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Mathematical Physics

Investigation of Static Two-Dimensional Models for Thermoelastic Prismatic Shells with Microtemperatures

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ABSTRACT. In this paper a boundary value problem for thermoelastic prismatic shell with microtemperatures is considered. A hierarchy of two-dimensional models for a static three-dimensional model for prismatic shell with surface force, the normal component of heat flux and the first heat flux moment given on the upper and the lower faces of the prismatic shell is constructed. The two-dimensional boundary value problems corresponding to the hierarchical models are investigated in suitable function spaces. The convergence of the sequence of vector-functions of three space variables, restored from the solutions of the two-dimensional boundary value problems of the constructed hierarchy to the exact solution of the original three-dimensional problem is proved and the rate of approximation is estimated provided that the solution satisfies additional regularity conditions. *© 2013 Bull. Georg. Natl. Acad. Sci.*

Key words: *boundary value problems for thermoelastic prismatic shells, theory of thermoelasticity with microtemperatures, Fourier-Legendre series.*

Investigation of mathematical models of continuum describing interaction of several physical fields is important both from theoretical and from the practical viewpoint due to numerous applications in chemical industry, biology, aviation, material science, etc. One of the theories of continua with microstructure was proposed by A.C. Eringen [1], where the particles of the continua are assumed to be composed of microelements which undergo microdeformations, and from the principles of conservation of mass, conservation of microinertia, balance of linear momentum, balance of first moment of momentum and the balance of energy the system of partial differential equations and boundary conditions for deformations, microdeformations and temperature are obtained. By extending Eringen's theory R. Grot [2] constructed the theory of thermoelasticity for thermoelastic materials with inner structure, where the concept of microtemperatures is introduced and it is assumed that the microelements have different temperatures. Further, a mathematical model for fluids with microtemperatures was proposed by P. Riha [3]. It should be pointed out that experimantal data for the silicone rubber containing spherical aluminium particles and for the human blood were found to conform closely to predicted theoretical thermal conductivity. Boundary value, initial-boundary value problems and problems of steady vibrations, fundamental solutions and exponential stability of solutons of the corresponding equations for linear models of the theory of thermoelasticity with microtemperatures were studied by various authors (see [4-6] and references given therein).

Mathematical models of various engineering structures along with three-dimensional problems often include two-dimensional approximations of three-dimensional boundary and initial-boundary value problems. One of the methods of constructing two-dimensional models for linearly elastic prismatic shells was proposed by I. Vekua in [7]. In this paper Vekua considered a three-dimensional linear model of an elastic prismatic shell and, expanding components of the displacement vector-function into orthogonal Fourier-Legendre series with respect to the variable of the prismatic shell thickness, a hierarchy of differential two-dimensional models was obtained. The estimates of accuracy for the two-dimensional hierarchical models for elastic prismatic shells were obtained in the spaces of classical regular functions in the paper [8], and the reduced two-dimensional models for thin shallow shells constructed by I. Vekua were investigated in Sobolev spaces in [9]. Later on, Vekua's dimensional reduction method, its generalizations and extensions for various problems of mathematical physics were studied in [10-15].

The present paper is devoted to the construction and investigation of two-dimensional hierarchical models of thermoelastic prismatic shells with microtemperatures by applying variational approach. We consider the variational formulation of three-dimensional boundary value problem for static linear model of thermoelastic prismatic shell within the theory of thermoelasticity with microtemperatures, and construct its two-dimensional hierarchical models in Sobolev spaces, when temperature and components of microtemperature and displacement vectors are equal to zero along a part of the lateral boundary of the body, and the surface forces, the normal component of heat flux and the first heat flux moment are given on the upper and the lower faces, and on the remaining part of the lateral boundary of the prismatic shell. We investigate the existence and uniqueness of solutions of the reduced two-dimensional problems in suitable weighted Sobolev spaces. Moreover, we prove the convergence of the sequence of vector-functions of three space variables restored from the solutions of the constructed two-dimensional problems to the solution of the original three-dimensional boundary value problem and if it possesses additional regularity we estimate the rate of convergence.

For any bounded domain $\Omega \subset \mathbf{R}^p, p \geq 1$, with Lipschitz boundary we denote by $L^2(\Omega)$ the space of square integrable functions in Ω in the Lebesgue sense. $W^{k,2}(\Omega) = H^k(\Omega)$, $k \geq 1$, is the Sobolev space of order k based on $L^2(\Omega)$, $\mathbf{H}^k(\Omega) = (H^k(\Omega))^3$, $\mathbf{L}^2(\Omega) = (L^2(\Omega))^3$ and $\mathbf{L}^k(\hat{\Gamma}) = [L^k(\hat{\Gamma})]^3$, where $\hat{\Gamma}$ is a Lipschitz surface.

Let us consider a thermoelastic body $\Omega \subset \mathbf{R}^3$ with microtemperatures, which consists of inhomogeneous, isotropic thermoelastic material with Lamé coefficients $\lambda(x)$, $\mu(x)$, mass density $\rho(x)$, thermal conductivity $\kappa(x)$, thermoelastic coefficient $\beta(x)$, and parameters $\kappa_1(x)$, $\kappa_2(x)$, $\kappa_3(x)$, $\kappa_4(x)$, $\kappa_5(x)$, $\kappa_6(x)$ which define thermal properties of the material. The applied body force density we denote by $\mathbf{f} = (f_i) : \Omega \to \mathbf{R}^3$ and the density of heat sources we denote by $f^{\theta} : \Omega \times (0,T) \to \mathbf{R}$ and the density of the first heat source moment vector we denote by $\mathbf{f}^M = (f_i^M) : \Omega \to \mathbf{R}^3$. The body is clamped along a part Γ_0 of the boundary $\Gamma_0 = \partial\Omega$ and on the remaining part $\Gamma_1 = \Gamma \setminus \overline{\Gamma_0}$ surface force with density $\mathbf{g} = (g_i): \Gamma_1 \to \mathbf{R}^3$ is given, the temperature θ vanishes along $\Gamma_0^\theta \subset \Gamma$ and on the remaining part $\Gamma_1^\theta = \Gamma \setminus \overline{\Gamma_0^\theta}$ the normal component of heat flux with density $g^\theta : \Gamma_1^\theta \to \mathbf{R}$ is given, and the components of the microtemperature $\mathbf{w} = (w_i)$ vanish along a part Γ_0^M of the boundary and on the remaining part $\Gamma_1^M = \Gamma \setminus \overline{\Gamma_0^M}$ the density of the first heat flux moment $\mathbf{g}^M = (g_i^M): \Gamma_1^M \to \mathbf{R}^3$ is given.

