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Hydrology

Generalized Method of Assessment of the Discharge of the Pressureless Uniform Motion of Newtonian and Non-Newtonian Fluids

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ABSTRACT. Generalized method of assessment of the discharge of the pressureless uniform motion of Newtonian and non-Newtonian fluids at laminar regime of motion is reported. *© 2013 Bull. Georg. Natl. Acad. Sci.*

Key words: *Newtonian fluid, non-Newtonian fluid, rheological model*

Derivation of the main equation

Motion of the fluid can be described both from axiomatic and phenomenological points of view. Axiomatic approach is a purely mathematical approach which gives the possibility to solve only a narrow part of practical problems. Phenomenological approach is a purely pragmatic approach enabling to approximately solve detailed practical engineering problems.

In the present paper prevalence is given to the phenomenological approach in which some assumptions are often of intuitive character and not based on strict mathematical and physical postulates. At this approach sometimes the assumptions seem to be opposite at first sight are admitted for consideration of different problems, which is done for establishment of a concrete target in solution of hydraulic problems. For illustration of this assumption it is enough to refer to the cases of description of the motion of hyperconcentrated (structural) debris flow, when attempts to combine as if opposite positions in relation to “solid” and “fluid” (viscous) bodies (motion of the quasi-solid body) are made. Such an assumption about motion of non-Newtonian bodies leads us to not strict (approximate) definition of these concepts which is not very important for engineers. However, it is important that the approach successfully works from the point of practical estimations.

Conception about “solid” body implies that the value of deformation depends on the value of acting force, whereas the conception of “viscous” body of the value of deformation depends on the velocity of deformation. In the former case a body saves its primary form, while in the latter case the body does not possess or possesses this property partially. Despite the contradiction, from practical point of view in phenomenological approach the study of the prob-

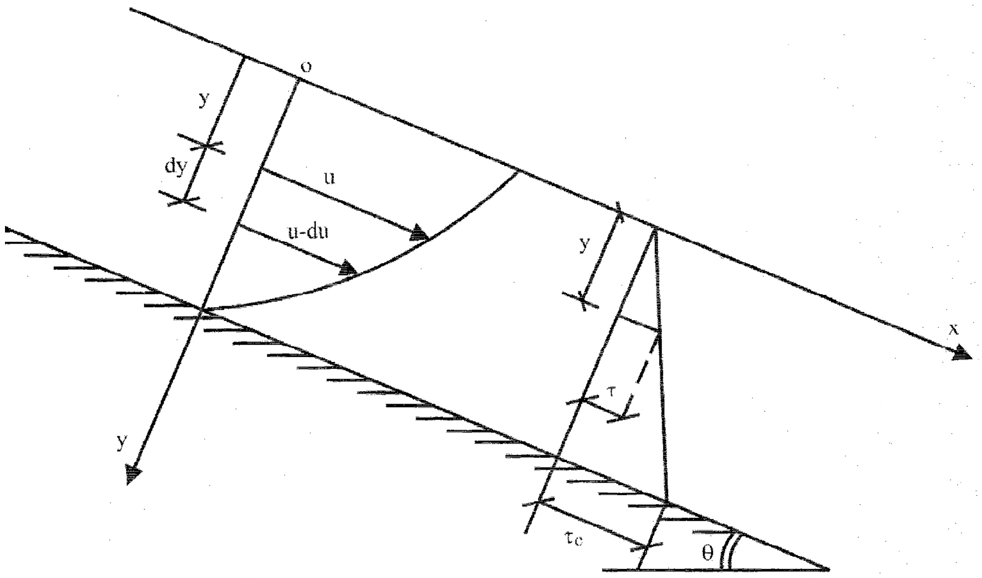

Fig. 1. Scheme of distribution of velocities and tangent stresses in pressureless uniform flow of Newtonian fluid.

lems of dynamics of non-Newtonian fluids including debris flows, as if incompatible, seems to be possible.

