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Mathematics

On Approximation of Lord-Shulman Model for Thermoelastic Plates with Variable Thickness by Two-Dimensional Problems

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ABSTRACT. In the present paper thermoelastic solid is considered within the framework of Lord-Shulman non-classical theory of thermoelasticity. Applying variational approach initial-boundary value problem corresponding to the three-dimensional model is investigated in suitable spaces of vector-valued distributions with values in Sobolev spaces. An algorithm of approximation by two-dimensional problems of the three-dimensional dynamical model for plate with variable thickness is constructed, when densities of surface force and normal component of heat flux are given on the upper and the lower face surfaces of the plate. The obtained two-dimensional initial-boundary value problems are investigated in suitable function spaces. Moreover, convergence of the sequence of vector-functions of three space variables restored from the solutions of the constructed two-dimensional problems to the solution of the original three-dimensional initial-boundary value problem is proved and under additional conditions the rate of convergence is estimated. *© 2014 Bull. Georg. Natl. Acad. Sci.*

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Various mathematical models for thermoelastic solids were developed to eliminate shortcomings of the classical thermoelasticity, particularly, infinite velocity of thermoelastic disturbances. One of such type models was obtained by H. Lord and Y. Shulman [1], where instead of the classical Fourier law of heat conduction Maxwell-Cataneo law is used, which is a generalization of the classical law and depends on one relaxation time. For Lord-Shulman nonclassical model the problem of propagation of a thermoelastic wave was studied and domain of influence result was obtained [2] in the classical spaces of twice continuously differentiable functions. Applying method of potential and theory of integral equations the problems of stable and pseudo oscillations for Lord-Shulman nonclassical model were studied in [3]. Note that solution of the three-dimensional initial-boundary value problems is a rather difficult task and it is important to construct algorithms of approximation of them by two-dimensional or one-dimensional problems. One of the dimensional reduction methods for plates with variable thickness in the classical theory of elasticity was suggested by I. Vekua in [4], where a hierarchy of initial-boundary value problems defined on two-dimensional space domain in differential form was obtained. Mathematical results on the relationship between the two-dimensional hierarchical models constructed in [4] and three-dimensional one in static case first were obtained in the spaces of classical regular functions in the paper [5], and the reduced two-dimensional models for thin shallow shells were investigated in Sobolev spaces in [6]. Later on, various hierarchical models were constructed and investigated applying Vekua's reduction method and its generalizations (see [7-11] and references given therein).

In this paper we study Lord-Shulman nonclassical three-dimensional dynamical model for thermoelastic solid and in the case of plate with variable thickness we construct and investigate an algorithm of approximation by two-dimensional problems. We consider initial-boundary value problem corresponding to Lord-Shulman three-dimensional model and applying variational approach we obtain the existence and uniqueness result in suitable spaces of vector-valued distributions with values in Sobolev spaces. We construct a hierarchy of two-dimensional problems approximating the original three-dimensional one for plate with variable thickness, when densities of surface force and normal component of heat flux are given on the upper and the lower face surfaces of the plate. We investigate the constructed two-dimensional initial-boundary value problems in suitable function spaces. Moreover, we prove that the sequence of vector-functions of three space variables restored from the solutions of the constructed two-dimensional problems converges to the solution of the original three-dimensional problem and under additional regularity conditions we estimate the rate of convergence.

We denote by $W^{r,2}(D) = H^r(D),\ r \geq 1$, $r \in \mathbf{R}$, the Sobolev space of order *r* based on the space $L^2(D)$ of square-integrable functions in $D \subset \mathbf{R}^p$, $p \in \mathbf{N}$, in Lebesgue sense, $\mathbf{H}^r(D) = [H^r(D)]^3$, $\mathbf{L}^2(D) = [L^2(D)]^3$ and $\mathbf{L}^s(\hat{\Gamma}) = [L^s(\hat{\Gamma})]^3$, $s \geq 1$, $s \in \mathbf{R}$, where $\hat{\Gamma}$ is a Lipschitz surface. For any Banach space *X*, $C^0([0,T];X)$ denotes the space of continuous functions on $[0,T]$ with values in *X*, $L^2(0,T;X)$ is the space of such functions $g:(0,T) \to X$ that $\|g(t)\|_X \in L^2(0,T)$. We denote by $g' = dg/dt$ the generalized derivative of $g \in L^2(0,T;X)$.