The static linear three-dimensional model of the stress-strain state of the thermoelastic body with microtemperatures in differential form is given by the following system of partial differential equations

$$-\sum_{j=1}^{3} \frac{\partial}{\partial x_j}\left(\lambda \sum_{p=1}^{3} e_{pp}(\mathbf{u})\delta_{ij} + 2\mu e_{ij}(\mathbf{u}) - \beta\theta\delta_{ij}\right) = \rho f_i \qquad \text{in } \Omega,\ i = 1,2,3, \quad (1)$$

$$-\sum_{j=1}^{3} \frac{\partial}{\partial x_j}\left(\kappa \frac{\partial \theta}{\partial x_j} + \kappa_1 w_j\right) = \rho f^\theta \qquad \text{in } \Omega, \quad (2)$$

$$-\sum_{j=1}^{3} \frac{\partial}{\partial x_j}\left(\kappa_4 \sum_{p=1}^{3} e_{pp}(\mathbf{w})\delta_{ij} + \kappa_5 \frac{\partial w_j}{\partial x_i} + \kappa_6 \frac{\partial w_i}{\partial x_j}\right) + \kappa_3 \frac{\partial \theta}{\partial x_i} + \kappa_2 w_i = -\rho f_i^M \qquad \text{in } \Omega,\ i = 1,2,3, \quad (3)$$

$$\sum_{j=1}^{3}\left(\lambda \sum_{p=1}^{3} e_{pp}(\mathbf{u})\delta_{ij} + 2\mu e_{ij}(\mathbf{u}) - \beta\theta\delta_{ij}\right)\nu_j = g_i \text{ on } \Gamma_1,\ i = 1,2,3,\ \mathbf{u}(0) = \mathbf{0} \text{ on } \Gamma_0, \quad (4)$$

$$-\sum_{j=1}^{3}\left(\kappa \frac{\partial \theta}{\partial x_j} + \kappa_1 w_j\right)\nu_j = g^\theta \quad \text{on} \quad \Gamma_1^\theta, \theta(0) = 0 \quad \text{on} \quad \Gamma_0^\theta, \quad (5)$$

$$-\sum_{j=1}^{3}\left(\kappa_4 \sum_{p=1}^{3} e_{pp}(\mathbf{w})\delta_{ij} + \kappa_5 \frac{\partial w_j}{\partial x_i} + \kappa_6 \frac{\partial w_i}{\partial x_j}\right)\nu_j = g_i^M \text{ on } \Gamma_1^M,\ i = 1,2,3,\ \mathbf{w}(0) = \mathbf{0} \text{ on } \Gamma_0^M, \quad (6)$$

where $\mathbf{u} = (u_i): \Omega \to \mathbf{R}^3$ is the displacement vector, $\theta : \Omega \to \mathbf{R}$ is the temperature distribution, $\mathbf{w} = (w_i): \Omega \to \mathbf{R}^3$ is the microtemperature vector, $e_{ij}(\mathbf{v}) = 1/2(\partial_i v_j + \partial_j v_i)$, $i, j = 1,2,3$. If we multiply equations (1) by smooth enough functions v_i, which vanish on Γ_0, multiply equation (2) by smooth enough function φ vanishing on Γ_0^θ, multi ply equations (3) by smooth enough functions z_i vanishing on Γ_0^M, integrate the obtained equations over the domain Ω and apply integration by parts, taking into account boundary conditions (4), (5), (6), we obtain

$$\int_\Omega \left(\lambda(x) \sum_{p=1}^{3} e_{pp}(\mathbf{u}) \sum_{q=1}^{3} e_{qq}(\mathbf{v}) + 2\mu(x) \sum_{i,j=1}^{3} e_{ij}(\mathbf{u}) e_{ij}(\mathbf{v})\right) dx =$$

$$= \int_{\Omega} \beta(x)\theta \sum_{i=1}^{3} \frac{\partial v_i}{\partial x_i} dx + \sum_{i=1}^{3} \int_{\Omega} \rho(x) f_i(x) v_i(x) dx + \sum_{i=1}^{3} \int_{\Gamma_1} g_i v_i d\Gamma, \tag{7}$$

$$\sum_{j=1}^{3} \int_{\Omega} \kappa(x) \frac{\partial \theta}{\partial x_j} \frac{\partial \varphi}{\partial x_j} dx + \sum_{j=1}^{3} \int_{\Omega} \kappa_1(x) w_j \frac{\partial \varphi}{\partial x_j} dx = \int_{\Omega} \rho(x) f^{\theta}(x) \varphi(x) dx - \int_{\Gamma_1^{\theta}} g^{\theta} \varphi d\Gamma, \tag{8}$$

$$\int_{\Omega} \left(\kappa_4(x) \sum_{p=1}^{3} e_{pp}(\mathbf{w}) \sum_{q=1}^{3} e_{qq}(\mathbf{z}) + \kappa_5(x) \sum_{i,j=1}^{3} \frac{\partial w_j}{\partial x_i} \frac{\partial z_i}{\partial x_j} + \kappa_6(x) \sum_{i,j=1}^{3} \frac{\partial w_i}{\partial x_j} \frac{\partial z_i}{\partial x_j} \right) dx + \sum_{j=1}^{3} \int_{\Omega} \kappa_3(x) \frac{\partial \theta}{\partial x_j} z_j dx +$$

$$+ \sum_{j=1}^{3} \int_{\Omega} \kappa_2(x) w_j z_j dx = -\sum_{j=1}^{3} \int_{\Omega} \rho(x) f_j^M(x) z_j(x) dx - \sum_{j=1}^{3} \int_{\Gamma_1^M} g_j^M z_j d\Gamma, \tag{9}$$

for all $\mathbf{v} = (v_i)$, φ and $\mathbf{z} = (z_i)$, which are smooth enough and equal to zero on Γ_0, Γ_0^{θ} and Γ_0^M, respectively. Note that if $\mathbf{u} = (u_i)$, θ and $\mathbf{w} = (w_i)$ are solutions of the equations (7)-(9) and are smooth enough, then they also satisfy differential equations (1)-(3) and boundary conditions (4)-(6). So, the problem (1)-(6) is equivalent to the problem (7)-(9), which can be used to define the weak solution of the three-dimensional boundary value problem for thermoelastic prismatic shell with microtemperatures.

Hereafter we consider the following variational formulation of the three-dimensional initial boundary value problem (1)-(6): find $\mathbf{u} \in \mathbf{V}(\Omega)$, $\theta \in V^{\theta}(\Omega)$, $\mathbf{w} \in \mathbf{V}^M(\Omega)$, which satisfy the following equations

$$A(\mathbf{u}, \mathbf{v}) = \int_{\Omega} \beta(x)\theta \sum_{i=1}^{3} \frac{\partial v_i}{\partial x_i} dx + \sum_{i=1}^{3} \int_{\Omega} \rho(x) f_i(x) v_i(x) dx + \sum_{i=1}^{3} \int_{\Gamma_1} g_i v_i d\Gamma, \ \forall \mathbf{v} \in \mathbf{V}(\Omega), \tag{10}$$

$$B((\theta, \mathbf{w}), (\varphi, \mathbf{z})) = \int_{\Omega} \rho(x) f^{\theta}(x) \varphi(x) dx - \int_{\Gamma_1} g^{\theta} \varphi d\Gamma - \sum_{j=1}^{3} \int_{\Omega} \rho(x) f_j^M(x) z_j(x) dx - \sum_{j=1}^{3} \int_{\Gamma_1} g_j^M z_j d\Gamma,$$

$$\forall \varphi \in V^{\theta}(\Omega), \mathbf{z} \in \mathbf{V}^M(\Omega), \tag{11}$$

