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On Hierarchical Models of Prismatic Shells within the Framework of the Chandrasekharaiah-Tzou Nonclassical Theory of Thermoelasticity

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ABSTRACT. In the present paper the Chandrasekharaiah-Tzou dynamical nonclassical model for thermoelastic prismatic shell is studied. The initial-boundary value problem corresponding to the dynamical three-dimensional model is investigated applying variational approach in suitable spaces of vector-valued distributions. A hierarchy of two-dimensional models is constructed for thermoelastic prismatic shell, when surface forces and the normal component of heat flux are given along the upper and the lower faces of the prismatic shell. The two-dimensional initial-boundary value problems corresponding to the models of the hierarchy are investigated in suitable function spaces. Moreover, the convergence of the sequence of approximate solutions of three space variables, constructed by means of the solutions of the reduced two-dimensional problems, to the exact solution of the original three-dimensional problem is proved and under suitable regularity conditions the rate of convergence is estimated. *© 2013 Bull. Georg. Natl. Acad. Sci.*

Key words: *nonclassical models for thermoelastic solids, prismatic shells, modeling error estimates.*

The classical theory of thermoelasticity is based on Fourier's law of heat conduction, which predicts the infinite propagation speed of thermal signals. To eliminate this unrealistic feature of the classical theory of thermoelasticity various generalizations were proposed. One of the first nonclassical models with one relaxation time parameter for thermoelastic bodies was constructed by H. Lord and Y. Shulman [1], where the classical Fourier's law of heat conduction was replaced by its modification originally proposed by Maxwell in the context of theory of gases, and later by Cattaneo in the context of heat conduction in rigid bodies. Later on, D. Tzou [2] proposed a dual-phase-lag heat conduction model, where one phase-lag corresponding to temperature gradient is caused by microstructural interactions such as phonon scattering or phonon-electron interactions, while the second phase-lag is interpreted as the relaxation time due to fast-transient effects of thermal inertia. Further, Chandrasekharaiah [3] constructed nonclassical model for thermoelastic bodies, where the classical Fourier's law of heat conduction was replaced with its generalization proposed by Tzou.

In this model the equation describing the temperature field involves the third order derivative with respect to the time variable of the temperature and divergence of the third order derivative with respect to the time variable of the displacement. Note that the Chandrasekharaiah-Tzou model is an extension of the Lord-Shulman [1] nonclassical model for thermoelastic bodies, which depends on one phase-lag. Particular one-dimensional initial-boundary value problems have been solved within the framework of the Chandrasekharaiah-Tzou theory in [4] and spatial behavior of solutions of the dual-phase-lag heat conduction equation and problems of stability of dual-phase-lag heat conduction models have been investigated in [5,6].

In this paper we construct and investigate a hierarchy of two-dimensional mathematical models for prismatic shells with variable thickness, when the stress-strain state of thermoelastic body is described by the Chandrasekharaiah-Tzou nonclassical three-dimensional model with two phase-lags. We employ generalization and extension of dimensional reduction method proposed by I. Vekua in the paper [7]. To construct two-dimensional models of plate I. Vekua considered differential formulation of the three-dimensional initial-boundary value problem and approximating components of the displacement vector-function by partial sums of orthogonal Fourier-Legendre series with respect to the variable of plate thickness a hierarchy of initial-boundary value problems defined on two-dimensional space domain was obtained. The relationship between the two-dimensional hierarchical models for plates and three-dimensional one in static case first was investigated in the spaces of classical regular functions in the paper [8], and the reduced two-dimensional models for thin shallow shells were investigated in Sobolev spaces in [9]. Later on, various hierarchical models were constructed and investigated applying Vekua's reduction method and its generalizations (see [10-14] and references given therein).

We consider three-dimensional initial-boundary value problem corresponding to the Chandrasekharaiah-Tzou dynamical model and applying variational approach and suitable a priori estimates we prove existence and uniqueness of solution in corresponding spaces of vector-valued distributions with values in Sobolev spaces. We construct hierarchical two-dimensional models for prismatic shell with variable thickness which may vanish on a part of the lateral boundary, when the densities of surface force and the normal component of heat flux are given along the upper and the lower faces of the prismatic shell. We investigate the initial-boundary value problems corresponding to the constructed dynamical two-dimensional models in suitable function spaces. Moreover, we prove that the sequence of vector-functions of three space variables restored from the solutions of the constructed two-dimensional problems converges to the exact solution of the original three-dimensional problem and under suitable regularity conditions of the solution we estimate the rate of convergence.

Let $W^{r,2}(D) \quad H^r(D),\ r \quad 1,\ r \quad \mathbf{R}$, be the Sobolev space of order r based on the space $L^2 \quad D$ of square-integrable functions in $D \quad \mathbf{R}^p,\ p \quad \mathbf{N}$, in Lebesgue sense, $\mathbf{H}^r(D) \quad [H^r(D)]^3$, $\mathbf{L}^2(D) \quad [L^2(D)]^3$ and $\mathbf{L}^s(\hat{\ }) \quad [L^s(\hat{\ })]^3$, $s \quad 1,\ s \quad \mathbf{R}$, where $\hat{\ }$ is a Lipschitz surface. For any Banach space X, $C^0([0,T];X)$ denotes the space of continuous functions on $[0,T]$ with values in X, $L^2(0,T;X)$ is the space of such functions $g:(0,T) \quad X$ that $\|g(t)\|_X \quad L^2(0,T)$. We denote by $g \quad dg/dt$ the generalized derivative of $g \quad L^2(0,T;X)$.

Let us consider a thermoelastic prismatic shell with thickness which may vanish on a part of its boundary, i.e. prismatic shell with initial configuration which is the following three-dimensional Lipschitz domain

$$\{(x_1,x_2,x_3) \quad \mathbf{R}^3;\, h \ (x_1,x_2) \quad x_3 \quad h \ (x_1,x_2),\ \ (x_1,x_2) \quad \},$$

where $\mathbf{R}^2$ is a two-dimensional bounded Lipschitz domain with boundary , h $C^0(\bar{\ })$ $C^{3,1}_{loc}(\ )$ are Lipschitz continuous in the interior of the domain and on together with their derivatives up to the third order, $h\ (x_1,x_2)$ $h\ (x_1,x_2)$, for (x_1,x_2) , is a Lipschitz curve, $h\ (x_1,x_2)$ $h\ (x_1,x_2)$, for (x_1,x_2) $\setminus$. The upper and the lower faces of , defined by the equations x_3 $h\ (x_1,x_2)$ and x_3 $h\ (x_1,x_2)$, (x_1,x_2) , we denote by and , respectively, and the lateral face, where the thickness of is positive, we denote by $\setminus(\overline{\qquad})$ $\{(x_1,x_2,x_3)$ $\mathbf{R}^3; h\ (x_1,x_2)$ x_3 $h\ (x_1,x_2)$, (x_1,x_2) $\}$. We assume that the prismatic shell consists of homogeneous, isotropic thermoelastic material. The applied body force density we denote by $\mathbf{f}$ (f_i): $(0,T)$ $\mathbf{R}^3$ and the density of heat sources we denote by f : $(0,T)$ $\mathbf{R}$. The prismatic shell is clamped and the temperature vanishes along a part $_0$ $\{(x_1,x_2,x_3)$ $\mathbf{R}^3; h\ (x_1,x_2)$ x_3 $h\ (x_1,x_2)$, (x_1,x_2) $_0\}$ of the lateral face , $_0$ is a Lipschitz curve, and on the remaining part $_1$ $\setminus\overline{\ }_0$ of the boundary the surface forces with density $\mathbf{g}$ (g_i): $_1$ $(0,T)$ $\mathbf{R}^3$ and the normal component of the heat flux with density g : $_1$ $(0,T)$ $\mathbf{R}$ are given.

