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Physics

On Quantum Description of Parametric Instability of Coherently Precessing Spin Mode of Superfluid ^{3}He-A

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ABSTRACT. Among actual problems of the spin dynamics of superfluid ^{3}*He* the anomalously fast (catastrophic) decay of coherently spin-precessing mode is under active investigation. Experimentally this low-temperature phenomenon is observed in ^{3}*He-B*. One of the possible mechanisms of the fast relaxation of the phase-coherent spin precession was attributed to the parametric instability realized in the background of a non-Leggett spin-orbital configuration of the Cooper condensate. An analogous process of the decay of coherent spin precession can be realized in ^{3}*He-A*. The quantum description of this event is considered in some detail. *© 2012 Bull. Georg. Natl. Acad. Sci.*

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After the discovery [1, 2] of superfluid phases of liquid ^{3}*He* in the millikelvin temperature range a huge amount of information about the properties of this exotic ordered state of condensed matter has been collected. It was firmly established that superfluid phases of ^{3}*He* are characterized by the formation at $T < T_c(P)$ of the spin-triplet p-wave Cooper pair's condensate. The main body of experimental information on the properties of superfluid phases was obtained by means of CW and pulsed NMR techniques. Of fundamental importance is Leggett's theoretical approach [3] which proved to be extremely efficient in interpreting many peculiarities of observed spin-dynamical phenomena of superfluid A and B phases.

An important role in Leggett's theory is played by the coupling between magnetic dipole moments of ^{3}*He* nuclei which in the presence of the Cooper condensate acquires a coherent contribution with far-reaching consequences. One of the manifestations of this coherent part of the dipole-dipole (spin-orbital) coupling in superfluid ^{3}He is observed as a shift of the transverse NMR frequency from the Larmor value gH. Another exotic behaviour (also due to coherent dipole-dipole coupling) is manifested as a linear (and non-linear) longitudinal ringing of the magnetization.

A very informative experimental tool is the pulsed NMR by which the spin-dynamical regime is generated by the application of a short RF pulse (of suitable duration) deviating the spin by an angle β from its initial equilibrium orientation and free induction signal is registered. The measurements of the dependence of

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dipole-shift of the transverse NMR frequency from the Larmor value was the main goal of pioneering experiments [4, 5]. Unusually long-lived induction decay signal (LLIDS) was registered in *^{3}He-B* in spite of the presence of (uncontrolled) inhomogeneity of the applied steady magnetic field which induces the dephasing of the coherent spin-precessing mode.

The mechanism of the formation of LLIDS in *^{3}He-B* was established due to joint experimental and theoretical efforts [6-9]. The experiments were performed by using a long cylindrical cell with the magnetic field $\vec{H} = H(z)\hat{z}$ oriented along cylinder axis $\hat{z}$. The controlled field inhomogeneity was given as $H(z) = H(z_0) + (\nabla H)(z - z_0)$ with constant gradient. The scenario which develops after the application of magnetization tipping RF pulse is based on the observation that due to the specific character of the dipole-dipole coupling potential of *^{3}He-B* on the angle β the spin dynamics follows the route where the dipole forces are effectively eliminated. This is an important condition of the appearance of the magnetization-carrying spin current against the background of the Cooper condensate[10,11] which redistributes the magnetization in a way as to restore the phase coherency lost immediately after the RF tipping pulse. As a result, after the longitudinal spin current dies away, in the closed experimental cell the two-domain structure is stabilized where the static domain (SD) with equilibrium magnetization is separated by a narrow wall from the homogeneously precessing domain (HPD). The two-domain structure is subject to relaxation effects taking place in the volume of HPD due to an intrinsic Leggett-Takagi mechanism [12]active in bulk superfluid ^{3}He and spin diffusion across the domain boundary. It is evident that the decay of the two-domain spin structure should be observed as the growth of SD at the expense of the shrinking of HPD. In this way the parameters of Leggett-Takagi mechanism and spin-diffusion coefficient have been measured.