where $\mathbf{V}(\Omega) = \{\mathbf{v} \in \mathbf{H}^1(\Omega); \mathbf{tr}(\mathbf{v}) = \mathbf{0} \text{ on } \Gamma_0\}$, $\mathbf{V}^M(\Omega) = \{\mathbf{v} \in \mathbf{H}^1(\Omega); \mathbf{tr}(\mathbf{v}) = \mathbf{0} \text{ on } \Gamma_0^M\}$, $\mathbf{tr}$ is the trace operator from $\mathbf{H}^1(\Omega)$ to $\mathbf{H}^{1/2}(\Gamma)$, $V^{\theta}(\Omega) = \{v \in H^1(\Omega); tr(v) = 0 \text{ on } \Gamma_0^{\theta}\}$, tr is the trace operator from $H^1(\Omega)$ to $H^{1/2}(\Gamma)$,

$$A(\tilde{\mathbf{v}}, \mathbf{v}) = \int_{\Omega} \left(\lambda(x) \sum_{p=1}^{3} e_{pp}(\tilde{\mathbf{v}}) \sum_{q=1}^{3} e_{qq}(\mathbf{v}) + 2\mu(x) \sum_{i,j=1}^{3} e_{ij}(\tilde{\mathbf{v}}) e_{ij}(\mathbf{v}) \right) dx, \qquad \forall \mathbf{v}, \tilde{\mathbf{v}} \in \mathbf{V}(\Omega),$$

$$B((\tilde{\varphi},\tilde{\mathbf{z}}),(\varphi,\mathbf{z}))=\sum_{j=1}^{3}\int_{\Omega}\kappa(x)\frac{\partial\tilde{\varphi}}{\partial x_j}\frac{\partial\varphi}{\partial x_j}dx+\sum_{j=1}^{3}\int_{\Omega}\kappa_1(x)\tilde{z}_j\frac{\partial\varphi}{\partial x_j}dx+$$

$$+\int_{\Omega}\left(\kappa_4(x)\sum_{p=1}^{3}e_{pp}(\tilde{\mathbf{z}})\sum_{q=1}^{3}e_{qq}(\mathbf{z})+\kappa_5(x)\sum_{i,j=1}^{3}\frac{\partial\tilde{z}_j}{\partial x_i}\frac{\partial z_i}{\partial x_j}+\kappa_6(x)\sum_{i,j=1}^{3}\frac{\partial\tilde{z}_i}{\partial x_j}\frac{\partial z_i}{\partial x_j}\right)dx+$$

$$+\sum_{j=1}^{3}\int_{\Omega}\kappa_3(x)\frac{\partial\tilde{\varphi}}{\partial x_j}z_j dx+\sum_{j=1}^{3}\int_{\Omega}\kappa_2(x)\tilde{z}_j z_j dx,\quad \forall\varphi,\tilde{\varphi}\in V^{\theta}(\Omega),\ \mathbf{z},\tilde{\mathbf{z}}\in\mathbf{V}^{M}(\Omega).$$

For the three-dimensional boundary value problem (10), (11) the following theorem is valid.

Theorem 1. *Let Ω be a Lipschitz domain and the parameters characterizing mechanical and thermal properties of the thermoelastic prismatic shell Ω with microtemperatures be such that ρ, β, λ, μ, κ, κ_1, κ_2, κ_3, κ_4, κ_5, $\kappa_6\in L^{\infty}(\Omega)$, $\rho(x)\geq c_{\rho}>0$, $\mu(x)\geq c_{\mu}>0$, $3\lambda(x)+2\mu(x)\geq c_{\lambda\mu}>0$, $\kappa(x)\geq c_{\kappa}>0$, $\kappa_6(x)\geq c_{\kappa_6}>0$, $|\kappa_5(x)|\leq\kappa_6(x)-\varepsilon_6$, $\varepsilon_6=const>0$, $3\kappa_4(x)+\kappa_5(x)+\kappa_6(x)\geq 0$, for almost all $x\in\Omega$, and there exists $\alpha>0$, for which the following condition is valid*

$$\kappa(x)\xi^2+(\kappa_1(x)+\alpha\kappa_3(x))\xi\eta+\alpha\kappa_2(x)\eta^2\geq c_{\kappa_1}(\xi^2+\eta^2),\ \forall\xi,\eta\in\mathbf{R},\ \text{a.e. in }\Omega.$$

If Γ_0, Γ_0^{θ}, Γ_0^{M} are Lipschitz surfaces with positive areas and $\mathbf{f}=(f_i)_{i=1}^{3}\in\mathbf{L}^{6/5}(\Omega)$, $\mathbf{g}=(g_i)_{i=1}^{3}\in\mathbf{L}^{4/3}(\Gamma_1)$, $\mathbf{f}^{M}=(f_i^{M})_{i=1}^{3}\in\mathbf{L}^{6/5}(\Omega)$, $\mathbf{g}^{M}=(g_i^{M})_{i=1}^{3}\in\mathbf{L}^{4/3}(\Gamma_1^{M})$, $f^{\theta}\in L^{6/5}(\Omega)$, $g^{\theta}\in L^{4/3}(\Gamma_1^{\theta})$, then the three-dimensional problem (10), (11) *possesses a unique solution* $(\mathbf{u},\theta,\mathbf{w})\in\mathbf{V}(\Omega)\times V^{\theta}(\Omega)\times\mathbf{V}^{M}(\Omega)$.

Let us consider the particular case of the thermoelastic body with microtemperatures, when Ω is a thermoelastic prismatic shell with thickness vanishing on a part of its lateral boundary, i.e. prismatic shell with initial configuration, which is a Lipschitz domain Ω of the following form

$$\Omega=\{(x_1,x_2,x_3)\in\mathbf{R}^3;\ h^-(x_1,x_2)<x_3<h^+(x_1,x_2),\quad (x_1,x_2)\in\omega\},$$

where $\omega\subset\mathbf{R}^2$ is a two-dimensional bounded Lipschitz domain with boundary $\partial\omega$, $h^{\pm}\in C^0(\overline{\omega})\cap C^{0,1}_{loc}(\omega\cup\tilde{\gamma})$ are continuous on $\overline{\omega}$, Lipschitz continuous in ω and on $\tilde{\gamma}\subset\partial\omega$, $h^+(x_1,x_2)>h^-(x_1,x_2)$, for $(x_1,x_2)\in\omega\cup\tilde{\gamma}$, $\tilde{\gamma}\subset\partial\omega$ is a Lipschitz curve, $h^+(x_1,x_2)=h^-(x_1,x_2)$, for $(x_1,x_2)\in\partial\omega\setminus\tilde{\gamma}$. The upper and the lower faces of Ω, defined by the equations $x_3=h^+(x_1,x_2)$ and $x_3=h^-(x_1,x_2)$, $(x_1,x_2)\in\omega$, we denote by Γ^+ and Γ^-, respectively, and the lateral face, where the thickness of Ω is positive, we denote by $\tilde{\Gamma}=\partial\Omega\setminus\overline{(\Gamma^+\cup\Gamma^-)}=\{(x_1,x_2,x_3)\in\mathbf{R}^3;\ h^-(x_1,x_2)<x_3<h^+(x_1,x_2),$ $(x_1,x_2)\in\tilde{\gamma}\}$. We assume that the temperature θ, the components of the displacement vector-function $\mathbf{u}=(u_i)$ and the components of the microtemperature $\mathbf{w}=(w_i)$ vanish along a part $\tilde{\Gamma}_0=\{(x_1,x_2,x_3)\in\mathbf{R}^3;$ $h^-(x_1,x_2)<x_3<h^+(x_1,x_2),\ (x_1,x_2)\in\tilde{\gamma}_0\}$, $\tilde{\gamma}_0\subset\tilde{\gamma}$, of the lateral face $\tilde{\Gamma}$ of the prismatic shell and on the remaining part $\Gamma_1=\Gamma\setminus\overline{\tilde{\Gamma}_0}$ of the boundary the normal component of heat flux with density $g^{\theta}:\Gamma_1\to\mathbf{R}$,