The nonclassical dynamical linear three-dimensional model of stress-strain state of thermoelastic body obtained by D. Chandrasekharaiah and D. Tzou in differential form is given by

$$\frac{{}^2u_i}{t^2}\ \sum_{j\ 1}^{3}\frac{}{x_j}\ \sum_{p\ 1}^{3} e_{pp}(\mathbf{u})\ _{ij}\ \ 2\ \ e_{ij}(\mathbf{u})\ \ \ _{ij}\ \ f_i \text{ in } (0,T),\ i\ \ 1,2,3, \quad (1)$$

$$\frac{}{t}\ \ _0\frac{2}{t^2}\ \frac{{}^2_0}{2}\frac{3}{t^3}\ \ \sum_{j\ 1}^{3}\frac{2}{x_j^2}\ \ \ _1\frac{}{t}\ \ \ _0\frac{}{t}\sum_{p\ 1}^{3} e_{pp}\ \ \mathbf{u}\ \ _0\frac{\mathbf{u}}{t}\ \frac{{}^2_0}{2}\frac{{}^2\mathbf{u}}{t^2}$$

$$f\ \ _0\frac{f}{t}\ \frac{{}^2_0}{2}\frac{{}^2f}{t^2} \text{ in } (0,T), \quad (2)$$

$$\mathbf{u}\ \ \mathbf{0} \text{ on } _0\ (0,T),\ \sum_{j\ 1}^{3}\ \sum_{p\ 1}^{3} e_{pp}(\mathbf{u})\ _{ij}\ \ 2\ \ e_{ij}(\mathbf{u})\ \ \ _{ij}\ \ _j\ \ g_i \text{ on } _1\ (0,T) \quad (3)$$

$$0 \text{ on } _0\ (0,T),\ \sum_{j\ 1}^{3}\frac{}{x_j}\ \ \ _1\frac{}{t}\ \ _j\ \ g \text{ on } _1\ (0,T), \quad (4)$$

$$\mathbf{u}(x,0)\ \ \mathbf{u}_0(x),\ \frac{\mathbf{u}}{t}(x,0)\ \ \mathbf{u}_1(x),\ \ (x,0)\ \ _0(x),\ \frac{}{t}(x,0)\ \ _1(x),\ \frac{2}{t^2}(x,0)\ \ _2(x) \text{ in } , \quad (5)$$

where $_{ij}$ is the Kronecker's delta, $e_{ij}(\mathbf{u})$ $1/2(\ u_i/\ x_j\ \ u_j/\ x_i)$, i,j $1,2,3$, $\mathbf{u}$ (u_i): $(0,T)$ $\mathbf{R}^3$ is the displacement vector-function of thermoelastic body, : $(0,T)$ $\mathbf{R}$ is the temperature distribution, , are Lamé constants, is the mass density, 0 is the thermal conductivity coefficient, 0 is the specific heat at zero strain, is the stress-temperature coefficient, $_0$ 0 is a constant reference temperature, and $_0$, $_1$ are two different phase-lags. Note, that in the case of $_0 \quad _1 \quad 0$ the nonclassical three-dimensional model (1)-(5) coincides with the classical linear three-dimensional model for thermoelastic bodies.

We investigate the existence and uniqueness of weak solution of the three-dimensional initial-boundary value problem (1)-(5) and therefore we employ the following variational formulation of the differential problem: Find $\mathbf{u}, \mathbf{u}, \mathbf{u} \quad C^0([0,T];\mathbf{V}(\ ))$, $\mathbf{u} \quad L^2(0,T;\mathbf{V}(\ ))$, $\mathbf{u}^{(4)} \quad L^2(0,T;\mathbf{L}^2(\ ))$, $\ ,\ \quad C^0([0,T]; V(\ ))$, $\quad L^2(0,T;V(\ )) \quad L(0,T;L^2(\ ))$, $\quad L^2(0,T;L^2(\ ))$, which satisfies the following equations in the sense of distributions on $(0,T)$,

$$(\mathbf{u}(.),\mathbf{v})_{\mathbf{L}^2(\ )} \quad a(\mathbf{u}(.),\mathbf{v}) \qquad , \sum_{p\ 1}^{3} e_{pp}(\mathbf{v}) \quad _{L^2(\ )} \qquad (\mathbf{f},\mathbf{v})_{\mathbf{L}^2(\ )} \quad (\mathbf{g},\mathbf{v})_{\mathbf{L}^2(\ _1)}, \quad \mathbf{v} \quad \mathbf{V}(\ ), \tag{6}$$

$$(.) \quad _0 \ (.) \ \frac{^2_0}{2} \ (.), \qquad _{L^2(\ )} \quad a\ (\ (.) \quad _1 \ (.),\ )$$

$$_0 \sum_{p\ 1}^{3} e_{pp} \ \mathbf{u} \quad \tau_0 \mathbf{u} \quad \frac{\tau_0^2}{2}\mathbf{u} \ , \qquad _{L^2(\ )} \quad (f\ ,\ )_{L^2(\ )} \quad (g\ ,\ )_{L^2(\ _1)}, \qquad V(\ ), \tag{7}$$

together with the initial conditions

$$\mathbf{u}(0) \quad \mathbf{u}_0, \ \ \mathbf{u}(0) \quad \mathbf{u}_1, \quad (0) \quad _0, \quad (0) \quad _1, \quad (0) \quad _2, \tag{8}$$

where $\mathbf{u}_0, \mathbf{u}_1$ are the initial displacement and velocity vector-functions, $_0$, $_1$, $_2$ are the initial distributions of the temperature, its rate of change and the acceleration of change of the temperature, $f \quad f \quad _0 \frac{f}{t} \ \frac{^2_0}{2} \frac{^2 f}{t^2}$, $\mathbf{V}(\ ) \quad \{\mathbf{v} \quad \mathbf{H}^1(\ ); \mathbf{tr}(\mathbf{v}) \quad \mathbf{0}$ on $_0\}$, $V(\ ) \quad \{ \quad H^1(\ ); tr(\ ) \quad 0$ on $_0\}$, $\mathbf{tr}: \mathbf{H}^1(\ ) \quad \mathbf{H}^{1/2}(\ )$ and $tr: H^1(\ ) \quad H^{1/2}(\ )$ are the trace operators,

$$a(\mathbf{v},\mathbf{v}) \qquad \sum_{p\ 1}^{3} e_{pp}(\mathbf{v}) \sum_{q\ 1}^{3} e_{qq}(\mathbf{v}) \ \ 2 \sum_{i,j\ 1}^{3} e_{ij}(\mathbf{v}) e_{ij}(\mathbf{v}) \ dx, \qquad \mathbf{v},\mathbf{v} \quad \mathbf{V}(\ ),$$

$$a\ (\ ,\ ) \qquad \sum_{j\ 1}^{3} \frac{}{x_j}\frac{}{x_j} dx, \qquad , \quad V(\ ),$$

$(.,.)_{\mathbf{L}^2(\ )}$, $(.,.)_{L^2(\ )}$, $(.,.)_{\mathbf{L}^2(\ _1)}$ and $(.,.)_{L^2(\ _1)}$ are scalar products in the spaces $\mathbf{L}^2(\ )$, $L^2(\ )$, $\mathbf{L}^2(\ _1)$ and $L^2(\ _1)$, respectively.

For the Chandrasekharaiah-Tzou nonclassical dynamical three-dimensional model for thermoelastic prismatic shell (6)-(8) the following existence and uniqueness theorem is valid.