In [13] an attempt was made to continue the investigation of relaxation effects in the two-domain structure at low temperatures. Down to $T \cong 0.4T_c$ the experimental data changed with temperature rather slowly and were in accordance with known theoretical predictions. At a further lowering of the temperature the induction decay signal abruptly became so short that the induction signal typical of HPD could not be measured. Subsequently new measurements were carried out [14] in order to verify whether the above-mentioned low temperature instability of coherent spin precession was a specific feature of the presence of HPD in the experimental cell. The measurements were carried out in the pulsed NMR regime in the absence of HPD structure and the low-temperature instability was still found. This result clearly showed that the catastrophic relaxation found in [13] is just a particular example of a more general phenomenon. The adequate mechanism of this puzzling low-temperature catastrophic relaxation could not be found for a long period of time.

An important contribution in elucidating the mechanism of the catastrophic relaxation of coherent spin precession observed in *^{3}He-B* at low temperatures was made in [15, 16]. In [15] it was proposed that an abrupt shortening of the relaxation time is due to the parametric instability of the spin-precessing mode with respect to the excitation of a pair of the longitudinal spin-wave quanta. This process is governed by the dipole-dipole forces acting against the background of a non-Leggett spin-orbital configuration of the Cooper condensate in the restricted geometry case. Another channel of the sudden decay of coherently precessing spin mode was attributed to the anisotropy of the spin-wave velocity [16].

An attempt to describe the parametric instability of spin-precessing mode in superfluid *^{3}He* was undertaken in [17] where this process is considered for the case of *^{3}He-A*. Although this ordered state considerably differs from *^{3}He-B*, the mechanism of the decay of coherently precessing mode seems to be quite general in the presence of a non-Leggett spin-orbital background of the Cooper condensate. The essence of the parametric instability in *^{3}He-A* is connected with the excitation of the longitudinal spin-waves due to the non-linear coupling of the components of magnetic anisotropy axis $\hat{d}$ provided by the dipole-dipole forces against the background of non-Leggett spin-orbital configuration. This scenario was considered in the framework of the Mathieu equation. In what follows we are going to investigate, somewhat retrospectively, the quantum approach to the process of disintegration of coherently precessing mode (long-wavelength transverse magnon) via the decay onto a pair of the short-wavelength longitudinal magnons.

The spin dynamics of superfluid ^3He in the presence of an external magnetic field $\vec{H}$ is governed by the Leggett Hamiltonian

$$H = \frac{1}{2} g^2 \left(\hat{\chi}^{-1}\right)_{\mu\nu} S_\mu S_\nu - g\vec{S}\cdot\vec{H} + U_D , \tag{1}$$

where $\vec{S}$ (measured in the units of $\hbar$) stands for the total spin of the system, $\hat{\chi}$ – for the magnetic susceptibility tensor and g - for the gyromagnetic ratio of ^3He nucleus. An important contribution to the Hamiltonian (1) is the spin-orbital coupling (of dipole-dipole origin) which for ^3He-A is described by the potential

$$U_D(\hat{d}) = -\frac{1}{2}\left(\frac{\chi_\perp}{g^2}\right)\Omega_A^2 \left(\hat{d}\cdot\hat{l}\right)^2 , \tag{2}$$

where Ω_A is linear longitudinal NMR frequency and the unit vectors $\hat{d}$ and $\hat{l}$ mark the anisotropy axis of the spin-triplet Cooper condensate in the spin and orbital spaces, respectively. The axis $\hat{d}$ appears in the magnetic susceptibility tensor components

$$\chi_{\mu\nu} = \chi_\perp \left(\delta_{\mu\nu} - a d_\mu d_\nu\right), \quad a = \frac{\chi_\perp - \chi_{II}}{\chi_\perp} > 0 . \tag{3}$$