Let us consider a thermoelastic body with initial configuration $\Omega \subset \mathbf{R}^3$, which consists of homogeneous isotropic thermoelastic material and the body is clamped along a part Γ_0 of the boundary $\Gamma = \partial\Omega$ and on the remaining part $\Gamma_1 = \Gamma \setminus \overline{\Gamma_0}$ surface force with density $\mathbf{g} = (g_i) : \Gamma_1 \to \mathbf{R}^3$ is given, the temperature θ vanishes along $\Gamma_0^\theta \subset \Gamma$ and on the remaining part $\Gamma_1^\theta = \Gamma \setminus \overline{\Gamma_0^\theta}$ the normal component of heat flux with density $g^\theta : \Gamma_1^\theta \to \mathbf{R}$ is given. The nonclassical dynamical three-dimensional model of the thermoelastic body Ω obtained by H. Lord and Y. Shulman in differential form is given by the following initial-boundary value problem

$$\rho \frac{\partial^2 u_i}{\partial t^2} - \sum_{j=1}^{3} \frac{\partial}{\partial x_j}\left(\lambda \sum_{p=1}^{3} e_{pp}(\mathbf{u})\delta_{ij} + 2\mu e_{ij}(\mathbf{u}) - \eta\theta\delta_{ij} \right) = f_i \text{ in } \Omega\times(0,T),\ i = 1,2,3, \tag{1}$$

$$\chi\left(\frac{\partial\theta}{\partial t} + \tau_0 \frac{\partial^2\theta}{\partial t^2} \right) - \kappa \sum_{j=1}^{3} \frac{\partial^2\theta}{\partial x_j^2} + \Theta_0\eta \frac{\partial}{\partial t} \sum_{p=1}^{3} e_{pp}\left(\mathbf{u} + \tau_0 \frac{\partial \mathbf{u}}{\partial t} \right) = f^\theta \text{ in } \Omega\times(0,T), \tag{2}$$

$$\sum_{j=1}^{3}\left(\lambda\sum_{p=1}^{3}e_{pp}(\mathbf{u})\delta_{ij}+2\mu e_{ij}(\mathbf{u})-\eta\theta\delta_{ij}\right)\nu_j=g_i \quad \text{on} \quad \Gamma_1\times(0,T), \tag{3}$$

$$\mathbf{u}=\mathbf{0} \quad \text{on} \quad \Gamma_0\times(0,T), \quad \kappa\sum_{j=1}^{3}\frac{\partial\theta}{\partial x_j}\nu_j=g^{\theta} \quad \text{on} \quad \Gamma_1^{\theta}\times(0,T), \quad \theta=0 \quad \text{on} \quad \Gamma_0^{\theta}\times(0,T), \tag{4}$$

$$u_i(x,0)=u_{0i}(x), \quad \frac{\partial u_i}{\partial t}(x,0)=u_{1i}(x), \quad \theta(x,0)=\theta_0(x), \quad \frac{\partial\theta}{\partial t}(x,0)=\theta_1(x) \ \text{in} \ \Omega, \quad i=1,2,3, \tag{5}$$