Theorem 1. *Let* $\mathbf{u}_0 \quad \mathbf{H}^4(\ ) \ \mathbf{V}(\ )$, $\mathbf{u}_1 \quad \mathbf{H}^3(\ ) \ \mathbf{V}(\ )$, $_0 \quad H^3(\ ) \quad V(\ )$, $_1 \quad H^2(\ ) \quad V(\ )$, $_2 \quad V(\ )$, $\mathbf{f} \quad C^0([0,T];\mathbf{H}^2(\ ))$, $\mathbf{f} \quad C^0([0,T];\mathbf{H}^1(\ ))$, $\mathbf{f}, \mathbf{f} \quad L^2(0,T;\mathbf{L}^2(\ ))$, $\mathbf{g}, \mathbf{g}, \mathbf{g}, \mathbf{g}, \mathbf{g}^{(4)} \quad L^2(0,T;\mathbf{L}^{4/3}(\ _1))$, $f, f \quad L^2(0,T;L^2(\ ))$, $g, g, g \quad L^2(0,T;L^{4/3}(\ _1))$, *and the following compatibility conditions are valid:*

$$
\begin{aligned}
&g_i(0) \sum_{j=1}^{3} \sum_{p=1}^{3} e_{pp}(\mathbf{u}_0)\ _{ij} \quad 2\ e_{ij}(\mathbf{u}_0) \quad _{0\ ij} \quad _j \Big|_{1}, \quad i \quad 1,2,3,\\
&g_i(0) \sum_{j=1}^{3} \sum_{p=1}^{3} e_{pp}(\mathbf{u}_1)\ _{ij} \quad 2\ e_{ij}(\mathbf{u}_1) \quad _{1\ ij} \quad _j \Big|_{1}, \quad i \quad 1,2,3,\\
&g_i(0) \sum_{j=1}^{3} \sum_{p=1}^{3} e_{pp}(\mathbf{u}_2)\ _{ij} \quad 2\ e_{ij}(\mathbf{u}_2) \quad _{2\ ij} \quad _j \Big|_{1}, \quad i \quad 1,2,3,\\
&\qquad g\ (0) \sum_{j=1}^{3} \frac{_0}{x_j} \quad _1 \frac{_1}{x_j} \quad _j \Big|_{1},
\end{aligned}
\qquad (9)
$$

where $u_{2i} \quad \frac{1}{} \sum_{j=1}^{3} \frac{}{x_j} \ 2\ e_{ij}(\mathbf{u}_0) \quad \sum_{p=1}^{3} e_{pp}(\mathbf{u}_0)\ _{ij} \quad _{0\ ij} \quad f_i(0)$, $i \quad 1,2,3$. If $\quad 0$, $\quad 0$, $3 \quad 2 \quad 0$, $\quad 0$, $\quad 0$, *and* $_0 \quad 0$, $_1 \quad 0$, *then the initial-boundary value problem* (6)-(8) *possesses a unique solution.*

To construct an algorithm of approximation of the Chandrasekharaiah-Tzou nonclassical three-dimensional model for thermoelastic prismatic shells by a sequence of two-dimensional models let us consider the subspaces $\mathbf{V}_\mathbf{N}^4(\ )$, $\mathbf{V}_\mathbf{N}^3(\ )$, $\mathbf{H}_\mathbf{N}^2(\ )$, $\mathbf{V}_\mathbf{N}^2(\ )$, $\mathbf{H}_\mathbf{N}^1(\ )$, $\mathbf{V}_\mathbf{N}(\ )$ and $\mathbf{H}_\mathbf{N}(\ )$ of $\mathbf{H}^4(\ ) \quad \mathbf{V}(\ )$, $\mathbf{H}^3(\ ) \quad \mathbf{V}(\ )$, $\mathbf{H}^2(\ )$, $\mathbf{H}^2(\ ) \quad \mathbf{V}(\ )$, $\mathbf{H}^1(\ )$, $\mathbf{V}(\ )$ and $\mathbf{L}^2(\ )$, respectively, $\mathbf{N} \quad (N_1, N_2, N_3)$, consisting of vector-functions whose components are polynomials with respect to the variable x_3 of thickness of the prismatic shell,

$$
\mathbf{v}_\mathbf{N} \quad (v_{\mathbf{N}i}),\ v_{\mathbf{N}i} \sum_{r_i=0}^{N_i} \frac{1}{h}(r_i \quad \frac{1}{2}) \overset{r_i}{v}_{\mathbf{N}i} P_{r_i}(z),\ \overset{r_i}{v}_{\mathbf{N}i} \quad L^2(\ ),\ 0 \quad r_i \quad N_i,\ i \quad 1,2,3,
$$

where $z \quad \frac{x_3 \ \bar{h}}{h}$, $h \quad \frac{h \quad h}{2}$, $\bar{h} \quad \frac{h \quad h}{2}$. In addition, we consider the subspaces $V_N^{,3}(\ )$, $V_N^{,2}(\ )$, $V_N(\ )$ and $H_N(\ )$ of $H^3(\ ) \quad V(\ )$, $H^2(\ ) \quad V(\ )$, $V(\ )$ and $L^2(\ )$, respectively, which consist of the following functions

$$
_N \sum_{r=0}^{N} \frac{1}{h}(r \quad \frac{1}{2}) \overset{r}{}_N P_r(z),\ \overset{r}{}_N \quad L^2(\ ),\ 0 \quad r \quad N\ .
$$

Since the functions h and h are Lipschitz continuous together with their derivatives up to the third order in the interior of the domain , from Rademacher's theorem [15] it follows that h , h , $_1 h$, $_1\ _2 h$ are differentiable almost everywhere in * and $_1\ _2\ _3 h \quad L\ (\ ^*)$ for all subdomains *, $\overline{^*}$, , $_1$, $_2$, $_3 \quad 1,2$. Therefore, the positiveness of h in implies that for any vector-function

$\mathbf{v}_{\mathbf{N}} \quad (v_{\mathbf{N}i})_{i\ 1}^{3} \quad \mathbf{V}_{\mathbf{N}}^{4}(\ )$ the corresponding functions $\overset{r_i}{v}_{\mathbf{N}i} \quad H^4(\ ^*)$ for all $\ ^*, \ \overline{\ ^*}$, i.e. $\overset{r_i}{v}_{\mathbf{N}i} \quad H^4_{loc}(\ )$, $0 \quad r_i \quad N_i, \ i \quad 1,2,3$. Similarly, if $\mathbf{v}_{\mathbf{N}} \quad (v_{\mathbf{N}i})_{i\ 1}^{3} \quad \mathbf{V}_{\mathbf{N}}^{3}(\ )$, $\hat{\mathbf{v}}_{\mathbf{N}} \quad (\hat{v}_{\mathbf{N}i})_{i\ 1}^{3} \quad \mathbf{V}_{\mathbf{N}}^{2}(\ )$, $\overline{\mathbf{v}}_{\mathbf{N}} \quad (\overline{v}_{\mathbf{N}i})_{i\ 1}^{3} \quad \mathbf{V}_{\mathbf{N}}(\ )$, then $\overset{r_i}{v}_{\mathbf{N}i} \quad H^3(\ ^*)$, $\overset{r_i}{\hat{v}}_{\mathbf{N}i} \quad H^2(\ ^*)$, $\overset{r_i}{\overline{v}}_{\mathbf{N}i} \quad H^1(\ ^*)$, for all $\ ^*, \ \overline{\ ^*}$, i.e. $\overset{r_i}{v}_{\mathbf{N}i} \quad H^3_{loc}(\ )$, $\overset{r_i}{\hat{v}}_{\mathbf{N}i} \quad H^2_{loc}(\ )$, $\overset{r_i}{\overline{v}}_{\mathbf{N}i} \quad H^1_{loc}(\ )$, $0 \quad r_i \quad N_i, \ i \quad 1,2,3$. For functions from the spaces $V_N^{\ ,3}(\ )$, $V_N^{\ ,2}(\ )$ and $V_N(\ )$ we also have $\overset{r}{\ }_N \quad H^3_{loc}(\ )$, if $\ _N \quad V_N^{\ ,3}(\ )$, $\overset{r}{\ }_N \quad H^2_{loc}(\ )$, if $\ _N \quad V_N^{\ ,2}(\ )$, and $\overset{r}{\ }_N \quad H^1_{loc}(\ )$, if $\ _N \quad V_N(\ )$, $0 \quad r \quad N$. Moreover, the norms $\|\cdot\|_{\mathbf{H}^4(\ )}$, $\|\cdot\|_{\mathbf{H}^3(\ )}$, $\|\cdot\|_{\mathbf{H}^2(\ )}$, $\|\cdot\|_{\mathbf{H}^1(\ )}$ and $\|\cdot\|_{H^3(\ )}$, $\|\cdot\|_{H^2(\ )}$, $\|\cdot\|_{H^1(\ )}$ in the spaces $\mathbf{H}^4(\ )$, $\mathbf{H}^3(\ )$, $\mathbf{H}^2(\ )$, $\mathbf{H}^1(\ )$ and $H^3(\ )$, $H^2(\ )$, $H^1(\ )$ define weighted norms $\|\cdot\|_{****}$, $\|\cdot\|_{***}$, $\|\cdot\|_{**}$, $\|\cdot\|_{*}$ and $\|\cdot\|_{\ ***}$, $\|\cdot\|_{\ **}$, $\|\cdot\|_{\ *}$ of vector-functions $v_{\mathbf{N}} \quad [H^4_{loc}(\ )]^{N_{1,2,3}}$, $v_{\mathbf{N}} \quad [H^3_{loc}(\ )]^{N_{1,2,3}}$, $v_{\mathbf{N}} \quad [H^2_{loc}(\ )]^{N_{1,2,3}}$, $v_{\mathbf{N}} \quad [H^1_{loc}(\ )]^{N_{1,2,3}}$, $N_{1,2,3} \quad N_1 \quad N_2 \quad N_3 \quad 3$, with components $\overset{r_i}{v}_{\mathbf{N}i}$, $v_{\mathbf{N}} \quad (\overset{r_i}{v}_{\mathbf{N}i})$, and $\ _N \quad [H^3_{loc}(\ )]^{N\ \ 1}$, $\ _N \quad [H^2_{loc}(\ )]^{N\ \ 1}$, $\ _N \quad [H^1_{loc}(\ )]^{N\ \ 1}$, with components $\overset{r}{\ }_N$, $\ _N \quad (\overset{r}{\ }_N)$, such that $\| v_{\mathbf{N}} \|_{****} \quad \| \mathbf{v}_{\mathbf{N}} \|_{\mathbf{H}^4(\ )}$, $\| v_{\mathbf{N}} \|_{***} \quad \| \mathbf{v}_{\mathbf{N}} \|_{\mathbf{H}^3(\ )}$, $\| v_{\mathbf{N}} \|_{**} \quad \| \mathbf{v}_{\mathbf{N}} \|_{\mathbf{H}^2(\ )}$, $\| v_{\mathbf{N}} \|_{*} \quad \| \mathbf{v}_{\mathbf{N}} \|_{\mathbf{H}^1(\ )}$ and $\| \ _N \|_{\ ***} \quad \| \ _N \|_{H^3(\ )}$, $\| \ _N \|_{\ **} \quad \| \ _N \|_{H^2(\ )}$, $\| \ _N \|_{\ *} \quad \| \ _N \|_{H^1(\ )}$. Using the properties of the Legendre polynomials, we can obtain explicit expressions for the norms $\|\cdot\|_{****}$, $\|\cdot\|_{***}$, $\|\cdot\|_{**}$, $\|\cdot\|_{*}$, $\|\cdot\|_{\ ***}$, $\|\cdot\|_{\ **}$ and $\|\cdot\|_{\ *}$, particularly, the norm $\|\cdot\|_{*}$ is given by