Decomposing the spin vector $\vec{S}$ onto the transverse and longitudinal components with respect to $\hat{d}$ as

$$g\vec{S}_\perp = \chi_\perp \left[\vec{H} - \left(\hat{d}\cdot\vec{H}\right)\hat{d}\right], \tag{4a}$$

$$g\vec{S}_{II} = \chi_{II}\left(\hat{d}\cdot\vec{H}\right)\hat{d} = S_{II}\hat{d} , \tag{4b}$$

the Hamiltonian (1) is reduced to

$$H = H_\perp + H_{II} + U_D\left(\hat{d}\right) , \tag{5}$$

where

$$H_\perp = \frac{g^2}{2\chi_\perp}\vec{S}_\perp^2 - g\vec{S}_\perp\cdot\vec{H} , \tag{6a}$$

$$H_{II} = \frac{g^2}{2\chi_\perp}\left[\vec{S}_{II}^2 + \frac{a}{1-a}\left(\hat{d}\cdot\vec{S}_{II}\right)^2\right] - g\vec{S}_{II}\cdot\vec{H}. \tag{6b}$$

Now, it is convenient to transform Eqs. (6a, b) in passing to new representation for $\vec{S}_\perp$ and $S_{II} = \hat{d}\cdot\vec{S}_{II}$.

Since $\vec{S}_\perp \perp \hat{d}$ one can introduce a new set of $2D$ variables $\vec{\xi}$ following the formula [18]

$$\vec{S}_\perp = \hat{d} \times \vec{\xi} \ . \tag{7}$$

On the other hand, taking into account that $(\hat{d} \cdot \vec{S})$ is a conserved quantity, it is concluded that $S_{II} = S_{II}^{(0)} = \frac{\chi_{II}}{g}\left(\hat{d}_0 \cdot \vec{H}\right)$, where $\hat{d}_0$ is the equilibrium value of $\hat{d}$. As a result it is found that

$$H_\perp = \frac{g^2}{2\chi_\perp}\left[\vec{\xi}^{\,2} - \left(\hat{d} \cdot \vec{\xi}\right)^2\right] - g\left(\hat{d} \times \vec{\xi}\right)\vec{H} \ , \tag{8a}$$

$$H_{II} = -\chi_{II}\left[\left(\hat{d}_0 \cdot \vec{H}\right)\left(\hat{d} \cdot \vec{H}\right) - \frac{1}{2}\left(\hat{d}_0 \cdot \vec{H}\right)^2\right]. \tag{8b}$$

In what follows we choose the coordinate frame with $\vec{H} = H\,\hat{z}$ and consider the case of an externally fixed orientation of the orbital axis $\hat{l} = (0, \cos\alpha, \sin\alpha)$. At $\alpha \neq 0$ the equilibrium orientation of the spin axis $\hat{d}_0$ is not confined to (x, y) plane because the equilibrium configuration is established at the balance of the torque imposed on $\hat{d}$ from the anisotropic part of the magnetic energy and the torque acting on $\hat{d}$ from a fixed orbital field $\hat{l}$ via the dipole-dipole potential:

$$a\left(\hat{d}_0 \cdot \vec{H}\right)\left(\hat{d}_0 \times \vec{H}\right) = \left(\frac{\Omega_A}{g}\right)^2\left(\hat{d}_0 \cdot \hat{l}\right)\left(\hat{d}_0 \times \hat{l}\right), \tag{9}$$

from which it is concluded that $\hat{d}_0 = (0, \cos\beta, \sin\beta)$ with $\beta = \beta\,(\alpha)$ given by an equation (here and in what follows the Larmor frequency $\omega_L = gH$ is introduced):

$$\tan 2\beta = \frac{\left(\dfrac{\Omega_A}{\omega_L}\right)^2 \sin 2\alpha}{a + \left(\dfrac{\Omega_A}{\omega_L}\right)^2 \cos 2\alpha} \ . \tag{10}$$

It can by shown that $\vec{\xi}_0 = -\left(\frac{\chi_\perp}{g^2}\right)\omega_L \cos\beta\ \hat{x}$.

