_i, \beta_i, \delta_i,\ i = 1,2$ *are fixed positive real numbers,* c_1, c_2 *are fixed real numbers.* $A_{\mu\tau N}$, $\varphi_{\tau N}(t;k,q,\alpha,\beta,\delta,a,b)$, $\psi_{\tau N N_0}(t;q,\alpha,\beta,\delta,a,b,c)$ *are determined according to formulas (2), (3) and (4). In special cases from (19) formulas which are given in [3-5,8] turn out. When* $p_i = 0$, $q_i = 0$, $N_i = 0$, $i = 1,2$, *then from (19) the following is received*

$$\xi(t_1,t_2) = \prod_{i=1}^{2} (t_i - c_i)^{N_{0i}+1} \left(a_i e^{\delta_i t_i} + b_i e^{-\delta_i t_i}\right) \sum_{k_1,k_2=-\infty}^{\infty} \prod_{i=1}^{2} \frac{\sin \alpha_i \left(t_i - \frac{k_i \pi}{\alpha_i}\right)}{\alpha_i \left(t_i - \frac{k_i \pi}{\alpha_i}\right)} \cdot \frac{\xi\left(\frac{k_1\pi}{\alpha_1}, \frac{k_2\pi}{\alpha_2}\right)}{\left(\frac{k_i\pi}{\alpha_i} - c_i\right)^{N_{0i}+1}} +$$

$$+ \sin(\alpha_1 t_1) \sin(\alpha_2 t_2) \sum_{\tau_1=0}^{N_{01}} \sum_{\tau_2=0}^{N_{02}} \left[\frac{\partial^{\tau_1+\tau_2} \xi(t_1,t_2)}{\partial t_1^{\tau_1} \partial t_2^{\tau_2}}\right]_{\substack{t_1=c_1 \\ t_2=c_2}} \cdot \prod_{i=1}^{2} \sum_{\mu_i=0}^{\tau_i} \frac{\tilde{\psi}_{\tau_i N_{0i}}(\alpha_i, c_i)}{\mu_i!\,(c_i - t_i)^{\tau_i - \mu_i}}, \qquad (20)$$

where the function $\tilde{\psi}_{\tau N_0}(\alpha, c)$ *is determined from* (14).

მათემატიკა

ანათვლების ერთი განზოგადებული სწრაფადკრებადი მწკრივის შესახებ

ზ. ფირანაშვილი

ვლ. ჭავჭანიძის კიბერნეტიკის ინსტიტუტი, თბილისი

(წარმოდგენილია აკადემიის წევრის ე. ნადარაიას მიერ)

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