$$\| v_{\mathbf{N}} \|_*^2 \quad \sum_{i\ 1}^{3} \sum_{r_i\ 0}^{N_i} (r_i \quad \frac{1}{2}) \left\| \sum_{s_i\ \ r_i}^{N_i} (s_i \quad \frac{1}{2})(1 \quad (\ 1)^{r_i\ \ s_i}) h^{\ 3/2} \overset{s_i}{v}_{\mathbf{N}i} \right\|_{L^2(\ )}^2 \quad \| h^{\ 1/2} \overset{r_i}{v}_{\mathbf{N}i} \|_{L^2(\ )}^2$$

$$\sum_{\ 1}^{2} \left\| \sum_{s_i\ \ r_i\ \ 1}^{N_i} (s_i \quad \frac{1}{2})(\ \ h \quad (\ 1)^{r_i\ \ s_i} \quad h\ ) h^{\ 3/2} \overset{s_i}{v}_{\mathbf{N}i} \quad h^{\ 1/2} \quad \overset{r_i}{v}_{\mathbf{N}i} \quad (r_i \quad 1) h^{\ 3/2} \quad h \overset{r_i}{v}_{\mathbf{N}i} \right\|_{L^2(\ )}^2 ,$$

where we assume that the sum with the lower limit greater than the upper one equals to zero.

For components $\overset{r_i}{v}_{\mathbf{N}i}$ and $\overset{r}{\ }_N$ of vector-functions $v_{\mathbf{N}} \quad [H^1_{loc}(\ )]^{N_{1,2,3}}$ and $\ _N \quad [H^1_{loc}(\ )]^{N\ \ 1}$, which satisfy the conditions $\| v_{\mathbf{N}} \|_{*}$ and $\| \ _N \|_{\ *}$ we can define the traces on . Indeed, the corresponding vector-function $\mathbf{v}_{\mathbf{N}} \quad (v_{\mathbf{N}i})_{i\ 1}^{3}$ and function $\ _N$ of three space variables belong to the space $\mathbf{V}_{\mathbf{N}}(\ ) \quad \mathbf{H}^1(\ )$ and $V_N(\ ) \quad H^1(\ )$, respectively. Consequently, applying the trace operator on the space $H^1(\ )$ we define the traces on for functions $\overset{r_i}{v}_{\mathbf{N}i}$ and $\overset{r}{\ }_N$,

$$tr\ (\overset{r_i}{v}_{\mathbf{N}i}) \quad \int_{\ h}^{\ h} tr(v_{\mathbf{N}i})| \quad P_{r_i}(z) dx_3, \ tr\ (\overset{r}{\ }_N) \quad \int_{\ h}^{\ h} tr(\ _N)| \quad P_r(z) dx_3, \ r_i \quad \overline{0, N_i}, \ i \quad \overline{1,3}, \ r \quad \overline{0, N}.$$

Since the vector-functions $\mathbf{v}_N \quad (v_{N_i})$ from the subspaces $\mathbf{V}_N(\ )$ and $\mathbf{H}_N(\ )$, and the functions $\ _N$

from $V_N(\ )$ and $H_N(\ )$ are uniquely defined by functions $\overset{r_i}{v}_{N_i}$ and $\overset{r}{\ }_N$ of two space variables, therefore considering the original three-dimensional problem (6)-(8) on these subspaces, we obtain the following hierarchy of two-dimensional initial-boundary value problems: Find $w_{\mathbf{N}}$, $w_{\mathbf{N}}$, $w_{\mathbf{N}}$ $C^0([0,T];\ V_{\mathbf{N}}(\ ))$, $w_{\mathbf{N}}$ $L^2(0,T;\ V_{\mathbf{N}}(\ ))$, $w_{\mathbf{N}}^{(4)}$ $L^2(0,T;\ H_{\mathbf{N}}(\ ))$, ${}_N$, ${}_N$ $C^0([0,T];\ V_N(\ ))$, ${}_N$ $L^2(0,T;V_N(\ ))$ $L\ (0,T;H_N(\ ))$, ${}_N$ $L^2(0,T;H_N(\ ))$, which satisfy the following equations in the sense of distributions on $(0,T)$,

$$\frac{d}{dt}R_{\mathbf{N}}(w_{\mathbf{N}},v_{\mathbf{N}})\quad a_{\mathbf{N}}(w_{\mathbf{N}},v_{\mathbf{N}})\quad b_{\mathbf{N}N}(\ _N,v_{\mathbf{N}})\quad L_{\mathbf{N}}(v_{\mathbf{N}}),\qquad v_{\mathbf{N}}\quad V_{\mathbf{N}}(\ ), \tag{10}$$

$$R_N(\ _N\quad {}_0\ _N\quad \frac{{}^2_0}{2}\ _N,\ _N)\quad a_N(\ _N\quad {}_1\ _N,\ _N)$$

$${}_0b_{\mathbf{N}N}(w_{\mathbf{N}}\quad {}_0w_{\mathbf{N}}\quad \frac{{}^2_0}{2}w_{\mathbf{N}},\ _N)\quad L_N(\ _N),\qquad {}_N\quad V_N(\ ), \tag{11}$$

together with the initial conditions

$$w_{\mathbf{N}}(0)\quad w_{\mathbf{N}0},\quad w_{\mathbf{N}}(0)\quad w_{\mathbf{N}1},\quad {}_N(0)\quad {}_{N\,0},\quad {}_N(0)\quad {}_{N\,1},\quad {}_N(0)\quad {}_{N\,2}, \tag{12}$$

where $V_{\mathbf{N}}(\ )\ \ \{v_{\mathbf{N}}\ \ (\overset{r_i}{v}_{\mathbf{N}i})\ \ [H^1_{loc}(\ )]^{N_{1,2,3}};\|v_{\mathbf{N}}\|_*\ \ ,tr\,(\overset{r_i}{v}_{\mathbf{N}i})\ \ 0 \text{ on }\ {}_0,r_i\ \ \overline{0,N_i},i\ \ \overline{1,3}\}$,

