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Mathematics

On the Regular Convergence by Rectangles of Multiple Fourier-Haar Series

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ABSTRACT. It is sharpened the known result on almost everywhere convergence by rectangles of Fourier-Haar series of functions from $L(\ln^+ L)^{n-1}([0,1]^n)$. Namely, it is proved that the condition $f \in L(\ln^+ L)^k([0,1]^n)$, where $0 \le k \le n-1$, implies almost everywhere convergence of each subseries of Fourier-Haar series of *f* having dimension not more than *k*+1. *© 2012 Bull. Georg. Natl. Acad. Sci*

Key words: *Fourier-Haar series, multiple series, convergence, subseries.*

Definitions and notation. Below everywhere if something else is not said we will assume that $n \in \mathbb{N}$ and $n \ge 2$. For a given *n*-tuple Haar series

$$\sigma = \sum_{k\in \;{}^{n}} c_k h_k$$

by $S_m(\sigma)(x)$, $S_r(\sigma)(x)$, and $G_m(\sigma)(x)$, where $x \in \mathbb{I}^n \equiv [0,1]^n$, $m \in \mathbb{N}^n$ and $r > 0$, denote, respectively, a rectangular partial sum, a spherical partial sum, and a rectangular general term of σ, i.e.,

$$S_m(\sigma)(x) = \sum_{k_1=1}^{m_1} \cdots \sum_{k_1=1}^{m_1} c_k h_k(x),$$

$$S_r(\sigma)(x) = \sum_{k_1^2+\cdots+k_n^2 \le r} c_k h_k(x),$$

$$G_m(\sigma)(x) = c_m h_m(x),$$

where $m = (m_1,\ldots,m_n)$, $k = (k_1,\ldots,k_n)$. If σ is the Fourier-Haar series of a function $f \in L(\mathbb{I}^n)$, then we will use *f* instead of σ in the above notation.

Fourier-Haar series of a function $f \in L(\mathbb{I}^n)$ at a point $x \in \mathbb{I}^n$ denote by $\sigma(f)(x)$.

Let $1 \le k \le n-1$. Any sequence obtained from $(a_m)_{m\in\mathbb{N}^n}$ by fixing some $n-k$ among n coordinates of the index $m=(m_1,\dots,m_n)$ is called a k-dimensional section of a sequence $(a_m)_{m\in\mathbb{N}^n}$. By n-dimensional section of $(a_m)_{m\in\mathbb{N}^n}$ we mean $(a_m)_{m\in\mathbb{N}^n}$.

Let $1 \le k \le n$. Any series composed by some k-dimensional section of the sequence $(a_m)_{m\in\mathbb{N}^n}$ is called k-dimensional subseries of the series $\sum_{m\in\mathbb{N}^n} a_m$.

Let us say that a series $\sum_{m\in\mathbb{N}^n} a_m$ regularly converge by rectangles if its every subseries converges by rectangles (here convergence by rectangles of single series is understood as its ordinary convergence). Note that such type of convergence of multiple series was considered by Móricz [1, 2].

Saying that a sequence $(a_m)_{m\in\mathbb{N}^n}$ converges (strongly converges) we mean the convergence of a_m to the limit as $\min\{m_1,\dots,m_n\}\to\infty$ ($\max\{m_1,\dots,m_n\}\to\infty$).

Note that for a sequence $(a_m)_{m\in\mathbb{N}^n}$ the following statements are equivalent: 1) $(a_m)_{m\in\mathbb{N}^n}$ strongly converges to a; 2) $(a_m)_{m\in\mathbb{N}^n}$ converges to a and every (n–1)-dimensional section of $(a_m)_{m\in\mathbb{N}^n}$ strongly converges to a; 3) every section of $(a_m)_{m\in\mathbb{N}^n}$ converges to a.

It is clear that if series $\sum_{m\in\mathbb{N}^n} a_m$ regularly converges by rectangles, then its general term $(a_m)_{m\in\mathbb{N}^n}$ strongly converges to 0.

Result. It is known that for every $n \ge 2$, $L(\ln^+ L)^{n-1}(\mathbb{I}^n)$ is the widest integral class in which the almost everywhere convergence of Fourier-Haar series is provided both for rectangular partial sums (see [3,4]) and for spherical partial sums (see [5-7]). Here we note that cubic partial sums of Fourier-Haar series of every summable function f (i.e. $S_m(f)(x)$ with $m_1 = m_2 = \cdots = m_n$) converge almost everywhere to f (see e.g. [4] for details).

The following theorem is true.

Theorem. *If* $0 \le k \le n-1$ *and* $f \in L(\ln^+ L)^k(\mathbb{I}^n)$*, then every* $(k+1)$*-dimensional subseries of* $\sigma(f)(x)$ *regularly converges by rectangles for almost every* $x \in \mathbb{I}^n$.

Corollary 1. *If* $f \in L(\ln^+ L)^{n-1}(\mathbb{I}^n)$*, then* $\sigma(f)(x)$ *regularly converges by rectangles for almost every* $x \in \mathbb{I}^n$.

Corollary 2 ([3,4]). *If* $f \in L(\ln^+ L)^{n-1}(\mathbb{I}^n)$*, then* $\sigma(f)(x)$ *converges by rectangles for almost every* $x \in \mathbb{I}^n$.

Corollary 3 ([8]). *If* $f \in L(\ln^+ L)^{n-2}(\mathbb{I}^n)$*, then every* ($n$–1)*-dimensional section of the sequence* $(G_m(f)(x))_{m\in\mathbb{N}^n}$ *strongly converges to* 0 *for almost every* $x \in \mathbb{I}^n$.

Remark. In [8] it was proved that: If an n-tuple Haar series σ converges by rectangles at a diadic-irrational point x and the rectangular general term $(G_m(\sigma)(x))_{m\in\mathbb{N}^n}$ strongly converges to 0, then σ

spherically converges at x to $\lim\limits_{m\to\infty} S_m(\sigma)(x)$.

From this result follows the corollary: *If an n-tuple Haar series* σ *regularly converges by rectangles at a diadic-irrational point x, then* σ *spherically converges at x to* $\lim\limits_{m\to\infty} S_m(\sigma)(x)$.

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გაძლიერებულია ცნობილი შედეგი $L(\ln^+ L)^{n-1}([0,1]^n)$ ფუნქციის ფურიე-ჰაარის მწკრივის მართკუთხედების მიხედვით თითქმის ყველგან კრებადობის შესახებ. სახელდობრ, დამტკიცებულია, რომ $f \in L(\ln^+ L)^k([0,1]^n)$ პირობა, სადაც $0 \le k \le n-1$, იწვევს f-ის ფურიე-ჰაარის მწკრივის, k+1-ზე არაუმეტესი განზომილების მქონე ყოველი ქვემწკრივის თითქმის ყველგან კრებადობას.

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