In the given case the main attention is focused on the fact that fluid "sticks" to the wall of the riverbed as a result of which at contact surface of the flow with the bed the gradient of velocity is observed.

In the present work there is an attempt to express discharge of pressureless uniform motion of both Newtonian and non-Newtonian fluids with the help of the model $Q=f(\tau)$,where Q is fluid discharge, τ - tangent stress. Determination of the dependences between parameters describing phenomenon finally is the construction of the model. Universal models are considered to be the laws.

Method of expression of discharge via elasticity can be found in [1-3]. Inserting into dependence $Q=f(\tau)$ concrete values $f(\tau)$ and integrating the obtained equation with account of boundary conditions the dependence for definition of fluid discharge with different rheological characteristics can be received.

The existing dependences of rheological character, which connect velocity gradient with shift stress, are divided into two groups. The first group implies the so-called "stationary fluids" From the rheological point of view these are fluids for which shift velocity depends only on the value of tangent stress. The second group implies "non-stationary fluids". These are fluids in which shift velocity is the function both of value of the tangent stress and time, i.e. duration of the force action on the body.

The present paper implies only the first group, i.e. the group of rheological "stationary fluids", which are divided into Newtonian and non-Newtonian fluids.

Discharge of the pressureless uniform motion flow with full depth H and under condition of "sticking" of the fluid on the wall of the riverbed can be determined by the dependence:

$$Q = B\int_{H}^{0} y du\,, \qquad (1)$$

where B is the width of the riverbed with straight angle longitudinal cross-section; u – local velocity of the flow.

Fig.1 presents distribution schemes of the velocity and tangent stresses in pressureless uniform flow. If τ_c is tangent stress on the bottom of the flow (i.e. at the contact surface of the flow and riverbed), then according to the condition of equilibrium of acting forces and with account of boundary condition we shall have:

$$\tau = \gamma y i\,, \qquad (2)$$

$$\tau_c = \gamma H i\,, \tag{3}$$

where y is specific weight of uniform fluid, i=sinθ - riverbed bottom gradient or

$$\tau = \tau_c \frac{y}{H}. \tag{4}$$

Taking into account that $\frac{du}{dy} = \frac{\tau}{\mu}$, where μ is dynamic coefficient of viscosity, i.e. $\frac{du}{dy} = f(\tau)$, therefore

$$du=f(\tau)dy. \tag{5}$$

Out of (4) it follows

$$y = \frac{\tau}{\tau_c} H \tag{6}$$

or $dy = \frac{H}{\tau_c} d\tau$.

Taking into account (5),(6) and (7) dependence (1) will be as follows:

$$Q = B\frac{H^2}{\tau_c}\int_{\tau_c}^{0} \tau f(\tau) d\tau. \tag{8}$$

Expression (8) allows to determine discharge of the fluid at pressureless motion of the established uniform flow. Inserting concrete values of $f(\tau)$ into (8) we can obtain corresponding values of the fluid discharge with different rheological characteristics.

Determination of the fluid discharge of Newtonian fluids

At laminar regime of the motion of Newtonian fluid

$$f(\tau) = \frac{du}{dy} = -\frac{\tau}{\mu}.$$

If we insert this expression into dependence (8) we shall get:

$$Q = -B\frac{H^2}{\tau_c \mu}\int_{\tau_c}^{0} \tau^2 d\tau = \frac{BH^2\tau_c}{3\mu}.$$

Taking into account (3) we shall have

$$Q = \frac{gH^3 iB}{3\nu}. \tag{9}$$

Here $\nu = \frac{\mu}{\rho}$ is kinematic coefficient of viscousity, μ - dynamic coefficient of viscousity, $\rho = \frac{\gamma}{g}$ - density of uniform fluid, g - acceleration of free fall.

Designate $q = \frac{Q}{B}$ as discharge per one linear metre of the width, then

$$q = \frac{gH^3 i}{3\nu}. \tag{9'}$$

The obtained dependences (9), (9') are well known for characteristics of laminar motion of Newtonian fluid [4].