$V^2_{\mathbf{N}}(\ )\ \ H^2_{\mathbf{N}}(\ )\ \ V_{\mathbf{N}}(\ ),\ H^2_{\mathbf{N}}(\ )\ \ \{v_{\mathbf{N}}\ \ (\overset{r_i}{v}_{\mathbf{N}i})\ \ [H^2_{loc}(\ )]^{\mathbf{N}_{1,2,3}};\ \|v_{\mathbf{N}}\|_{**}\ \ \}$,

$V^3_{\mathbf{N}}(\ )\ \ H^3_{\mathbf{N}}(\ )\ \ V_{\mathbf{N}}(\ ),\ H^3_{\mathbf{N}}(\ )\ \ \{v_{\mathbf{N}}\ \ (\overset{r_i}{v}_{\mathbf{N}i})\ \ [H^3_{loc}(\ )]^{N_{1,2,3}};\|v_{\mathbf{N}}\|_{***}\ \ \},V^4_{\mathbf{N}}(\ )\ \ H^4_{\mathbf{N}}(\ )\ \ V_{\mathbf{N}}(\ )$,

$H^4_{\mathbf{N}}(\ )\ \ \{v_{\mathbf{N}}\ \ (\overset{r_i}{v}_{\mathbf{N}i})\ \ [H^4_{loc}(\ )]^{N_{1,2,3}};\|v_{\mathbf{N}}\|_{****}\ \ \},H_{\mathbf{N}}(\ )\ \ \{v_{\mathbf{N}}\ \ (\overset{r_i}{v}_{\mathbf{N}i})\ \ [L^2(\ )]^{N_{1,2,3}};$

$\|v_{\mathbf{N}}\|^2_{H_{\mathbf{N}}(\ )}\ \ \sum_{i\ 1}^{3}\sum_{r_i\ 0}^{N_i}\|h^{\ 1/2}\overset{r_i}{v}_{\mathbf{N}i}\|^2_{L^2(\ )}\ \ \},\ V_N(\ )\ \ \{\ _N\ \ (\overset{r}{\ }_N)\ \ [H^1_{loc}(\ )]^{N\ \ 1};\|\ _N\|_*\ \ ,$

$tr\,(\overset{r}{\ }_N)\ \ 0 \text{ on }\ {}_0,r\ \ \overline{0,N}\},\ V_N^{\ ,2}(\ )\ \ \{\ _N\ \ (\overset{r}{\ }_N)\ \ [H^2_{loc}(\ )]^{N\ \ 1}\ \ V_N(\ );\|\ _N\|_{**}\ \ \}$,

$V_N^{\ ,3}(\ )\ \ \{\ _N\ \ (\overset{r}{\ }_N)\ \ [H^3_{loc}(\ )]^{N\ \ 1}\ \ V_N(\ );\|\ _N\|_{***}\ \ \},\ H_N(\ )\ \ \{\ _N\ \ (\overset{r}{\ }_N)\ \ [L^2(\ )]^{N\ \ 1};$

$\|\ _N\|^2_{H_N(\ )}\ \ \sum_{r\ 0}^{N}\left\|h^{\ 1/2}\overset{r}{\ }_N\right\|^2_{L^2(\ )}\ \ \}$,

the bilinear forms $R_{\mathbf{N}}$, R_N, $a_{\mathbf{N}}$, a_N, $b_{\mathbf{N}N}$, $b_{\mathbf{N}N}$ are defined by the corresponding forms in the left-hand sides of the equations (10), (11) and by taking account of the properties of the Legendre polynomials, we obtain the following expressions

$$R_{\mathbf{N}}(y_{\mathbf{N}},v_{\mathbf{N}})\quad \sum_{i\ 1}^{3}\sum_{r_i\ 0}^{N_i} r_i\ \frac{1}{2}\quad \frac{1}{h}\overset{r_i}{y}_{\mathbf{N}i}\overset{r_i}{v}_{\mathbf{N}i}\,d\ \ ,\ R_N(\ _N,\ _N)\quad \sum_{r\ 0}^{N} r\ \frac{1}{2}\quad \frac{1}{h}\overset{r}{\ }_N\overset{r}{\ }_N\,d\ \ ,$$

$$a_{\mathbf{N}}(y_{\mathbf{N}},v_{\mathbf{N}})\quad \sum_{r\ 0}^{N_{\max}} r\ \frac{1}{2}\ \frac{1}{h}\quad \sum_{p\ 1}^{3}\overset{r}{e}_{pp}(y_{\mathbf{N}})\sum_{q\ 1}^{3}\overset{r}{e}_{qq}(v_{\mathbf{N}})\ \ 2\sum_{i,j\ 1}^{3}\overset{r}{e}_{ij}(y_{\mathbf{N}})\overset{r}{e}_{ij}(v_{\mathbf{N}})\ d\ \ ,$$

$$a_N(\ _N,\ _N)\ \sum_{r\,0}^{N} r\ \frac{1}{2}\ \frac{1}{h^3}\sum_{s\ r}^{N}(s\ \frac{1}{2})^{s}_{N}(1\ (\ 1)^{r\ s})\ \sum_{\hat{s}\ r}^{N}(\hat{s}\ \frac{1}{2})^{\hat{s}}_{N}(1\ (\ 1)^{r\ \hat{s}})$$

$$^{2}_{1}\frac{1}{h}\ \ ^{r}_{N}\ (r\ \ 1)\frac{h}{h}{}^{r}_{N}\ \sum_{s\ r\ 1}^{N}\frac{^{s}_{N}}{h}\ s\ \frac{1}{2}\ \ h\ \ (\ 1)^{r\ s}\ \ h$$

$$^{r}_{N}\ (r\ \ 1)\frac{h}{h}{}^{r}_{N}\ \sum_{\hat{s}\ r\ 1}^{N}\frac{^{\hat{s}}_{N}}{h}\ \hat{s}\ \frac{1}{2}\ \ h\ \ (\ 1)^{r\ \hat{s}}\ \ h\ \ \ d\ ,$$

$$b_{\mathbf{N}N}(\ _N, v_{\mathbf{N}})\ \ b_{\mathbf{N}N}(v_{\mathbf{N}},\ _N)\ \sum_{r\,0}^{N} r\ \frac{1}{2}\ \frac{1}{h^2}\sum_{s\ r}^{N_3}(s\ \frac{1}{2})v^{s}_{\mathbf{N}3}(1\ (\ 1)^{r\ s})$$

$$^{2}_{1}\frac{1}{h}\ \ v^{r}_{\mathbf{N}}\ \ (r\ \ 1)\frac{h}{h}v^{r}_{\mathbf{N}}\ \sum_{s\ r\ 1}^{N}\frac{v^{s}_{\mathbf{N}}}{h}\ s\ \frac{1}{2}\ \ h\ \ (\ 1)^{r\ s}\ \ h\ \ \ ^{r}_{N}\ d\ ,$$

where $N_{\max}\ \ \max\{N_1,N_2,N_3\}$, $\overset{r}{e}_{ij}(v_{\mathbf{N}})\ \ \frac{1}{2}\ \ _i(\overset{r}{v}_{\mathbf{N}j})\ \ _j(\overset{r}{v}_{\mathbf{N}i})\ \ \overset{r}{e}_{ij}(v_{\mathbf{N}})$, $i,j\ \ 1,2,3$,

$$\overset{r}{e}_{ij}(v_{\mathbf{N}})\ \ \frac{r\ \ 1}{h}\ \ _ih\,\overset{r}{v}_{\mathbf{N}j}\ \ _jh\,\overset{r}{v}_{\mathbf{N}i}\ \sum_{s\ r\ 1}^{N_{\max}}\frac{1}{h}\ s\ \frac{1}{2}\ \ \overset{s}{v}_{\mathbf{N}j}(\ _ih\ \ (\ 1)^{r\ s}\ \ _ih\ )$$

$$\overset{s}{v}_{\mathbf{N}i}(\ _jh\ \ (\ 1)^{r\ s}\ \ _jh\ )\ \sum_{s\ r}^{N_{\max}}\frac{1}{h}\ s\ \ \frac{1}{2}\ (1\ \ (\ 1)^{r\ s})\ \frac{(i\ \ 1)(i\ \ 2)}{2}\overset{s}{v}_{\mathbf{N}j}\ \ \frac{(j\ \ 1)(j\ \ 2)}{2}\overset{s}{v}_{\mathbf{N}i}\ \ .$$