where δ_{ij} is the Kronecker's delta, $\mathbf{u}=(u_i):\Omega\times(0,T)\to\mathbf{R}^3$ is the displacement vector-function of thermoelastic body, $\theta:\Omega\times(0,T)\to\mathbf{R}$ is the temperature distribution, λ,μ are Lamé constants, ρ is a mass density, $\kappa>0$ is the thermal conductivity coefficient, $\chi>0$ is the specific heat at zero strain, η is the stress-temperature coefficient, $\Theta_0>0$ is a constant reference temperature and τ_0 is the relaxation time, $\mathbf{f}=(f_i):\Omega\to\mathbf{R}^3$ is the density of applied body force and $f^{\theta}:\Omega\times(0,T)\to\mathbf{R}$ is the density of heat sources, $\mathbf{u}_0=(u_{0i})$ and $\mathbf{u}_1=(u_{1i})$ are initial displacement and velocity vector-functions, θ_0 and θ_1 are initial distributions of temperature and the rate of change of temperature. Note that, in the case of $\tau_0=0$ the nonclassical three-dimensional model (1)-(5) coincides with the classical linear three-dimensional model for thermoelastic bodies.

To investigate the existence and uniqueness of weak solution of the three-dimensional initial-boundary value problem (1)-(5) we consider the following variational formulation: Find the unknown vector-function $\mathbf{u}\in C^0([0,T];\mathbf{V}(\Omega))$, $\mathbf{u}'\in C^0([0,T];\mathbf{V}(\Omega))$, $\mathbf{u}''\in L^{\infty}(0,T;\mathbf{V}(\Omega))$, $\mathbf{u}'''\in L^{\infty}(0,T;\mathbf{L}^2(\Omega))$, and function $\theta\in C^0([0,T];V^{\theta}(\Omega))$, $\theta'\in L^{\infty}(0,T;V^{\theta}(\Omega))$, $\theta''\in L^{\infty}(0,T;L^2(\Omega))$, which satisfies the following equations in the sense of distributions on $(0,T)$,

$$\frac{d}{dt}(\rho\mathbf{u}'(.),\mathbf{v})_{\mathbf{L}^2(\Omega)}+a(\mathbf{u}(.),\mathbf{v})-\eta\left(\theta,\sum_{p=1}^{3}\frac{\partial v_p}{\partial x_p}\right)_{L^2(\Omega)}=(\mathbf{f},\mathbf{v})_{\mathbf{L}^2(\Omega)}+(\mathbf{g},\mathbf{v})_{\mathbf{L}^2(\Gamma_1)}, \quad \forall\mathbf{v}\in\mathbf{V}(\Omega), \tag{6}$$

$$\begin{aligned}&\frac{d}{dt}(\chi\tau_0\theta'(.),\varphi)_{L^2(\Omega)}+(\chi\theta'(.),\varphi)_{L^2(\Omega)}+a^{\theta}(\theta(.),\varphi)+\\&+\Theta_0\eta\left(\sum_{p=1}^{3}\frac{\partial u'_p}{\partial x_p}+\tau_0\sum_{p=1}^{3}\frac{\partial u''_p}{\partial x_p},\varphi\right)_{L^2(\Omega)}=(f^{\theta},\varphi)_{L^2(\Omega)}+(g^{\theta},\varphi)_{L^2(\Gamma_1^{\theta})}, \quad \forall\varphi\in V^{\theta}(\Omega),\end{aligned} \tag{7}$$

together with the initial conditions

$$\mathbf{u}(0)=\mathbf{u}_0, \ \mathbf{u}'(0)=\mathbf{u}_1, \ \theta(0)=\theta_0, \quad \theta'(0)=\theta_1, \tag{8}$$

where $\mathbf{V}(\Omega)=\{\mathbf{v}\in\mathbf{H}^1(\Omega);\mathbf{tr}(\mathbf{v})=\mathbf{0} \ \text{on} \ \Gamma_0\}$, $V^{\theta}(\Omega)=\{\varphi\in H^1(\Omega);tr(\varphi)=0 \ \text{on} \ \Gamma_0^{\theta}\}$, $\mathbf{tr}:\mathbf{H}^1(\Omega)\to\mathbf{H}^{1/2}(\Gamma)$ and $tr:H^1(\Omega)\to H^{1/2}(\Gamma)$ are the trace operators,

