Hydraulic Engineering

Determination of the Darcy Coefficient at Pressure Flow of Non-Newtonian Fluid in the Pipe

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ABSTRACT. The methodology of hydraulic calculation of the pressure losses along the length at the motion of non-Newtonian fluid with flow core in a round pipe is presented in the paper. *© 2012 Bull. Georg. Natl. Acad. Sci.*

***Key words**: non-Newtonian fluid, flow core, pressure loss along the length.*

Non-Newtonian fluids, where together with viscosity some additional properties appear, such as limit pressure shift, at the reaching of which the medium begins to flow as Newtonian fluid, attract special interest due to their wide spread.

An attempt is made to determine the value of Darcy coefficient at the motion of non-Newtonian fluid with flow core.

In the Figure the calculation scheme of the distribution of the velocities of the pressure motions of non-Newtonian fluid with core of the flow in pipeline of round form is presented. Such a motion is necessary to consider in the system of (x,r) where "Ox" is directed along the axis of the pipe. Let us denote the radius of the pipe r_c, core radius – r_o.

The rheologic law of the non-Newtonian fluid motion is often described by the Shvedov-Bingham equation [1]:

$$\tau = \tau_0 \pm \mu \frac{du}{dr}, \tag{1}$$

where τ – friction stress at the point of the pipe cross-section, τ_0 – limit stress of the shift after reaching of which the flow of the medium begins, μ – dynamic coefficient of structural viscosity, $\frac{du}{dr}$ – shift speed, r – flowing radius of the point.

Physical explanation of specific properties of such media is based on the presence in them of some inner hard structure at rest, which resists outside impact up to $\tau < \tau_0$ (i.e. fluidity is absent) and the medium behaves as a solid body. The medium begins "to flow" when $\tau > \tau_0$, making it unlike viscous non-Newtonian fluids. At the motion of these flows the medium sticks to the wall, as the result of which the gradient of velocity is observed in the contact surface of the flow with the pipe. Not rarely such a medium is characterized by flow core with undestroyed structure. Flow core on straight-line parts of the pipe behaves as a pivot (quasi-solid body). To such media existing in practice we refer sewage hyperconcen-

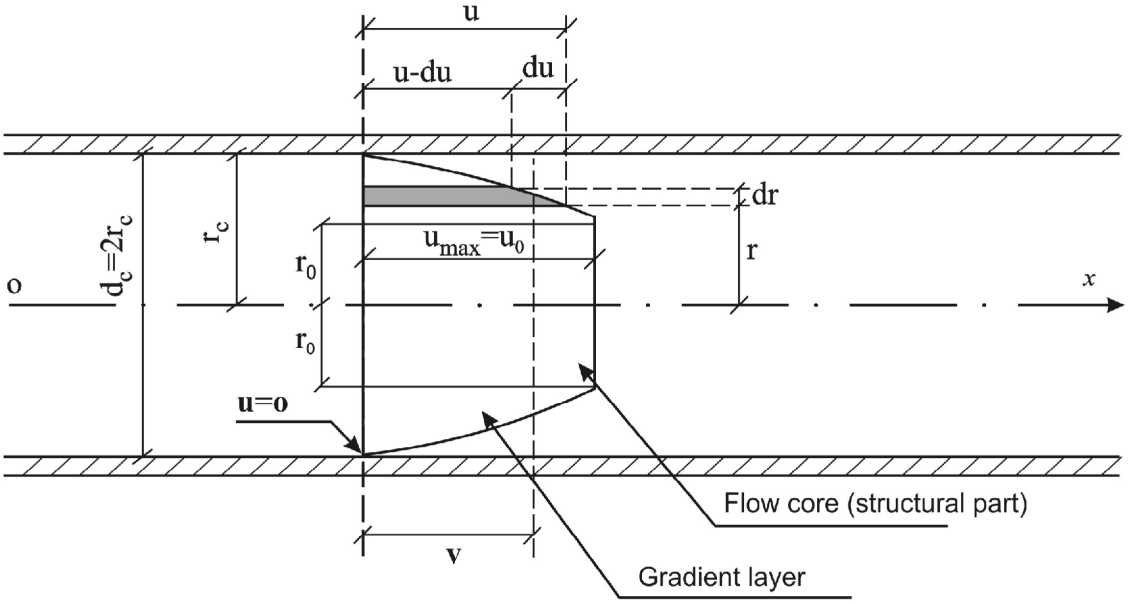

Fig. Scheme of distribution of the pressure flow velocities of non-Newtonian fluid with core of the flow in the pipeline of round cross-section.

trated alluvial debris flows (in draw-off tunnels), clayish and cement slurry (used for washing out of the oil field wells), concrete motion in pressure pipelines of concrete drift, etc.

Naturally, it is not possible to calculate Darcy (λ) coefficient in the mentioned cases according to the known dependences of hydraulics.