The linear forms $L_{\mathbf{N}}$, L_N are defined by the right-hand sides of the equations (6), (7) and are given by

$$L_{\mathbf{N}}(v_{\mathbf{N}})\ \sum_{i\,1}^{3}\sum_{r_i\,0}^{N_i} r_i\ \frac{1}{2}\ \ \frac{1}{h}\overset{r_i}{v}_{\mathbf{N}i}\ \overset{r_i}{f}_{\mathbf{N}i}\ \ g_{\mathbf{N}i}\ \ \ g_{\mathbf{N}i}\ \ (\ 1)^{r_i}\ \ d\ \ \ _1\frac{1}{h}\overset{r_i}{v}_{\mathbf{N}i}\,\overset{r_i}{g}_{\mathbf{N}i}\,d\ _1\ ,$$

$$L_N(\ _N)\ \sum_{r\,0}^{N} r\ \frac{1}{2}\ \ \frac{1}{h}{}^{r}_{N}\ \ \overset{r}{f}_N\ \ g_N\ \ \ g_N\ \ (\ 1)^{r}\ \ d\ \ \ _1\frac{1}{h}{}^{r}_{N}\ \overset{r}{g}_N\ d\ _1\ ,$$

where $_1\ \ \setminus\ _0$, $\sqrt{1\ \ (\ _1h\ )^2\ \ (\ _2h\ )^2}$, $\overset{r}{v}\ \int_{h}^{h} vP_r(z)dx_3$, for all functions $v\ \ L^2(\ )$, $r\ \ \mathbf{N}\ \ \{0\}$,

$g_{\mathbf{N}i}$, g_N and $g_{\mathbf{N}i}$, g_N are restrictions of

$$g_{\mathbf{N}i}(t)\ \ g_i(t)\ \ \sum_{j\,1}^{3}\ \ \sum_{p\,1}^{3} e_{pp}(\mathbf{w}_{\mathbf{N}0})\ _{ij}\ \ 2\ \ e_{ij}(\mathbf{w}_{\mathbf{N}0})\ \ \ _{N\,0\ ij}\ \ \ _j\Big|_1$$

$$t\ \sum_{j\,1}^{3}\ \ \sum_{p\,1}^{3} e_{pp}(\mathbf{w}_{\mathbf{N}1})\ _{ij}\ \ 2\ \ e_{ij}(\mathbf{w}_{\mathbf{N}1})\ \ \ _{N\,1\ ij}\ \ \ _j\Big|_1$$

$$\frac{t^2}{2}\sum_{j=1}^{3}\Big(\sum_{p=1}^{3} e_{pp}(\mathbf{w}_{\mathbf{N}2})\,{}_{ij}\;\;2\;\;e_{ij}(\mathbf{w}_{\mathbf{N}2})\qquad {}_{N\,2\;ij}\Big)\;\;{}_j\Big|_{1}\qquad g_i(0)\quad g_i(0)t\quad g_i(0)\frac{t^2}{2},\;\; i\quad 1,2,3,$$

$$w_{\mathbf{N}2i}\quad \frac{1}{}\;\sum_{j=1}^{3}\frac{}{x_j}\Big(2\;\;e_{ij}(\mathbf{w}_{\mathbf{N}0})\quad \sum_{p=1}^{3} e_{pp}(\mathbf{w}_{\mathbf{N}0})\,{}_{ij}\qquad {}_{N\,0\;ij}\Big)\quad f_{\mathbf{N}i}(0)\;\;,\;\; i\quad 1,2,3\,,$$

$$g_N\,(t)\quad g\,(t)\quad \sum_{j=1}^{3}\Big(\frac{{}_{N\,0}}{x_j}\quad {}_{1}\frac{{}_{N\,1}}{x_j}\Big)\;\;{}_j\Big|_{1}\quad g\;\;(0),$$

on the upper and the lower faces of the prismatic shell, respectively, $\mathbf{w}_{\mathbf{N}0}\quad \mathbf{V}^4_{\mathbf{N}}(\;\;)$, $\mathbf{w}_{\mathbf{N}1}\quad \mathbf{V}^3_{\mathbf{N}}(\;\;)$, ${}_{N\,0}\quad V_N^{,3}(\;\;)$, ${}_{N\,1}\quad V_N^{,2}(\;\;)$, ${}_{N\,2}\quad V_N\,(\;\;)$ correspond to the initial data $w_{\mathbf{N}0}$, $w_{\mathbf{N}1}$, ${}_{N\,0}$, ${}_{N\,1}$, ${}_{N\,2}$ of the two-dimensional problem.

For the two-dimensional initial-boundary value problems (10)-(12) of the constructed hierarchy the following theorem is proved.

Theorem 2. *If two-dimensional domain and functions* h , h *are such that* $\mathbf{R}^3$ *is a Lipschitz domain,* 0, 0, $3\quad 2\quad 0$, 0, 0, ${}_0\;\;0$, ${}_1\;\;0$, *the functions* $\overset{r_i}{f}_{\mathbf{N}i}$, $\overset{r_i}{g}_{\mathbf{N}i}$, $g_{\mathbf{N}i}$ $(r_i\quad 0,...,N_i, i\quad 1,2,3)$, $\overset{r}{f}_N$, $\overset{r}{g}_N$ $(r\quad 0,...,N\;)$, g_N *satisfy the following conditions*

$$f_{\mathbf{N}}\quad (\overset{r_i}{f}_{\mathbf{N}i})\quad C^0([0,T];H^2_{\mathbf{N}}(\;\;)), (f_{\mathbf{N}})\quad C^0([0,T];H^1_{\mathbf{N}}(\;\;)),\;\; h^{\;1/2}(\overset{r_i}{f}_{\mathbf{N}i})\;,h^{\;1/2}(\overset{r_i}{f}_{\mathbf{N}i})\quad L^2(0,T;L^2(\;\;)),$$

$${}^{3/4}g_{\mathbf{N}i},\;\;{}^{3/4}(g_{\mathbf{N}i}),\;\;{}^{3/4}(g_{\mathbf{N}i})\;,\;\;{}^{3/4}(g_{\mathbf{N}i})\;,\;\;{}^{3/4}(g_{\mathbf{N}i})^{(4)}\quad L^2(0,T;L^{4/3}(\;\;)),$$

$$h^{\;1/4}\overset{r_i}{g}_{\mathbf{N}i},h^{\;1/4}(\overset{r_i}{g}_{\mathbf{N}i})\;,h^{\;1/4}(\overset{r_i}{g}_{\mathbf{N}i})\;,h^{\;1/4}(\overset{r_i}{g}_{\mathbf{N}i})\;,h^{\;1/4}(\overset{r_i}{g}_{\mathbf{N}i})^{(4)}\quad L^2(0,T;L^{4/3}(\;{}_1)),\; r_i\quad \overline{0,N_i},\, i\quad \overline{1,3},$$

$$h^{\;1/2}\overset{r}{f}_N\;,h^{\;1/2}(\overset{r}{f}_N\;)\quad L^2(0,T;L^2(\;\;)),\quad {}^{3/4}g_N\;,\quad {}^{3/4}(g_N\;),\quad {}^{3/4}(g_N\;)\quad L^2(0,T;L^{4/3}(\;\;)),$$

$$h^{\;1/4}\overset{r}{g}_N\;,h^{\;1/4}(\overset{r}{g}_N\;)\;,h^{\;1/4}(\overset{r}{g}_N\;)\quad L^2(0,T;L^{4/3}(\;{}_1))\;,\; r\quad 0,...,N\;\;,$$

and $w_{\mathbf{N}0}\quad V^4_{\mathbf{N}}(\;\;)$, $w_{\mathbf{N}1}\quad V^3_{\mathbf{N}}(\;\;)$, ${}_{N\,0}\quad V_N^{,3}(\;\;)$, ${}_{N\,1}\quad V_N^{,2}(\;\;)$, ${}_{N\,2}\quad V_N\,(\;\;)$, *then the dynamical two-dimensional problem* (10)-(12) *possesses a unique solution.*