Determination of the fluid discharge of non-Newtonian fluids

a) Shvedov-Bingham model

Let us take into consideration that according to Shvedov-Bingham

$$\tau = \tau_0 - \mu\frac{du}{dy}, \tag{10}$$

where τ_0 is dynamic stress of the shift, in fact it expresses stress at the depth h (Fig.2), h – depth of the flow core ("structural part of the flow), i.e. depth of the flow from free surface up to gradient layer (it is determined according to the presented method in [5]). If "static" stress of the shift characterizes shift value at the moment of the beginning of the motion of the system, then "dynamic" stress of the shift is a conditional concept and expresses constant part of full tangent stress (not depending on velocity) during the motion.

Then

$$\frac{du}{dy} = \frac{\tau_0 - \tau}{\mu} = f(\tau). \tag{11}$$

With account of (11) dependence (8) will be:

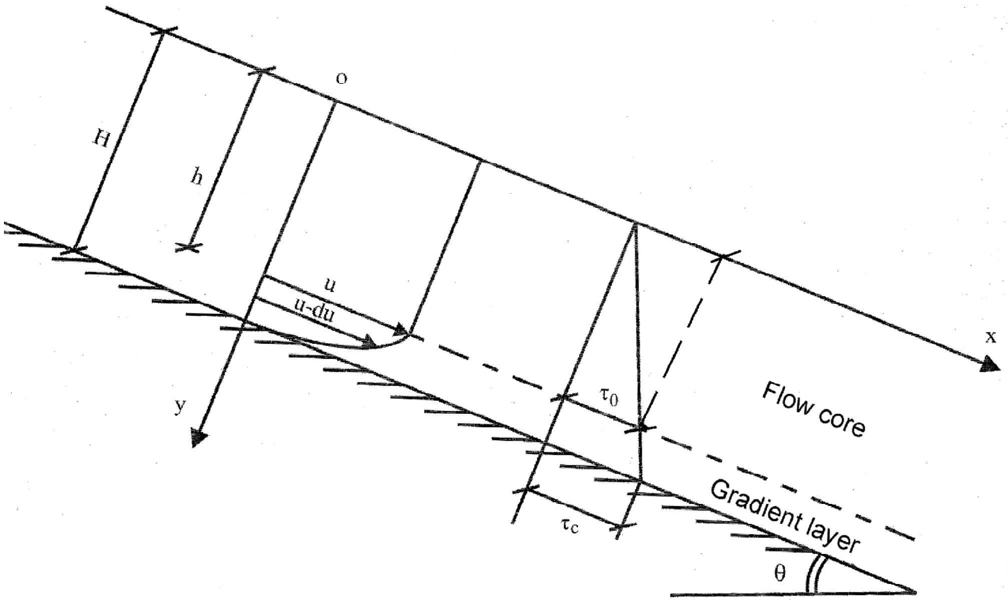

Fig. 2. Scheme of distribution of velocities and tangent stresses in pressureless uniform flow of non-Newtonian fluid.

$$Q = B\frac{H^2}{\tau_c^2}\int_{\tau_c}^{0}\frac{\tau(\tau_0-\tau)}{\mu} = \frac{BH^2\tau_c}{6\mu}\left(2-3\frac{\tau_0}{\tau_c}\right).$$

Taking into account (3) we get:

$$Q = \frac{BgH^3 i}{6\nu}\left(2-\frac{3\tau_0}{\rho gHi}\right) \quad (12)$$

In the case when $\tau_0 = 0$ dependence (12) coincides with (9).

From dependence (12) it follows that such a liquid will start motion under condition

$$2 > \frac{3\tau_0}{\rho gHi}$$

i.e.

$$\tau_0 < \frac{2}{3}\tau_c \quad (13)$$

as $\tau_0 = \gamma hi$ at the presence of flow core the fluid begin the motion when

$$h < \frac{2}{3}H. \quad (14)$$

Considering the given model, it is advisable to do the integration in the boundaries of gradient layer, and not along the whole depth of the flow as the velocity in the core is constant.