surface force with density $\mathbf{g}=(g_i):\Gamma_1\to\mathbf{R}^3$ and the density of the first heat flux moment $\mathbf{g}^M=(g_i^M):\Gamma_1\to\mathbf{R}^3$ are given.

In order to construct the hierarchy of two-dimensional models let us consider the subspaces $\mathbf{V_N}(\Omega)=\mathbf{V_N}^M(\Omega)$ of $\mathbf{V}(\Omega)=\mathbf{V}^M(\Omega)$, $\mathbf{N}=(N_1,N_2,N_3)$, consisting of vector-functions whose components are polynomials with respect to the variable x_3,

$$\mathbf{v_N}=(v_{\mathbf{N}i}),\quad v_{\mathbf{N}i}=\sum_{r_i=0}^{N_i}\frac{1}{h}(r_i+\frac{1}{2})\overset{r_i}{v}_{\mathbf{N}i}P_{r_i}(y),\quad \overset{r_i}{v}_{\mathbf{N}i}\in L^2(\omega),\quad 0\le r_i\le N_i,\quad i=1,2,3, \tag{12}$$

where $y=\dfrac{x_3-\overline{h}}{h}$, $h=\dfrac{h^+-h^-}{2}$, $\overline{h}=\dfrac{h^++h^-}{2}$, $P_r(y)$ denotes the Legendre polynomial of order $r\in\mathbf{N}\cup\{0\}$. We also consider the subspaces $V_{N_\theta}^\theta(\Omega)$ of $V^\theta(\Omega)$, respectively, which consist of the following functions

$$\varphi_{N_\theta}=\sum_{r=0}^{N_\theta}\frac{1}{h}(r+\frac{1}{2})\overset{r}{\varphi}_{N_\theta}P_r(y),\ \overset{r}{\varphi}_{N_\theta}\in L^2(\omega),\ r=0,...,N_\theta. \tag{13}$$

Note that the functions h^+ and h^- are Lipschitz continuous in ω and hence, due to Rademacher's theorem [16], h^+ and h^- are differentiable almost everywhere in ω and $\partial_\alpha h^\pm\in L^\infty(\omega^*)$, for all subdomains ω^*, $\overline{\omega^*}\subset\omega$, $\alpha=1,2$. Therefore, the positiveness of h in ω implies that for any vector-function $\mathbf{v_N}=(v_{\mathbf{N}i})_{i=1}^3\in\mathbf{V_N}(\Omega)$ the corresponding functions $\overset{r_i}{v}_{\mathbf{N}i}\in H^1(\omega^*)$ for all ω^*, $\overline{\omega^*}\subset\omega$, i.e. $\overset{r_i}{v}_{\mathbf{N}i}\in H^1_{loc}(\omega)$, $0\le r_i\le N_i$, $i=1,2,3$. Similarly, for all functions $\varphi_{N_\theta}\in V_{N_\theta}^\theta(\Omega)$, the functions $\overset{r}{\varphi}_{N_\theta}$ of two space variables in the expressions of φ_{N_θ} belong to $H^1(\omega^*)$, $\overline{\omega^*}\subset\omega$, i.e. $\overset{r}{\varphi}_{N_\theta}\in H^1_{loc}(\omega)$, $r=0,...,N_\theta$. Moreover, the norms $\|\cdot\|_{\mathbf{H}^1(\Omega)}$ and $\|\cdot\|_{H^1(\Omega)}$ in the spaces $\mathbf{H}^1(\Omega)$ and $H^1(\Omega)$ define weighted norms $\|\cdot\|_*$ and $\|\cdot\|_{\theta*}$ of vector-functions $\vec{v}_\mathbf{N}\in[H^1_{loc}(\omega)]^{N_{1,2,3}}$, $N_{1,2,3}=N_1+N_2+N_3+3$, with components $\overset{r_i}{v}_{\mathbf{N}i}$, $\vec{v}_\mathbf{N}=(\overset{r_i}{v}_{\mathbf{N}i})$, and $\vec{\varphi}_{N_\theta}\in[H^1_{loc}(\omega)]^{N_\theta+1}$, with components $\overset{r}{\varphi}_{N_\theta}$, $\vec{\varphi}_{N_\theta}=(\overset{r}{\varphi}_{N_\theta})$, such that $\|\vec{v}_\mathbf{N}\|_*=\|\mathbf{v_N}\|_{\mathbf{H}^1(\Omega)}$ and $\|\vec{\varphi}_{N_\theta}\|_{\theta*}=\|\varphi_{N_\theta}\|_{H^1(\Omega)}$. Using (12), (13) and properties of the Legendre polynomials [17], we obtain explicit expressions of the norms $\|\cdot\|_*$ and $\|\cdot\|_{\theta*}$,

$$\|\vec{v}_\mathbf{N}\|_*^2=\sum_{i=1}^3\sum_{r_i=0}^{N_i}(r_i+\frac{1}{2})\left[\left\|\sum_{s_i=r_i}^{N_i}(s_i+\frac{1}{2})(1-(-1)^{r_i+s_i})h^{-3/2}\overset{s_i}{v}_{\mathbf{N}i}\right\|^2_{L^2(\omega)}+\left\|h^{-1/2}\overset{r_i}{v}_{\mathbf{N}i}\right\|^2_{L^2(\omega)}+\right.$$

$$\left.+\sum_{\alpha=1}^2\left\|\sum_{s_i=r_i+1}^{N_i}(s_i+\frac{1}{2})(\partial_\alpha h^+-(-1)^{r_i+s_i}\partial_\alpha h^-)h^{-3/2}\overset{s_i}{v}_{\mathbf{N}i}-h^{-1/2}\partial_\alpha\overset{r_i}{v}_{\mathbf{N}i}+(r_i+1)h^{-3/2}\partial_\alpha h\overset{r_i}{v}_{\mathbf{N}i}\right\|^2_{L^2(\omega)}\right],$$

$$\left\|\vec{\varphi}_{N_\theta}\right\|^2_{\theta*}=\sum_{r=0}^{N_\theta}(r+\frac{1}{2})\left[\left\|\sum_{s=r}^{N_\theta}(s+\frac{1}{2})(1-(-1)^{r+s})h^{-3/2}\overset{s}{\varphi}_{N_\theta}\right\|^2_{L^2(\omega)}+\left\|h^{-1/2}\overset{r}{\varphi}_{N_\theta}\right\|^2_{L^2(\omega)}+\right.$$

$$\left.+\sum_{\alpha=1}^{2}\left\|\sum_{s=r+1}^{N_\theta}(s+\frac{1}{2})(\partial_\alpha h^+-(-1)^{r+s}\partial_\alpha h^-)h^{-3/2}\overset{s}{\varphi}_{N_\theta}-h^{-1/2}\partial_\alpha\overset{r}{\varphi}_{N_\theta}+(r+1)h^{-3/2}\partial_\alpha h\overset{r}{\varphi}_{N_\theta}\right\|^2_{L^2(\omega)}\right],$$

where we assume that the sum with the lower limit greater than the upper one equals to zero.