So, we have constructed a hierarchy of dynamical two-dimensional models for thermoelastic prismatic shell with variable thickness on the basis of the Chandrasekharaiah-Tzou nonclassical three-dimensional model for thermoelastic bodies. In the following theorem we formulate the results on the relationship between the constructed two-dimensional and the original three-dimensional models for thermoelastic prismatic shells, where we assume that the functions h and h defining the upper and the lower faces of the prismatic shell and their derivatives up to the third order are Lipschtiz continuous on the domain $\mathbf{R}^2$, i.e. $h\quad C^{3,1}(\bar{\;\;})$, and we use the following anisotropic weighted Sobolev spaces

$H^{0,0,s}_h(\;\;)\quad \{v;\, h^k\;{}^k_3 v\quad L^2(\;\;), 0\quad k\quad s\}$, $s\quad \mathbf{N}$,

$H^{1,1,s}_h(\;\;)\quad \{v;\, h^{k\;\,1}\;{}^{k\;\,1}_3\;{}^r_i v\quad L^2(\;\;), 1\quad k\quad s, r\quad 0,1, i\quad 1,2,3\}$, $s\quad \mathbf{N}$,

$H^{2,2,s}_h(\;\;)\quad \{v;\, h^{k\;\,2}\;{}^{k\;\,2}_3\;{}^r_i\;{}^{r}_j v\quad L^2(\;\;), 1\quad k\quad s, r,r\quad 0,1, i,j\quad 1,2,3\}$, $s\quad \mathbf{N}, s\quad 2,$

$H_h^{3,3,s}(\ )\ \ \{v;\ h^{k\ 3}\ {}_3^{k\ 3}\ {}_i^r\ {}_j^r\ {}_p^{\hat r} v\ \ L^2(\ ), 1\ \ k\ \ s, r, r, \hat r\ \ 0,1, i, j, p\ \ 1,2,3\},\ s\ \ \mathbf{N}, s\ \ 3,$

$H_h^{4,4,s}(\ )\ \ \{v;\ h^{k\ 4}\ {}_3^{k\ 4}\ {}_i^r\ {}_j^r\ {}_p^{\hat r}\ {}_q^{\bar r} v\ \ L^2(\ ), 1\ \ k\ \ s, r, r, \hat r, \bar r\ \ 0,1, i, j, p, q\ \ 1,2,3\},\ s\ \ 4,\ s\ \ \mathbf{N},$

which are Hilbert spaces with respect to the corresponding norms.

Theorem 3. *Let* $0,\ \ 0,\ 3\ \ 2\ \ 0,\ \ 0,\ \ 0,\ {}_0\ 0,\ {}_1\ 0,\ \mathbf{u}_0\ \ \mathbf{H}^4(\ )\ \ \mathbf{V}(\ ),$ $\mathbf{u}_1\ \ \mathbf{H}^3(\ )\ \ \mathbf{V}(\ ),\ {}_0\ H^3(\ )\ \ V(\ ),\ {}_1\ H^2(\ )\ \ V(\ ),\ {}_2\ V(\ ),\ \mathbf{f}\ \ C^0([0,T];\mathbf{H}^2(\ )),$ $\mathbf{f}\ \ C^0([0,T];\mathbf{H}^1(\ )),\ \mathbf{f}, \mathbf{f}\ \ L^2(0,T;\mathbf{L}^2(\ )),\ \mathbf{g}, \mathbf{g}, \mathbf{g}, \mathbf{g}, \mathbf{g}^{(4)}\ L^2(0,T;\mathbf{L}^{4/3}(\ {}_1)),\ f, f\ \ L^2(0,T;L^2(\ )),$ $g, g, g\ \ L^2(0,T;L^{4/3}(\ {}_1))$ *and the compatibility conditions (9) are fulfilled. If the vector-functions of three space variables* $\mathbf{w}_{\mathbf{N}0}\ \ \mathbf{V}_{\mathbf{N}}^4(\ ),\ \mathbf{w}_{\mathbf{N}1}\ \ \mathbf{V}_{\mathbf{N}}^3(\ ),\ {}_{N\,0}\ V_N^{,3}(\ ),\ {}_{N\,1}\ V_N^{,2}(\ ),\ {}_{N\,2}\ V_N(\ )$ *corresponding to the initial conditions* $w_{\mathbf{N}0}\ \ V_{\mathbf{N}}^4(\ ),\ w_{\mathbf{N}1}\ \ V_{\mathbf{N}}^3(\ ),\ {}_{N\,0}\ V_N^{,3}(\ ),\ {}_{N\,1}\ V_N^{,2}(\ ),$ ${}_{N\,2}\ V_N(\ )$ *of the two-dimensional problems, tend to* $\mathbf{u}_0$, $\mathbf{u}_1$, ${}_0$, ${}_1$ *and* ${}_2$ *in the spaces* $\mathbf{H}^4(\ )$, $\mathbf{H}^2(\ )$, $H^3(\ )$, $H^2(\ )$ *and* $H^1(\ )$, *respectively, and the vector-functions of three space variables* $\mathbf{f}_{\mathbf{N}}\ \ C^0([0,T];\mathbf{H}_{\mathbf{N}}^2(\ ))$, $\mathbf{f}_{\mathbf{N}}\ \ C^0([0,T];\ \mathbf{H}_{\mathbf{N}}^1(\ ))$, $\mathbf{f}_{\mathbf{N}}\ \ L^2(0,T;\mathbf{H}_{\mathbf{N}}(\ ))$, $\mathbf{f}_{\mathbf{N}}(0)\ \ \mathbf{H}_{\mathbf{N}}^2(\ )$, $\mathbf{f}_{\mathbf{N}}(0)\ \ \mathbf{H}_{\mathbf{N}}^1(\ )$ *and* $f_N\ \ L^2(0,T;H_N(\ ))$ *corresponding to the vector-functions* $f_{\mathbf{N}}\ \ (f_{\mathbf{N}i}^{r_i})\ \ C^0([0,T];H_{\mathbf{N}}^2(\ ))$, $(f_{\mathbf{N}})\ \ C^0([0,T];\ H_{\mathbf{N}}^1(\ ))$, $(f_{\mathbf{N}})\ \ L^2(0,T;H_{\mathbf{N}}(\ ))$, $f_{\mathbf{N}}(0)\ \ H_{\mathbf{N}}^2(\ )$, $(f_{\mathbf{N}})(0)\ \ H_{\mathbf{N}}^1(\ )$ *and* $f_N\ \ (f_N^{\,r})\ \ L^2(0,T;H_N(\ ))$ *are such that* $\mathbf{f}_{\mathbf{N}}$, $\mathbf{f}_{\mathbf{N}}$, $\mathbf{f}_{\mathbf{N}}$ *tend to* $\mathbf{f}$, $\mathbf{f}$, $\mathbf{f}$ *in* $L^2(0,T;\mathbf{L}^2(\ ))$, $\mathbf{f}_{\mathbf{N}}(0)$ *tends to* $\mathbf{f}(0)$ *in* $\mathbf{H}^2(\ )$, $\mathbf{f}_{\mathbf{N}}(0)$ *tends to* $\mathbf{f}(0)$ *in* $\mathbf{L}^2(\ )$, *and* f_N *converges to* f *in the space* $L^2(0,T;L^2(\ ))$, *as* $N_{\min}\ \ \min\limits_{1\ i\ 3}\{N_i, N\ \}$ *, then the sequences of vector-functions* $\{\mathbf{w}_{\mathbf{N}}(t)\}$ *and functions* $\{\ {}_N(t)\}$ *constructed by means of the solutions* $w_{\mathbf{N}}(t)$ *and* ${}_N(t)$ *of the two-dimensional problems of the hierarchy* (10)-(12)*, tend to the solutions* $\mathbf{u}(t)$ *and* (t) *of the original three-dimensional problem* (6)-(8),