$$a(\tilde{\mathbf{v}},\mathbf{v})=\int_{\Omega}\left(\lambda\sum_{p=1}^{3}e_{pp}(\tilde{\mathbf{v}})\sum_{q=1}^{3}e_{qq}(\mathbf{v})+2\mu\sum_{i,j=1}^{3}e_{ij}(\tilde{\mathbf{v}})e_{ij}(\mathbf{v})\right)dx, \qquad \forall\mathbf{v},\tilde{\mathbf{v}}\in\mathbf{V}(\Omega),$$

$$a^{\theta}(\tilde{\varphi},\varphi)=\kappa\int_{\Omega}\sum_{j=1}^{3}\frac{\partial\tilde{\varphi}}{\partial x_j}\frac{\partial\varphi}{\partial x_j}dx,\qquad \forall\varphi,\tilde{\varphi}\in V^{\theta}(\Omega),\quad e_{ij}(\mathbf{v})=\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right), i,j=1,2,3,$$

$(.,.)_{\mathbf{L}^2(\Omega)}$, $(.,.)_{L^2(\Omega)}$, $(.,.)_{\mathbf{L}^2(\Gamma_1)}$ and $(.,.)_{L^2(\Gamma_1)}$ are scalar products in the spaces $\mathbf{L}^2(\Omega)$, $L^2(\Omega)$, $\mathbf{L}^2(\Gamma_1)$ and $L^2(\Gamma_1)$, respectively.

For initial-boundary value problem (6)-(8) corresponding to Lord-Shulman nonclassical dynamical three-dimensional model for thermoelastic body the following theorem is valid.

Theorem 1. *Suppose that,* $\mathbf{u}_0\in\mathbf{H}^3(\Omega)\cap\mathbf{V}(\Omega)$, $\mathbf{u}_1\in\mathbf{H}^2\cap\mathbf{V}(\Omega)$, $\theta_0\in H^2(\Omega)\cap V^{\theta}(\Omega)$, $\theta_1\in V^{\theta}(\Omega)$, $\mathbf{f}\in C^0([0,T];\mathbf{H}^1(\Omega))$, $\mathbf{f}',\mathbf{f}''\in L^2(0,T;\mathbf{L}^2(\Omega))$, $\mathbf{g},\mathbf{g}',\mathbf{g}'',\mathbf{g}'''\in L^2(0,T;\mathbf{L}^{4/3}(\Gamma_1))$, $f^{\theta},f^{\theta'}\in L^2(0,T;L^2(\Omega))$, $g^{\theta},g^{\theta'},g^{\theta''}\in L^2(0,T;L^{4/3}(\Gamma_1^{\theta}))$, *and the following compatibility conditions are valid*

$$\begin{gathered}g_i(0)=\sum_{j=1}^{3}\left(\lambda\sum_{p=1}^{3}e_{pp}(\mathbf{u}_0)\delta_{ij}+2\mu e_{ij}(\mathbf{u}_0)-\eta\theta_0\delta_{ij}\right)\nu_j,\\ g_i'(0)=\sum_{j=1}^{3}\left(\lambda\sum_{p=1}^{3}e_{pp}(\mathbf{u}_1)\delta_{ij}+2\mu e_{ij}(\mathbf{u}_1)-\eta\theta_1\delta_{ij}\right)\nu_j,\end{gathered}\quad i=1,2,3,\ g^{\theta}(0)=\sum_{j=1}^{3}\kappa\frac{\partial\theta_0}{\partial x_j}\nu_j\,. \tag{9}$$