For components $\overset{r_i}{v}_{\mathbf{N}i}$ and $\overset{r}{\varphi}_{N_\theta}$ of $\vec{v}_{\mathbf{N}}\in[H^1_{loc}(\omega)]^{N_{1,2,3}}$ and $\vec{\varphi}_{N_\theta}\in[H^1_{loc}(\omega)]^{N_\theta+1}$, which possess the properties $\left\|\vec{v}_{\mathbf{N}}\right\|_*<\infty$ and $\left\|\vec{\varphi}_{N_\theta}\right\|_{\theta*}<\infty$ we can define the trace on $\tilde{\gamma}$. Indeed, the corresponding vector-function of three space variables $\mathbf{v_N}=(v_{\mathbf{N}i})_{i=1}^3$ and function φ_{N_θ} belong to the space $\mathbf{V_N}(\Omega)\subset\mathbf{H}^1(\Omega)$ and $V^\theta_{N_\theta}(\Omega)\subset H^1(\Omega)$, respectively. Consequently, applying the trace operator $tr:H^1(\Omega)\to H^{1/2}(\Gamma)$ on the space $H^1(\Omega)$, we define the traces on $\tilde{\gamma}$ for $\overset{r_i}{v}_{\mathbf{N}i}$ and $\overset{r}{\varphi}_{N_\theta}$, $r_i=0,...,N_i$, $i=1,2,3$, $r=0,...,N_\theta$,

$$tr_{\tilde{\gamma}}(\overset{r_i}{v}_{\mathbf{N}i})=\int_{h^-}^{h^+}tr(v_{\mathbf{N}i})|_{\tilde{\Gamma}}\,P_{r_i}(z)dx_3,\ tr_{\tilde{\gamma}}(\overset{r}{\varphi}_{N_\theta})=\int_{h^-}^{h^+}tr(\varphi_{N_\theta})|_{\tilde{\Gamma}}\,P_r(z)dx_3\,.$$

Since the vector-functions $\mathbf{v_N}=(v_{\mathbf{N}i})$, $\mathbf{z_N}=(z_{\mathbf{N}i})$ from the subspaces $\mathbf{V_N}(\Omega)$ and the functions φ_{N_θ} from $V^\theta_{N_\theta}(\Omega)$ are defined by functions $\overset{r_i}{v}_{\mathbf{N}i}$, $\overset{r_i}{z}_{\mathbf{N}i}$ and $\overset{r}{\varphi}_{N_\theta}$ of two space variables, therefore considering the original three-dimensional problem (10), (11) on these subspaces, we obtain the following hierarchy of two-dimensional problems: find $\vec{u}_{\mathbf{N}}\in\vec{V}_{\mathbf{N}}(\omega)$, $\vec{\theta}_{N_\theta}\in\vec{V}^\theta_{N_\theta}(\omega)$, $\vec{w}_{\mathbf{N}}\in\vec{V}_{\mathbf{N}}(\omega)$, which satisfy the following equations

$$A_{\mathbf{N}}(\vec{u}_{\mathbf{N}},\vec{v}_{\mathbf{N}})=L_{\mathbf{N}N_\theta}(\vec{\theta}_{N_\theta},\vec{v}_{\mathbf{N}}),\qquad\forall\vec{v}_{\mathbf{N}}\in\vec{V}_{\mathbf{N}}(\omega),\tag{14}$$

$$B_{N_\theta\mathbf{N}}((\vec{\theta}_{N_\theta},\vec{w}_{\mathbf{N}}),(\vec{\varphi}_{N_\theta},\vec{z}_{\mathbf{N}}))=L^\theta_{N_\theta\mathbf{N}}((\vec{\varphi}_{N_\theta},\vec{z}_{\mathbf{N}})),\qquad\forall\vec{\varphi}_{N_\theta}\in\vec{V}^\theta_{N_\theta}(\omega),\vec{z}_{\mathbf{N}}\in\vec{V}_{\mathbf{N}}(\omega),\tag{15}$$

where $\vec{V}_{\mathbf{N}}(\omega)=\{\vec{v}_{\mathbf{N}}=(\overset{r_i}{v}_{\mathbf{N}i})\in[H^1_{loc}(\omega)]^{N_{1,2,3}};\ \left\|\vec{v}_{\mathbf{N}}\right\|_*<\infty,\ tr_{\tilde{\gamma}}(\overset{r_i}{v}_{\mathbf{N}i})=0$ on $\tilde{\gamma}_0$, $r_i=0,...,N_i$, $i=1,2,3\}$, $\vec{V}^\theta_{N_\theta}(\omega)=\{\vec{\varphi}_{N_\theta}=(\overset{r}{\varphi}_{N_\theta})\in[H^1_{loc}(\omega)]^{N_\theta+1};\left\|\vec{\varphi}_{N_\theta}\right\|_{\theta*}<\infty,tr_{\tilde{\gamma}}(\overset{r}{\varphi}_{N_\theta})\equiv0,r=0,...N\}$, the bilinear forms $A_{\mathbf{N}}$, $B_{N_\theta\mathbf{N}}$ are defined by the corresponding forms in the left-hand sides of the equations (10), (11), $A_{\mathbf{N}}(\vec{u}_{\mathbf{N}},\vec{v}_{\mathbf{N}})=A(\mathbf{u_N},\mathbf{v_N})$, $B_{N_\theta\mathbf{N}}((\vec{\theta}_{N_\theta},\vec{w}_{\mathbf{N}}),(\vec{\varphi}_{N_\theta},\vec{z}_{\mathbf{N}}))=B((\theta_{N_\theta},\mathbf{w_N}),(\varphi_{N_\theta},\mathbf{z_N}))$, for all $\vec{u}_{\mathbf{N}},\vec{v}_{\mathbf{N}}\in\vec{V}_{\mathbf{N}}(\omega)$, $\vec{\theta}_{N_\theta},\vec{\varphi}_{N_\theta}\in\vec{V}^\theta_{N_\theta}(\omega)$, $\vec{w}_{\mathbf{N}},\vec{z}_{\mathbf{N}}\in\vec{V}_{\mathbf{N}}(\omega)$, which correspond to $\mathbf{u_N},\mathbf{v_N}\in\mathbf{V_N}(\Omega)$, $\theta_{N_\theta},\varphi_{N_\theta}\in V^\theta_{N_\theta}(\Omega)$, $\mathbf{w_N},\mathbf{z_N}\in\mathbf{V_N}(\Omega)$. Taking into account the properties of the Legendre polynomials [17] we obtain explicit expressions for the bilinear forms $A_{\mathbf{N}}$ and $B_{N_\theta\mathbf{N}}$, when the parameters λ, μ, κ, κ_1, κ_2, κ_3, κ_4, κ_5, κ_6 are constants,