Then we shall have:

$$Q = B\frac{H^2}{\tau_c^2}\int_{\tau_c}^{\tau_0}\frac{\tau(\tau_0-\tau)}{\mu}d\tau$$

or after integration with account of $\tau_c = \gamma Hi$ and $\tau_0 = \gamma hi$ we get

$$Q = \frac{BgiH^3}{\nu}f(\beta), \quad (15)$$

where

$$f(\beta) = \frac{\beta}{2}(\beta^2-1) + \frac{1}{3}(1-\beta^3), \quad (16)$$

where $\beta = \frac{h}{H}$ is relative depth.

Out of the received dependences it follows that the motion of hyperconcentrated by alluvia flow (cohesive, structural) is provided by erosion incision, inset under condition

$$\frac{H}{3}\left(1-\frac{h^3}{H^3}\right) > \frac{h}{2}\left(1-\frac{h^2}{H^2}\right) \quad (17)$$

or at

$$h<0.9H. \qquad (17')$$

If we designate the condition via Q_H and Q_{HH}, hence the discharges of Newtonian and non-Newtonian fluids (with corresponding coefficients of kinematic viscosities) after (9) and (15) we get

$$Q_{HH} = 3Q_H f(\beta). \qquad (18)$$

In the case of h=0, β=0, from (16) we get $f(\beta)$=1/3 and we have $Q=Q_H=Q_{HH}$.

Model of De Vale-Ostvald

For evalution of tangent stress the model assumes the dependence:

$$\tau = k\left(\frac{du}{dy}\right)^n \qquad (19)$$

where k is measure of the mixture consistence (the more viscosity is, the more is k), n – index of non-Newtonian behavior.

When $n = 1$, then $k = \mu$ and we get the Newtonian fluid. In the case of n<1 with the increase of gradient velocity the decrease of so-called "effective" viscosity occurs. "Effective" viscosity seems to be the viscosity creating the impression as if we deal with plastic medium. Such bodies are called "pseudo-plastic" [3].

When n >1 with the increase of gradient velocity the increase of "effective" viscosity occurs. In such cases these fluids are called dilatant [3].

In the considered case

$$f(\tau) = \frac{du}{dy} = -\left(\frac{\tau}{k}\right)^{\frac{1}{n}} \qquad (20)$$

Taking into account (20) dependence (8) after integration with account of boundary conditions will be as follows:

$$Q = \frac{B\rho^{1/n} g^{1/n} i^{1/n} H^{2+1/n}}{\left(2+1/n\right)k^{1/n}}. \qquad (21)$$

At $n = 1$ and $k = \mu$ we get (9), i.e. expression for determination of discharge of Newtonian fluid.

Analogous transformations can be used for deterfmination of fluid bodies with excellent rheological indices [2,3].

ჰიდროლოგია

ნიუტონური და არანიუტონური სითხეებისთვის ხარჯის განსაზღვრის განზოგადებული მეთოდი ნაკადის თანაბარი რეჟიმით მოძრაობისას

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REFERENCES

1. *O.G. Natishvili, V.I. Tevzadze* (2001), Dvizhenie selei i ikh vzaimodeistvie s sooruzheniiami. Tbilisi, 148 p. (in Russian).
2. *J. Astarita, J. Marucci* (1978), Osnovy gidromekhaniki neniutonovskikh zhidkostei. M., 309 p. (in Russian). Foundations of Hydromechanics of Non-Newtonian Liquids
3. *U.L. Wilkinson* (1964), Neniutonovskie zhidkosti, M., 216 p. (in Russian).
4. *D.V. Shterenlikht* (1984), Gidravlika. M., 640 p. (in Russian).
5. *O.G. Natishvili, V.I. Tevzadze* (2011), Gidrotekhnicheskoe stroitel'stvo, 12: 57-59 (in Russian).

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