Finally, in choosing $\vec{\xi} = (\xi_x, 0, \xi_z)$ and remembering that $\hat{d}^2 = 1$ the Leggett Hamiltonian can be transcribed in terms of conjugated pairs of variables (ξ_x, d_x) and (ξ_z, d_z) as

$$H = \frac{g^2}{2\chi_\perp}\left[\xi_x^2 + \xi_z^2 - (\xi_x d_x + \xi_z d_z)^2\right] + \omega_L \xi_x \sqrt{1 - \left(d_x^2 + d_z^2\right)} - \chi_{II}\left[\left(\hat{d}_0 \cdot \vec{H}\right)\left(\hat{d} \cdot \vec{H}\right) - \frac{1}{2}\left(\hat{d}_0 \cdot \vec{H}\right)^2\right] - $$
$$- \frac{\chi_\perp}{2g^2}\Omega_A^2\left[\left(1 - d_x^2\right)\cos^2\alpha - d_z^2 \cos 2\alpha + d_z\sqrt{1 - \left(d_x^2 + d_z^2\right)}\ \sin 2\alpha\right]. \tag{11}$$

Starting from Eq. (11) the set of the Hamilton equations follows:

$$\dot{\xi}_x = -\frac{\partial H}{\partial d_x}, \quad \dot{d}_x = \frac{\partial H}{\partial \xi_x}; \tag{12a}$$

$$\dot{\xi}_z = -\frac{\partial H}{\partial d_z}, \quad \dot{d}_z = \frac{\partial H}{\partial \xi_z}. \tag{12b}$$

From Eqs. (12a, b) in detailed form it is obtained:

$$\dot{\xi}_x = \frac{g^2}{\chi_\perp}\left(\xi_x^2 d_x + \xi_x \xi_z d_z\right) + \omega_L \frac{\xi_x d_x}{\sqrt{1-\left(d_x^2+d_z^2\right)}} - \frac{\chi_\perp}{g^2}\Omega_A^2\left[\cos^2\alpha \; d_x + \frac{1}{2}\sin 2\alpha \frac{dxdz}{\sqrt{1-\left(d_x^2+d_z^2\right)}}\right], \tag{13a}$$

$$\dot{d}_x = \frac{g^2}{\chi_\perp}\left[\xi_x\left(1-d_x^2\right) - \xi_z d_x d_z\right] + \omega_L \sqrt{1-\left(d_x^2+d_z^2\right)}; \tag{13b}$$

$$\dot{\xi}_z = \frac{g^2}{\chi_\perp}\left(\xi_z^2 d_z + \xi_x \xi_z d_x\right) + \omega_L \frac{\xi_x d_z}{\sqrt{1-\left(d_x^2+d_z^2\right)}} - \frac{\chi_\perp}{g^2}\Omega_A^2\left[\cos 2\alpha \quad d_z - \right.$$
$$\left. -\frac{1}{2}\sin 2\alpha \frac{1-d_x^2-2d_z^2}{\sqrt{1-\left(d_x^2+d_z^2\right)}}\right] + \frac{\chi_{II}}{g^2}\omega_L^2 \sin\beta, \tag{14a}$$

$$\dot{d}_z = \frac{g^2}{\chi_\perp}\left[\xi_z\left(1-d_z^2\right) - \xi_x d_x d_z\right]. \tag{14b}$$

Expanding the right-hand side of Eqs. (13a, b) and Eqs (14a, b) in the small deviations $\delta\vec{\xi}\,(=\vec{p})$ and $\delta\hat{d}\,(=\vec{q})$ from their equilibrium values up to the second order it is found that

$$\dot{p}_x = -m\omega_x^2 q_x - \kappa_1 p_z + \left[\frac{\sin\beta}{m} p_x p_z - \frac{\omega_L}{\cos\beta}\left(\cos 2\beta \;\; p_x q_x + \cos^2\beta \;\; p_z q_z\right) - \right.$$
$$\left. -\frac{m}{2\cos^3\beta}(\omega_L^2 \sin 2\beta + +\Omega_A^2 \sin 2\alpha) \;\; q_x q_z\right], \tag{15a}$$