Considering that $\tau = \gamma RI = \gamma \frac{r}{2} I$, where R is hydraulic radius of the pipe, γ – specific weight of non-Newtonian fluid, I – hydraulic gradient, instead of (1) we get

$$\gamma \frac{r}{2} I = \tau_0 - \mu \frac{du}{dr}. \quad (2)$$

Integration of (2) with account of boundary conditions, when $r=r_c$ then $u=0$ gives

$$u = \frac{\gamma I}{4\mu}\left(r_c^2 - r^2\right) - \frac{\tau_0}{\mu}\left(r_c - r\right). \quad (3)$$

At $\tau_0 = 0$ dependence (3) takes generally known form of characteristics of laminar motion of non-Newtonian fluid [2]:

$$u = \frac{\gamma I}{4\mu}\left(r_c^2 - r^2\right). \quad (4)$$

At $r = r_0$ from (3) we obtain the velocity of core motion (structural part):

$$u = u_0 = u_{\max} = \frac{\gamma I}{4\mu}\left(r_c^2 - r_0^2\right) - \frac{\tau_0}{\mu}\left(r_c - r_0\right). \quad (5)$$

Elementary medium discharge in the pipe will be

$$dQ = ud\omega = \frac{\gamma I}{4\mu}\left(r_c^2 - r^2\right)2\pi r dr - \frac{\tau_0}{\mu}\left(r_c - r\right)2\pi r dr, \quad (6)$$

where $d\omega$ is area of elementary live cross-section.

Taking into account that the first member of the right part of (6) characterizes elementary discharge of gradient layer of the flow (i.e. discharge in the limits of a ring between radii r_0 and r_c) the second part presents the flow core discharge from 0 to r_0, then after integration of (6) we shall get the value of total medium discharge in the pipe with radius r_c:

$$Q = \frac{\pi}{\mu}\left[\frac{I\gamma}{8}\left(r_c^4 - 2r_c^2 r_0^2 + r_0^4\right) + 2\tau_0\left(\frac{r_c r_0^2}{2} - \frac{r_0^3}{3}\right)\right]. \quad (7)$$

At $r_0=0$, i.e. for non-Newtonian fluid, (7) takes the form of [2], i.e. we get the Poiseuille dependence:

$$Q = \frac{\pi I r_c^4 \gamma}{8\mu}. \quad (8)$$

Table. Number values of $N = f\left(\frac{d_0}{d_c}\right)$**:**

$\beta \approx d_0/d_c$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
N	0.98	0.95	0.91	0.893	0.896	0.928	0.992	1.085	1.2025

Taking into consideration that $\tau_0 = \gamma \frac{r_0}{2} I$, expression (7) takes the form:

$$Q = \frac{\gamma \pi I}{2\mu}\left[\frac{r_c^4}{4} - \frac{r_c^2 r_0^2}{2} + r_c r_0^3 - \frac{5}{12} r_0^4\right]. \qquad (7')$$

Average velocity of the motion of non-Newtonian fluid with flow core in the round pipe will be:

$$V = \frac{Q}{\pi \ r_c^2} = \frac{\gamma I}{2\mu}\left(\frac{r_c^2}{4} - \frac{r_0^2}{2} + \frac{r_0^3}{r_c} - \frac{5}{12}\frac{r_0^4}{r_c^2}\right). \qquad (9)$$

Changing the radius of the pipe via diameter of the pipe *d* and taking into account $\gamma = \rho g$, $\upsilon = \frac{\mu}{\rho}$, where υ is coefficient of kinematic viscosity, g – acceleration of gravity force, ρ – density, instead of (9) we shall get

$$V = \frac{Igd_c^2}{32\upsilon}\left(1 - 2\frac{d_0^2}{d_c^2} + 4\frac{d_0^3}{d_c^3} - \frac{5}{3}\frac{d_0^4}{d_c^4}\right). \qquad (10)$$

Taking into account that hydraulic gradient on straight-line parts of the pipe $I = \frac{h_l}{l}$, where l is the length of the straight-line pipe, h_l – the value of loss by friction along the length, we can write:

$$h_l = \frac{32lV\nu}{gd_c^2\left(1 - 2\beta^2 + 4\beta^3 - \frac{5}{3}\beta^4\right)}, \qquad (11)$$

where $\beta = \frac{d_0}{d_c}$ is a relative diameter.

Considering that the general expression for losses of the pressure by length [1,2]:

$$h_l = \lambda \frac{l}{d_c}\frac{V^2}{2g}, \qquad (12)$$

Comparing (11) and (12) and determining λ we obtain

$$\lambda = \frac{64\nu}{Vd_c N}, \qquad (13)$$

where

$$N = 1 - 2\beta^2 + 4\beta^3 - \frac{5}{3}\beta^4. \qquad (14)$$

Or expressing (13) via the Reynolds number $\mathrm{Re} = \frac{Vd_c}{\upsilon}$ instead of (13) we get

$$\lambda = \frac{16}{\mathrm{Re} \cdot N}. \qquad (15)$$

Thus, pressure losses at friction in straight-line parts of the pipe at the motion of non-Newtonian fluid with flow core reach the maximum at $d_0 = \frac{d_c}{2}$, which should be taken into consideration while choosing force devices (plants) of such systems.

ჰიდროტექნიკა

მილსადენებში არანიუტონური სითხის დაწნევიანი მოძრაობის დროს დარსის კოეფიციენტის განსაზღვრა

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