$$A_{\mathbf{N}}(\vec{y}_{\mathbf{N}},\vec{v}_{\mathbf{N}})=\sum_{r=0}^{N_{\max}}\left(r+\frac{1}{2}\right)\int_{\omega}\frac{1}{h}\left(\lambda\sum_{p=1}^{3}\overset{r}{e}_{pp}(\vec{y}_{\mathbf{N}})\sum_{q=1}^{3}\overset{r}{e}_{qq}(\vec{v}_{\mathbf{N}})+2\mu\sum_{i,j=1}^{3}\overset{r}{e}_{ij}(\vec{y}_{\mathbf{N}})\overset{r}{e}_{ij}(\vec{v}_{\mathbf{N}})\right)d\omega,$$

$$B_{N_\theta\mathbf{N}}((\vec{\psi}_{N_\theta},\vec{y}_{\mathbf{N}}),(\vec{\varphi}_{N_\theta},\vec{z}_{\mathbf{N}}))=\sum_{r=0}^{N_\theta}\sum_{i=1}^{3}\left(r+\frac{1}{2}\right)\kappa\int_{\omega}\frac{1}{h}\left(\frac{\partial\overset{r}{\psi}_{N_\theta}}{\partial x_i}-L_{ri}^{N_\theta}\vec{\psi}_{N}\right)\left(\frac{\partial\overset{r}{\varphi}_{N_\theta}}{\partial x_i}-L_{ri}^{N_\theta}\vec{\varphi}_{N}\right)d\omega+$$

$$+\sum_{i=1}^{3}\sum_{r=0}^{\min\{N_i,N_\theta\}}\left(r+\frac{1}{2}\right)\kappa_1\int_{\omega}\frac{1}{h}\overset{r}{y}_{\mathbf{N}i}\left(\frac{\partial\overset{r}{\varphi}_{N_\theta}}{\partial x_i}-L_{ri}^{N_\theta}\vec{\varphi}_{N_\theta}\right)d\omega+\sum_{r=0}^{N_{\max}}\left(r+\frac{1}{2}\right)\kappa_4\int_{\omega}\frac{1}{h}\sum_{p=1}^{3}\overset{r}{e}_{pp}(\vec{y}_{\mathbf{N}})\sum_{q=1}^{3}\overset{r}{e}_{qq}(\vec{z}_{\mathbf{N}})d\omega+$$

$$+\sum_{i,j=1}^{3}\sum_{r=0}^{\min\{N_i,N_j\}}\left(r+\frac{1}{2}\right)\kappa_5\int_{\omega}\frac{1}{h}\left(\frac{\partial\overset{r}{y}_{\mathbf{N}j}}{\partial x_i}-L_{ri}^{N_j}\vec{y}_{\mathbf{N}j}\right)\left(\frac{\partial\overset{r}{z}_{\mathbf{N}i}}{\partial x_j}-L_{rj}^{N_i}\vec{z}_{\mathbf{N}i}\right)d\omega+$$

$$+\sum_{i,j=1}^{3}\sum_{r=0}^{N_i}\left(r+\frac{1}{2}\right)\kappa_6\int_{\omega}\frac{1}{h}\left(\frac{\partial\overset{r}{y}_{\mathbf{N}i}}{\partial x_j}-L_{rj}^{N_i}\vec{y}_{\mathbf{N}i}\right)\left(\frac{\partial\overset{r}{z}_{\mathbf{N}i}}{\partial x_j}-L_{rj}^{N_i}\vec{z}_{\mathbf{N}i}\right)d\omega+$$

$$+\sum_{i=1}^{3}\sum_{r=0}^{\min\{N_i,N_\theta\}}\left(r+\frac{1}{2}\right)\kappa_3\int_{\omega}\frac{1}{h}\left(\frac{\partial\overset{r}{\psi}_{N_\theta}}{\partial x_i}-L_{ri}^{N_\theta}\vec{\psi}_{N_\theta}\right)^{i}\overset{r}{z}_{\mathbf{N}i}\,d\omega+\sum_{i=1}^{3}\sum_{r=0}^{N}\left(r+\frac{1}{2}\right)\kappa_2\int_{\omega}\frac{1}{h}\overset{r}{y}_{\mathbf{N}i}\,\overset{r}{z}_{\mathbf{N}i}\,d\omega,$$

where $N_{\max}=\max\{N_1,N_2,N_3,N_\theta\}$, $\vec{y}_{\mathbf{N}i}=(\overset{r_i}{y}_{\mathbf{N}i})_{r_i=0}^{N_i}$ $\vec{z}_{\mathbf{N}i}=(\overset{r_i}{z}_{\mathbf{N}i})_{r_i=0}^{N_i}$ $i=1,2,3$ $\overset{r}{y}_{\mathbf{N}i}=\overset{r}{z}_{\mathbf{N}i}=\overset{r}{v}_{\mathbf{N}i}=0$, for $r>N_i$,

$$\overset{r}{e}_{ij}(\vec{v}_{\mathbf{N}})=\frac{1}{2}\left(\partial_i(\overset{r}{v}_{\mathbf{N}j})+\partial_j(\overset{r}{v}_{\mathbf{N}i})+\overset{r}{\tilde{e}}_{ij}(\vec{v}_{\mathbf{N}})\right),\ i,j=1,2,3,$$

$$\overset{r}{\tilde{e}}_{ij}(\vec{v}_{\mathbf{N}})=-\frac{r+1}{h}\left(\partial_i h\overset{r}{v}_{\mathbf{N}j}+\partial_j h\overset{r}{v}_{\mathbf{N}i}\right)-\sum_{s=r+1}^{N_{\max}}\frac{1}{h}\left(s+\frac{1}{2}\right)\left(\overset{s}{v}_{\mathbf{N}j}(\partial_i h^{+}-(-1)^{r+s}\partial_i h^{-})+\right.$$

$$\left.+\overset{s}{v}_{\mathbf{N}i}(\partial_j h^{+}-(-1)^{r+s}\partial_j h^{-})\right)+\sum_{s=r}^{N_{\max}}\frac{1}{h}\left(s+\frac{1}{2}\right)(1-(-1)^{r+s})\left(\frac{(i-1)(i-2)}{2}\overset{s}{v}_{\mathbf{N}j}+\frac{(j-1)(j-2)}{2}\overset{s}{v}_{\mathbf{N}i}\right),$$

for any $\vec{\xi}=(\overset{k}{\xi})_{k=0}^{N}\in\mathbf{R}^{N+1}$, $r\in\mathbf{N}$, $0\le r\le N$