$$\begin{array}{lll} \mathbf{w}_{\mathbf{N}}(t)\ \ \mathbf{u}(t) & \text{in } \mathbf{H}^1(\ ),\ \ t\ \ [0,T], & \\ \mathbf{w}_{\mathbf{N}}(t)\ \ \mathbf{u}(t) & \text{in } \mathbf{H}^1(\ ),\ \ t\ \ [0,T], & \\ \mathbf{w}_{\mathbf{N}}(t)\ \ \mathbf{u}(t) & \text{in } \mathbf{L}^2(\ ),\ \ t\ \ [0,T], & \text{as } N_{\min}\quad , \\ \mathbf{w}_{\mathbf{N}}(t)\ \ \mathbf{u}(t) & \text{in } L^2(0,T;\mathbf{H}^1(\ )), & \\ \mathbf{w}_{\mathbf{N}}(t)\ \ \mathbf{u}(t) & \text{in } L^2(0,T;\mathbf{L}^2(\ )), & \end{array}$$

$$\begin{array}{lll} {}_N(t)\ \ (t) & \text{in } H^1(\ ),\ \ t\ \ [0,T], & \\ {}_N(t)\ \ (t) & \text{in } L^2(\ ),\ \ t\ \ [0,T], & \text{as } N_{\min}\quad . \\ {}_N(t)\ \ (t) & \text{in } L^2(0,T;H^1(\ )), & \\ {}_N(t)\ \ (t) & \text{in } L^2(0,T;L^2(\ )), & \end{array}$$

In addition, if $d^r\mathbf{u}/dt^r\ \ L^2(0,T;(H_h^{1,1,s_r}(\ ))^3),\ r\ \ 0,1,2,3,\ \mathbf{u}^{(4)}\ \ L^2(0,T;(H_h^{0,0,s_4}(\ ))^3),\ s_k\ \ \mathbf{N}, s_k\ \ 5,$ $k\ \ 0,1,2,3,\ s_4\ \ \mathbf{N},\ s_4\ \ 4,\ d^r\ \ /dt^r\ \ L^2(0,T;\ H_h^{1,1,s_r}(\ )),\ r\ \ 0,1,2,\ \ \ \ L^2(0,T;\ H_h^{0,0,s_3}(\ )),$

$s_k \in \mathbf{N}$, $s_k \geq 4$, $k = 0,1,2$, $s_3 \in \mathbf{N}$, $s_3 \geq 3$, and $\mathbf{u}_0 \in (H_h^{4,4,s_0}(\Omega))^3$, $\mathbf{u}_1 \in (H_h^{3,3,s_1}(\Omega))^3$, $\theta_0 \in H_h^{3,3,s_0}(\Omega)$, $\theta_1 \in H_h^{2,2,s_1}(\Omega)$, $\chi_2 \in H_h^{1,1,s_2}(\Omega)$, $s_0, s_1, s_0, s_1, s_2 \in \mathbf{N}$, $s_0 \geq 8$, $s_1 \geq 6$, $s_0 \geq 6$, $s_1 \geq 5$, $s_2 \geq 4$, $d^p\mathbf{f}/dt^p \in L^2(0,T;(H_h^{0,0,\hat{s}_p}(\Omega))^3)$, $p = 0, 1, 2$, $\hat{s}_0, \hat{s}_1, \hat{s}_2 \in \mathbf{N}$, $\hat{s}_0, \hat{s}_1, \hat{s}_2 \geq 2$, $\mathbf{f}(0) \in (H_h^{2,2,\overline{s}_0}(\Omega))^3$, $\overline{s}_0 \in \mathbf{N}$, $\overline{s}_0 \geq 4$, then for appropriate initial conditions $w_{\mathbf{N}0}$, $w_{\mathbf{N}1}$, θ_{N0}, θ_{N1}, χ_{N1} and $f_{\mathbf{N}}$ the following estimate is valid

$$\|\mathbf{u}-\mathbf{w}_{\mathbf{N}}\|_{C^0([0,T];\mathbf{H}^1(\Omega))}+\|\dot{\mathbf{u}}-\dot{\mathbf{w}}_{\mathbf{N}}\|_{C^0([0,T];\mathbf{H}^1(\Omega))}+\|\ddot{\mathbf{u}}-\ddot{\mathbf{w}}_{\mathbf{N}}\|_{C^0([0,T];\mathbf{L}^2(\Omega))}+\|\ddot{\mathbf{u}}-\ddot{\mathbf{w}}_{\mathbf{N}}\|_{L^2(0,T;\mathbf{H}^1(\Omega))}$$
$$+\|\dddot{\mathbf{u}}-\dddot{\mathbf{w}}_{\mathbf{N}}\|_{L^2(0,T;\mathbf{L}^2(\Omega))}+\|\theta-\theta_N\|_{C^0([0,T];H^1(\Omega))}+\|\dot\theta-\dot\theta_N\|_{C^0([0,T];L^2(\Omega))}$$
$$+\|\dot\theta-\dot\theta_N\|_{L^2(0,T;H^1(\Omega))}+\|\ddot\theta-\ddot\theta_N\|_{L^2(0,T;L^2(\Omega))}\leq\frac{1}{(N_{\min})^s}o(T,\Omega,\vartheta_0,h,\mathbf{N},N),$$

where $s = \min\{\min_{0\leq i\leq 3}\{s_i - 1\}, s_4, \min_{0\leq j\leq 2}\{s_j - 1, \hat{s}_j\}, s_3, s_0 - 4, s_1 - 2, s_0 - 3, s_1 - 2, s_2 - 1, \overline{s}_0 - 2\}$ and

$o(T,\Omega,\vartheta_0,h,\mathbf{N},N) \to 0$, as $N_{\min} \to \infty$.

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ნაშრომში შესწავლილია ჩანდრასექჰარაია-ცოუს დინამიკური არაკლასიკური მოდელი თერმოდრეკადი პრიზმული გარსებისათვის. ვარიაციული მიდგომის გამოყენებით სამგანზომილებიანი დინამიკური მოდელის შესაბამისი საწყის-სასაზღვრო ამოცანა გამოკვლეულია სათანადო ვექტორული მნიშვნელობების მქონე განაწილებების სივრცეებში. თერმოდრეკადი პრიზმული გარსი-

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REFERENCES

1. *H.W. Lord, Y. Shulman* (1967), J. Mech. Phys. Solids, **15**: 229-309.
2. *D.Y. Tzou* (1995), J. Heat Transfer, **117**: 8-16.
3. *D.S. Chandrasekharaiah* (1998), Appl. Mech. Review, **51**: 705-729.
4. *D.S. Chandrasekharaiah* (1996), J. Thermal Stresses, **19**: 695-710.
5. *R. Quintanilla* (2005), J. Thermal Stresses, **28**: 43-57.
6. *R. Quintanilla, R. Racke* (2007), Proc. Royal Society A, **463**: 659-674.
7. *I.N. Vekua* (1955), Trudy Tbil. Matem. Inst., **21**: 191-259 (in Russian).
8. *D.G. Gordeziani* (1974), Dokl. Acad. Nauk SSSR, **215**, 6: 1289-1292 (Russian).
9. *D.G. Gordeziani* (1974), Dokl. Acad. Nauk SSSR, **216**, 4: 751-754 (in Russian).
10. *G. Avalishvili, M. Avalishvili* (2004), Appl. Math. Inform. and Mech., **9**, 1: 1-16.
11. *G . Avalishvili, M. Avalishvili, D. Gordeziani, B. Miara* (2010), Anal. Appl., **8**: 125-159.
12. *M. Avalishvili, D. Gordeziani* (2003), Georgian Math. J., **10**, 1: 17-36.
13. *V. Vogelius*, *I. Babuška* (1981), Math. of Computation, **37**, 155: 31-68.
14. *M. Dauge, E. Faou, Z. Yosibash* (2004), Encyclopedia of Computational Mechanics, **1**: 199-236.
15. *H. Whitney* (1957), Geometric Integration Theory. Princeton.

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