$$\dot{q}_x = \frac{p_x}{m} - \kappa_2 q_z + \left[-\frac{\sin\beta}{m} p_z q_x + \frac{\omega_L}{2\cos\beta}\left(\cos 2\beta\; q_x^2 - \frac{q_z^2}{\cos^2\beta}\right)\right]; \tag{15b}$$

$$\dot{p}_z = -\frac{m}{\cos^2\beta}\omega_z^2 q_z + \kappa_2 p_x + \left[\frac{\sin\beta}{m} p_z^2 - \frac{\omega_L}{\cos\beta}\left(\cos^2\beta\; p_z q_x - \frac{p_x q_z}{\cos^2\beta}\right) - \right.$$
$$\left. -\frac{m}{4\cos^3\beta}\left(\omega_L^2 \sin 2\beta + +\Omega_A^2 \sin 2\alpha\right)\left(q_x^2 + \frac{3\,q_z^2}{\cos^2\beta}\right)\right], \tag{16a}$$

$$\dot{q}_z = \frac{\cos^2\beta}{m} p_z + \kappa_1 q_x + \left[-\frac{\sin\beta}{m} (2 p_z q_z + p_x q_x) + \omega_L \cos\beta \, q_x q_z \right]. \tag{16b}$$

In Eqs. (15a, b) and (16a, b) the following abbreviations are used:

$$m = \frac{\chi_\perp}{g^2} \;,\; \kappa_1 = \omega_L \sin\beta \;\; \cos\beta \;,\; \kappa_2 = \omega_L \tan\beta \;, \tag{17a}$$

$$\omega_x^2 = \omega_L^2 \sin^2\beta + \Omega_A^2 \left(\cos^2\alpha + \frac{1}{2} \sin 2\alpha \tan\beta \right) \tag{17b}$$

$$\omega_z^2 = \omega_L^2 + \Omega_A^2 \left(\cos 2\alpha \;\; \cos^2\beta + \sin 2\alpha \frac{1 + 2\cos^2\beta}{2} \tan\beta \right) . \tag{17c}$$

The same set of equations can be reproduced by means of Taylor expansion [19] of the Hamiltonian (11) to the corresponding order in $\delta\vec{\xi}$ and $\delta\hat{d}$. It can be verified that the linear part of Eqs. (15a, b) and (16a, b) is governed by

$$H_2 = \left(\frac{1}{2m} p_x^2 + \frac{m}{2} \omega_x^2 q_x^2 \right) + \left(\frac{\cos^2\beta}{m} p_z^2 + \frac{m\omega_z^2}{2\cos^2\beta} q_z^2 \right) + \kappa_1 p_z q_x - \kappa_2 p_x q_z \;, \tag{18}$$

and the non-linear contribution stems from

$$H_3 = -\frac{\sin\beta}{m} (p_x q_x + p_z q_z) p_z + \omega_L \left[\cos\beta \;\; p_z q_x q_z + \frac{1}{2\cos\beta} \left(\cos 2\beta \;\; q_x^2 - \frac{q_z^2}{\cos^2\beta} \right) p_x \right] +$$
$$+ \frac{m\omega_L^2}{2\cos^2\beta} \left[\sin\beta + \frac{1}{2} \left(\frac{\Omega_A}{\omega_L} \right)^2 \frac{\sin 2\alpha}{\cos\beta} \right] \left(q_x^2 + \frac{q_z^2}{\cos^2\beta} \right) \; q_z . \tag{19}$$