$$L_{ri}^{N}\vec{\xi}=(r+1)\frac{\partial_i h}{h}\overset{r}{\xi}+\sum_{s=r+1}^{N}\left(s+\frac{1}{2}\right)(\partial_i h^{+}-(-1)^{r+s}\partial_i h^{-})\frac{\overset{s}{\xi}}{h}-\sum_{s=r}^{N}\left(s+\frac{1}{2}\right)(1-(-1)^{r+s})\frac{(i-1)(i-2)}{2}\frac{\overset{s}{\xi}}{h},$$

and we assume that a sum with the upper limit less than the lower one equals zero. The linear forms $L_{\mathbf{N}N_\theta}$, $L_{N_\theta\mathbf{N}}^{\theta}$ are defined by the right-hand sides of the equations (10), (11) and are given by the following expressions

$$L_{\mathbf{N}N_\theta}(\vec{\theta}_{N_\theta},\vec{v}_{\mathbf{N}})=\sum_{r=0}^{N_\theta}\left(r+\frac{1}{2}\right)\int_\omega \overset{r}{\beta}\left[\frac{1}{h^2}\left(\sum_{s=r}^{N_3}(s+\frac{1}{2})\overset{s}{v}_{\mathbf{N}3}(1-(-1)^{r+s})\right)+\right.$$

$$\left.+\sum_{\alpha=1}^{2}\frac{1}{h}\left(\partial_\alpha \overset{r}{v}_{\mathbf{N}\alpha}-(r+1)\frac{\partial_\alpha h}{h}\overset{r}{v}_{\mathbf{N}\alpha}-\sum_{s=r+1}^{N_\alpha}\frac{\overset{s}{v}_{\mathbf{N}\alpha}}{h}\left(s+\frac{1}{2}\right)\left(\partial_\alpha h^+-(-1)^{r+s}\partial_\alpha h^-\right)\right)\right]\overset{r}{\theta}_{N_\theta}\,d\omega+$$

$$+\sum_{i=1}^{3}\sum_{r_i=0}^{N_i}\left(r_i+\frac{1}{2}\right)\left[\int_\omega\frac{1}{h}\overset{r_i}{v}_{\mathbf{N}i}\left(\overset{r_i}{\rho}\overset{r_i}{f}_i+g_i^+\lambda_++g_i^-\lambda_-(-1)^{r_i}\right)d\omega+\int_{\gamma_1}\frac{1}{h}\overset{r_i}{v}_{\mathbf{N}i}\overset{r_i}{g}_i\,d\gamma_1\right],$$

$$L^\theta_{N_\theta\mathbf{N}}((\vec{\varphi}_{N_\theta},\vec{z}_{\mathbf{N}}))=\sum_{r=0}^{N_\theta}\left(r+\frac{1}{2}\right)\left[\int_\omega\frac{1}{h}\overset{r}{\varphi}_{N_\theta}\left(\overset{r}{\rho}\overset{r}{f}{}^\theta-g^{\theta+}\lambda_+-g^{\theta-}\lambda_-(-1)^r\right)d\omega-\int_{\gamma_1}\frac{1}{h}\overset{r}{\varphi}_{N_\theta}\overset{r}{g}{}^\theta\,d\gamma_1\right]+$$

$$+\sum_{i=1}^{3}\sum_{r_i=0}^{N_i}\left(r_i+\frac{1}{2}\right)\left[\int_\omega\frac{1}{h}\overset{r_i}{z}_{\mathbf{N}i}\left(-\overset{r_i}{\rho}\overset{r_i}{f}{}_i^M-g_i^{M+}\lambda_+-g_i^{M-}\lambda_-(-1)^{r_i}\right)d\omega-\int_{\gamma_1}\frac{1}{h}\overset{r_i}{z}_{\mathbf{N}i}\overset{r_i}{g}{}_i^M\,d\gamma_1\right],$$

and $\gamma_1=\tilde{\gamma}\setminus\tilde{\gamma}_0$, $\lambda_\pm=\sqrt{1+(\partial_1 h^\pm)^2+(\partial_2 h^\pm)^2}$, $\overset{r}{\varphi}=\int_{h^-}^{h^+}\varphi P_r(z)dx_3$, for all functions $\varphi\in L^2(\Omega)$, $r\in\mathbf{N}\cup\{0\}$,

g_i^+, $g^{\theta+}$, g_i^{M+} and g_i^-, $g^{\theta-}$, g_i^{M-} are restrictions of g_i, g^θ, g_i^M, respectively, on the upper Γ^+ and the lower Γ^- faces of the prismatic shell.

For the constructed two-dimensional boundary value problems (14), (15) the following existence and uniqueness theorem is proved.

Theorem 2. *If ω and functions h^+, h^- are such that Ω is a Lipschitz domain, $\tilde{\gamma}_0$ is a Lipschitz curve with positive length, ρ, β, λ, μ, κ, κ_1, κ_2, κ_3, κ_4, κ_5, $\kappa_6\in L^\infty(\Omega)$ there exists $\alpha>0$ such that $\rho(x)\ge c_\rho>0$, $\mu(x)\ge c_\mu>0$, $3\lambda(x)+2\mu(x)\ge c_{\lambda\mu}>0$, $\kappa(x)\ge c_\kappa>0$, $\kappa_6(x)\ge c_{\kappa_6}>0$, $|\kappa_5(x)|\le\kappa_6(x)-\varepsilon_6$, $\varepsilon_6=const>0$, $3\kappa_4(x)+\kappa_5(x)+\kappa_6(x)\ge 0$, a.e. in Ω,*

$$\kappa(x)\xi^2+(\kappa_1(x)+\alpha\kappa_3(x))\xi\eta+\alpha\kappa_2(x)\eta^2\ge c_{\kappa_1}(\xi^2+\eta^2),\ \forall\xi,\eta\in\mathbf{R}$$

and the functions $\overset{r_i}{f}_i$, $\overset{r_i}{g}_i$, $g_i^\pm$, $\overset{r_i}{f}{}_i^M$, $\overset{r_i}{g}{}_i^M$, $g_i^{M\pm}$ $(r_i=0,...,N_i, i=1,2,3)$, $\overset{r}{f}{}^\theta$, $\overset{r}{g}{}^\theta$, $g^{\theta\pm}$ $(r=0,...,N_\theta)$ satisfy the following conditions

$$h^{-1/6}\overset{r_i}{f}_i\in L^{6/5}(\omega),\quad \lambda_\pm^{3/4}g_i^\pm\in L^{4/3}(\omega),\quad h^{-1/4}\overset{r_i}{g}_i\in L^{4/3}(\gamma_1),\quad r_i=0,...,N_i,\ i=1,2,3,$$

$$h^{-1/6}\overset{r}{f}{}^\theta\in L^{6/5}(\omega),\ \lambda_\pm^{3/4}g^{\theta\pm}\in L^{4/3}(\omega),\ h^{-1/4}\overset{r}{g}{}^\theta\in L^{4/3}(\gamma_1),\quad r=0,...,N_\theta,$$

$$h^{-1/6}\overset{r_i}{f}{}_i^M\in L^{6/5}(\omega),\ \lambda_\pm^{3/4}g_i^{M\pm}\in L^{4/3}(\omega),\ h^{-1/4}\overset{r_i}{g}{}_i^M\in L^{4/3}(\gamma_1),\ r_i=0,...,N_i,\ i=1,2,3,$$

then the static two-dimensional problem (14), (15) *possesses a unique solution.*