Eq. (19) contains the information on the mechanism of the interaction between various spin-excitation modes of ^3He-A. In order to specify this eigen-modes it is necessary to go back to the linear level and address Eqs. (18) from which it is evident that in the case of the non-Leggett spin-orbital configuration the coefficients κ_1 and κ_2 are non-zero and H_2 is non-diagonal in the initial variables $\vec{p}$ and $\vec{q}$. The diagonalization of Eq. (18) can be achieved by passing to new pairs ($\mathscr{P}_x$, Q_x) and ($\mathscr{P}_z$, Q_z) according to the transformation

$$p_x = \mathscr{P}_x \cos\lambda - Q_z \frac{\sin\lambda}{f} , \qquad q_x = Q_x \cos\lambda + \mathscr{P}_z f \sin\lambda \; ; \tag{20a}$$

$$p_z = \mathscr{P}_z \cos\lambda - Q_x \frac{\sin\lambda}{f} , \qquad q_z = Q_z \cos\lambda + \mathscr{P}_x f \sin\lambda \; . \tag{20b}$$

The diagonalization is realized by a suitable choice of the rotation angle λ and an amplitude f as

$$\tan 2\lambda = 2\sqrt{2}\omega_L \sin\beta \frac{\sqrt{\omega_z^2+\omega_x^2}}{\omega_z^2-\omega_x^2}\ ,\ f^2 = \frac{1}{m^2}\frac{2\cos^2\beta}{\omega_z^2+\omega_x^2}\ . \tag{21}$$

As a result of this procedure the second order Hamiltonian (18) acquires the following form:

$$H_2 = \left(\frac{1}{2m} I_x \mathscr{P}_x^2 + \frac{1}{2} m\Omega_x^2 Q_x^2\right) + \left(\frac{\cos^2\beta}{2m} I_z \mathscr{P}_z^2 + \frac{m\,\Omega_z^2}{2\cos^2\beta} Q_z^2\right), \tag{22}$$

where

$$I_x = 1 + \frac{\omega_z^2-\omega_x^2-\sqrt{\ }}{2\left(\omega_z^2+\omega_x^2\right)},\quad I_z = 1 - \frac{\omega_z^2-\omega_x^2-\sqrt{\ }}{2\left(\omega_z^2+\omega_x^2\right)}, \tag{23}$$

$$\Omega_x^2 = \frac{1}{2}\left[\omega_z^2+\omega_x^2 - \frac{1}{2}\left(\omega_z^2-\omega_x^2+\sqrt{\ }\right)\right],\quad \Omega_z^2 = \frac{1}{2}\left[\omega_z^2+\omega_x^2 + \frac{1}{2}\left(\omega_z^2-\omega_x^2+\sqrt{\ }\right)\right], \tag{24}$$

with

$$\sqrt{\ } = \sqrt{\left(\omega_z^2-\omega_x^2\right)^2 + 8\,\omega_L^2\left(\omega_z^2+\omega_x^2\right)\sin^2\beta}\ . \tag{25}$$

Let us pass to the normal coordinate representation according to the prescription:

$$\mathscr{P}_z = \sqrt{\frac{m\Omega_z}{2\cos^2\beta}}\left(\hat a + \hat a^+\right),\quad Q_z = i\sqrt{\frac{I_z\cos^2\beta}{2m\Omega_z}}\left(\hat a - \hat a^+\right); \tag{26a}$$

$$\mathscr{P}_x = \sqrt{\frac{m\Omega_x}{2}}\left(\hat b + \hat b^+\right),\quad Q_x = i\sqrt{\frac{I_x}{2m\Omega_x}}\left(\hat b - \hat b^+\right), \tag{26b}$$

where the commutators $\left[\hat a, \hat a^+\right] = \left[\hat b, \hat b^+\right] = 1$. As a result it is concluded that

$$H_2 = I_z\Omega_z \hat a^+ \hat a + I_x\Omega_x \hat b^+ \hat b + const. \tag{27}$$

This is an answer for the spectrum of the eigen-oscillations of ^3He-*A* developing against the background of the non-Leggett equilibrium spin-orbital configuration described by $\hat l = (0, \cos\alpha, \sin\alpha)$ and $\hat d_0 = (0, \cos\beta, \sin\beta)$ with $\beta = \beta(\alpha)$ found according to Eq.(10).

Now we are ready to transcribe the non-linear part of the Leggett Hamiltonian (Eq.19) in the terms of normal coordinates. Before considering the non-Leggett equilibrium configuration case (which is our main concern), it is instructive to analyze first the Leggett configuration with $\alpha = \beta = 0$ appropriate to bulk ^3He-*A*. In this case $\Omega_z^2 = \omega_L^2 + \Omega_A^2 = \omega_\perp^2$, $\Omega_x^2 = \Omega_A^2 = \omega_{II}^2$ and the second order contribution to Leggett Hamiltonian (Eq.18) is diagonal in the initial variables $\vec p$ and $\vec q$. As a result it is concluded that in the normal coordinates (here $\omega_\perp >> \omega_{II}$)

$$p_z = \sqrt{\frac{m\omega_\perp}{2}}\left(\hat{a}+\hat{a}^+\right),\ q_z = i\sqrt{\frac{1}{2m\omega_\perp}}\left(\hat{a}-\hat{a}^+\right), \tag{28a}$$

$$p_x = \sqrt{\frac{m\omega_{II}}{2}}\left(\hat{b}+\hat{b}^+\right),\ q_x = i\sqrt{\frac{1}{2m\omega_{II}}}\left(\hat{b}-\hat{b}^+\right). \tag{28b}$$

In order to construct the non-linear contribution H_3 Eqs.(28a, b) are to be used in Eq.(19). In this way it can be shown that for the Leggett spin-orbital configuration

$$H_3 = \omega_L\left[p_z q_x q_z + \frac{1}{2}p_x\left(q_x^2 - q_z^2\right)\right] = -\frac{\omega_L}{2\sqrt{2m\omega_{II}}}\left[\left(\hat{a}-\hat{a}^+\right)\left(\hat{a}+\hat{a}^+\right)\left(\hat{b}-\hat{b}^+\right)+\right.$$

$$\left.+\frac{1}{2}\left(\hat{b}-\hat{b}^+\right)^2\left(\hat{b}+\hat{b}^+\right)-\frac{1}{2}\frac{\omega_{II}}{\omega_\perp}\left(\hat{a}-\hat{a}^+\right)^2\left(\hat{b}+\hat{b}^+\right)\right]. \tag{29}$$

It is seen that in bulk ^3He-A (with the Leggett equilibrium configuration) the interaction term $\hat{a}\hat{b}^+\hat{b}^+$ corresponding to the decay of *a*-type excitation onto the pair of the *b*-type modes is absent.

On the other hand, in [17] it was shown that in a strong magnetic field ($\omega_L >> \Omega_A$) the high frequency coherently precessing transverse mode of the magnetization experiences parametric instability signaling the decay of this mode at sufficiently low temperatures. An important observation is that the decay onto a pair of longitudinal spin-waves happens against the background of a non-Leggett equilibrium spin-orbital configuration with externally fixed $\hat{l} = (0, \cos\alpha, \sin\alpha)$. In order to reproduce this process in terms of the Hamiltonian (19) we have to address the general case described by the variables ($\mathscr{P}_x$, Q_x) and ($\mathscr{P}_z$, Q_z) . According to the transformations (20a, b) it can be shown that in the strong magnetic field case

$$H_2 = \omega_\perp \hat{a}^+\hat{a} + \omega_{II}\hat{b}^+\hat{b}, \tag{30}$$

where now

$$\omega_\perp^2 = \omega_L^2 + \Omega_A^2\cos 2\alpha ,$$

$$\omega_{II}^2 = \Omega_A^2\cos^2\alpha , \tag{31}$$

and the contribution to H_3 corresponding to the process $\hat{a}\hat{b}^+\hat{b}^+$ is

$$H_3 = -iV(\alpha)\hat{b}^+\hat{b}^+\hat{a} \tag{32}$$

with the amplitude

$$V(\alpha) = \frac{1}{12\sqrt{2}}\sqrt{\frac{\omega_L}{m}}\frac{1+a}{a}\left(\frac{\Omega_A}{\omega_L}\right)^2\sin 2\alpha . \tag{33}$$