Along with the investigation of the boundary value problems corresponding to the obtained hierarchy of two-dimensional models it is very important to study the relationship between the constructed two-dimensional boundary value problems and the original three-dimensional one. In order to formulate the corresponding theorem let us define the following anisotropic weighted Sobolev space

$$H^{1,1,s}_{h^\pm}(\Omega)=\{v;\partial_3^{r-1}v\in H^1(\Omega),\partial_\alpha h^\pm\partial_3^r v\in L^2(\Omega),\alpha=1,2,r=1,...,s\},\qquad s\in\mathbf{N},$$

which is a Hilbert space equipped with the corresponding norm

$$\|v\|^2_{H^{1,1,s}_{h^\pm}(\Omega)}=\sum_{r=1}^{s}\left(\left\|\partial_3^{r-1}v\right\|_{H^1(\Omega)}+\sum_{\alpha=1}^{2}\left(\left\|\partial_\alpha h^+\partial_3^r v\right\|_{L^2(\Omega)}+\left\|\partial_\alpha h^-\partial_3^r v\right\|_{L^2(\Omega)}\right)\right),\ s\in\mathbf{N}.$$

Theorem 3. *Let* Ω *be a Lipschitz domain,* $\tilde\gamma_0$ *be a Lipschitz curve with positive length,* ρ, β, λ, μ, κ, κ_1, κ_2, κ_3, κ_4, κ_5, $\kappa_6\in L^\infty(\Omega)$, $\rho(x)\ge c_\rho>0$, $\mu(x)\ge c_\mu>0$, $3\lambda(x)+2\mu(x)\ge c_{\lambda\mu}>0$, $\kappa(x)\ge c_\kappa>0$, $\kappa_6(x)\ge c_{\kappa_6}>0$, $|\kappa_5(x)|\le\kappa_6(x)-\varepsilon_6$, $\varepsilon_6=const>0$, $3\kappa_4(x)+\kappa_5(x)+\kappa_6(x)\ge 0$, *for almost all* $x\in\Omega$, *and there exists* $\alpha>0$, *for which the following condition is valid*

$$\kappa(x)\xi^2+(\kappa_1(x)+\alpha\kappa_3(x))\xi\eta+\alpha\kappa_2(x)\eta^2\ge c_{\kappa_1}(\xi^2+\eta^2),\ \forall\xi,\eta\in\mathbf{R},\ \text{a.e. in }\Omega.$$

If $\mathbf{f}=(f_i)_{i=1}^3\in\mathbf{L}^{6/5}(\Omega)$, $\mathbf{g}=(g_i)_{i=1}^3\in\mathbf{L}^{4/3}(\Gamma_1)$, $\mathbf{f}^M=(f_i^M)_{i=1}^3\in\mathbf{L}^{6/5}(\Omega)$, $\mathbf{g}^M=(g_i^M)_{i=1}^3\in\mathbf{L}^{4/3}(\Gamma_1)$, $f^\theta\in L^{6/5}(\Omega)$, $g^\theta\in L^{4/3}(\Gamma_1)$, *then the sequences of vector-functions* $\mathbf{u_N}$, $\mathbf{w_N}$ *and functions* θ_{N_θ} *restored from the solutions* $\vec u_{\mathbf{N}}$, $\vec w_{\mathbf{N}}$ and $\vec\theta_{N_\theta}$ *of the reduced two-dimensional problems* (14), (15), *tend to the solutions* $\mathbf{u}$, $\mathbf{w}$ *and* θ *of the original three-dimensional problem* (10), (11),

$$\begin{aligned}&\mathbf{u_N}\to\mathbf{u} && \textit{in }\mathbf{H}^1(\Omega),\\ &\theta_{N_\theta}\to\theta && \textit{in }H^1(\Omega), \qquad \textit{as }N_{\min}=\min\{N_1,N_2,N_3,N_\theta\}\to\infty.\\ &\mathbf{w_N}\to\mathbf{w} && \textit{in }\mathbf{H}^1(\Omega),\end{aligned}$$

In addition, if $\mathbf{u}\in(H^{1,1,s_1}_{h^\pm}(\Omega))^3$, $\theta\in H^{1,1,s_2}_{h^\pm}(\Omega)$, $\mathbf{w}\in(H^{1,1,s_3}_{h^\pm}(\Omega))^3$, $s_1,s_2,s_3\in\mathbf{N}$, $s_1,s_2,s_3\ge 2$, *then the following estimate is valid*

$$\|\mathbf{u}-\mathbf{u_N}\|_{\mathbf{H}^1(\Omega)}+\left\|\theta-\theta_{N_\theta}\right\|_{H^1(\Omega)}+\|\mathbf{w}-\mathbf{w_N}\|_{\mathbf{H}^1(\Omega)}\le\frac{1}{(N_{\min})^s}o(T,\Omega,\tilde\Gamma_0,h^\pm,\mathbf{N},N_\theta),$$

where $s=\min\{s_1-1,s_2-1,s_3-1\}$, $o(T,\Omega,\tilde\Gamma_0,h^\pm,\mathbf{N},N_\theta)\to 0$, *as* $N_{\min}\to\infty$.

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REFERENCES:

1. *A.C. Eringen* (1967), Mechanics of Generalized Continua, 18-35.
2. *R. Grot* (1969), Int. J. Eng. Sci., **7**: 801-814.
3. *P. Riha* (1975), Acta Mech., **23**: 1-8.
4. *D. Iesan, R. Quintanilla* (2000), J. Thermal Stresses, **23**: 199-215.
5. *A. Scalia*, *M. Svanadze*, *R. Tracina* (2010), J. Thermal Stresses, **33**: 721-753.
6. *M. Svanadze* (2004), J. Thermal Stresses, **27**: 151-170.
7. *I.N. Vekua* (1955), Trudy Tbil. Matem. Inst., **21**: 191-259 (in Russian).
8. *D.G. Gordeziani* (1974), Dokl. Acad. Nauk SSSR, **215**, 6: 1289-1292 (in Russian).
9. *D.G. Gordeziani* (1974), Dokl. Acad. Nauk SSSR, **216**, 4: 751-754 (in Russian).
10. *G. Avalishvili, M. Avalishvili* (2007), Bull. Georg. Natl. Acad. Sci., **175**, 2: 31-34.
11. *G. Avalishvili, M. Avalishvili, D. Gordeziani, B. Miara* (2010), Anal. Appl., **8**: 125-159.
12. *M. Avalishvili, D. Gordeziani* (2003), Georgian Math. J., **10**, 1: 17-36.
13. *M. Dauge, E. Faou, Z. Yosibash* (2004), Encyclopedia of Computational Mechanics, **1**: 199-236.
14. *G.V. Jaiani* (2001), Z. Angew. Math. Mech., **81**, 3: 147-173.
15. *V. Vogelius*, *I. Babuška* (1981), Math. of Computation, **37**, 155: 31-68.
16. *H. Whitney* (1957), Geometric integration theory: Princeton University Press, Princeton.
17. *D. Jackson* (1941), Fourier series and orthogonal polynomials: Carus Math. Monographs, VI, Chicago.

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