In order to describe the decay of the transverse magnon onto the pair of longitudinal magnons it is necessary to take into account the contribution of the spin currents. This is achieved in supplementing the Leggett Hamiltonian (19) by the term

$$U_\nabla = \frac{1}{2} m c_{ij}^2 (\nabla_i \hat{d}) \cdot (\nabla_j \hat{d}), \tag{34}$$

where

$$c_{ij}^2 = c_\perp^2 (\delta_{ij} - l_i l_j) + c_{II}^2 l_i l_j, \tag{35}$$

with $c_\perp$ and c_{II} being the transverse and longitudinal spin-wave velocities. The gradient energy (34) generates a torque acting on $\vec{S}$ given as

$$m \hat{d} \times (c_\perp^2 \nabla_\perp^2 \hat{d} + c_{II}^2 \nabla_{II}^2 \hat{d}). \tag{36}$$

The results of this modification are simple. In the Fourier representation

$$H_2(\vec{k}) = \omega_\perp(\vec{k}) \hat{a}^+(\vec{k}) \hat{a}(\vec{k}) + \omega_{II}(\vec{k}) \hat{b}^+(\vec{k}) \hat{b}(\vec{k}), \tag{37}$$

where the magnon frequencies (squared) are given as

$$\omega_\perp^2(\vec{k}) = \omega_L^2 + \Omega_A^2 \cos 2\alpha + c_{ij}^2 k_i k_j,$$

$$\omega_{II}^2(\vec{k}) = \Omega_A^2 \cos^2 \alpha + c_{ij}^2 k_i k_j. \tag{38}$$

As to the third order term H_3, it takes the form

$$H_3(\vec{k}) = -iV(\alpha) \hat{b}^+(-\vec{k}) \hat{b}^+(\vec{k}) \hat{a}(0), \tag{39}$$

and describes the decay of transverse magnon (with $\vec{k} = 0$) into a pair of longitudinal magnons traveling in the opposite directions, so that the conservation law of the energy and momentum is fulfilled:

$$\omega_\perp(0) = \omega_{II}(\vec{k}) + \omega_{II}(-\vec{k}),$$

$$\vec{k} = 0 = \vec{k} + (-\vec{k}). \tag{40}$$

As can be seen from Eq. (33), the amplitude of this process is non-zero in the case of the non-Leggett spin-orbital configuration in accordance with the conclusion made in [17].

As a general conclusion it can be stated that, in spite of the fundamental difference in the structure of the order parameters of superfluid A and B phases, the catastrophic low temperature instability of homogeneously spin-precessing modes in both cases is developing in the background of the non-leggett orbital textures present in the restricted geometry.

ფიზიკა

ზედენად ^{3}He-A-ში კოჰერენტული სპინური პრეცესიის პარამეტრული არამდგრადობის კვანტური აღწერა

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ზედენად ^{3}He-ში სპინური დინამიკის თავისებურებათა შორის აქტიურად შეისწავლება კოჰერენტული სპინ-პრეცესიული მდგომარეობების ანომალურად სწრაფი (კატასტროფული) რელაქსაციის პროცესები. ექსპერიმენტულად ეს მოვლენა დეტალურად იქნა გამოკვლეული ^{3}He-ის B ფაზაში. ამ უჩვეულო მოვლენის შესაძლო ახსნა ეფუძნება მოსაზრებას, რომ სპინ-პრეცესიული მდგომარეობის სწრაფი რელაქსაცია დაკავშირებულია მის დაშლასთან გასწვრივ სპინურ ტალღებად. მსგავსი პროცესი შესაძლებელია ^{3}He-ის A ფაზაშიც. ნაშრომში ჩატარებულია ამ მექანიზმის კვანტური აღწერა.

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