If $\rho>0$, $\Theta_0>0$, $\mu>0$, $3\lambda+2\mu>0$, $\chi>0$, $\kappa>0$ *and* $\tau_0>0$, *then the initial-boundary value problem* (6)-(8) *possesses a unique solution and the following energy identity is valid*

$$\begin{aligned}&\rho\,\|\mathbf{u}'+\tau_0\mathbf{u}''\|^2_{\mathbf{L}^2(\Omega)}+a(\mathbf{u}+\tau_0\mathbf{u}',\mathbf{u}+\tau_0\mathbf{u}')+\frac{\chi}{\Theta_0}\|\theta+\tau_0\theta'\|^2_{L^2(\Omega)}+\frac{2}{\Theta_0}\int_0^t a^{\theta}(\theta(\tau),\theta(\tau))d\tau+\\&+\frac{\tau_0}{\Theta_0}a^{\theta}(\theta,\theta)=\rho\,\|\mathbf{u}'(0)+\tau_0\mathbf{u}''(0)\|^2_{\mathbf{L}^2(\Omega)}+a(\mathbf{u}(0)+\tau_0\mathbf{u}'(0),\mathbf{u}(0)+\tau_0\mathbf{u}'(0))+\\&+\frac{\chi}{\Theta_0}\|\theta(0)+\tau_0\theta'(0)\|^2_{L^2(\Omega)}+\frac{\tau_0}{\Theta_0}a^{\theta}(\theta(0),\theta(0))+2\int_0^t(\mathbf{f}(\tau)+\tau_0\mathbf{f}'(\tau),\mathbf{u}'(\tau)+\tau_0\mathbf{u}''(\tau))_{\mathbf{L}^2(\Omega)}d\tau+\\&+\frac{2}{\Theta_0}\int_0^t(f^{\theta}(\tau),\theta(\tau)+\tau_0\theta'(\tau))_{L^2(\Omega)}d\tau+\frac{2}{\Theta_0}\int_0^t(g^{\theta}(\tau),\theta(\tau))_{L^2(\Gamma_1^{\theta})}d\tau+\frac{2\tau_0}{\Theta_0}(g^{\theta},\theta)_{L^2(\Gamma_1^{\theta})}-\\&-\frac{2\tau_0}{\Theta_0}(g^{\theta}(0),\theta(0))_{L^2(\Gamma_1^{\theta})}-\frac{2\tau_0}{\Theta_0}\int_0^t(g^{\theta'}(\tau),\theta(\tau))_{L^2(\Gamma_1^{\theta})}d\tau+2(\mathbf{g}+\tau_0\mathbf{g}',\mathbf{u}+\tau_0\mathbf{u}')_{\mathbf{L}^2(\Gamma_1)}-\\&-2(\mathbf{g}(0)+\tau_0\mathbf{g}'(0),\mathbf{u}(0)+\tau_0\mathbf{u}'(0))_{\mathbf{L}^2(\Gamma_1)}-2\int_0^t(\mathbf{g}'(\tau)+\tau_0\mathbf{g}''(\tau),\mathbf{u}(\tau)+\tau_0\mathbf{u}'(\tau))_{\mathbf{L}^2(\Gamma_1)}d\tau,\ \forall t\in[0,T].\end{aligned}$$

Let us consider particular case of thermoelastic body, when Ω is a plate with variable thickness, which may vanish on a part of its boundary, i.e. body that occupies three-dimensional Lipschitz domain of the following form

$$\Omega=\{(x_1,x_2,x_3)\in\mathbf{R}^3;\ h^-(x_1,x_2)<x_3<h^+(x_1,x_2),\quad (x_1,x_2)\in\omega\},$$

where $\omega\subset\mathbf{R}^2$ is a two-dimensional bounded Lipschitz domain with boundary $\partial\omega$,

$h^{\pm} \in C^0(\bar{\omega}) \cap C^{2,1}_{loc}(\omega \cup \tilde{\gamma})$ are Lipschitz continuous in the interior of the domain ω and on $\tilde{\gamma} \subset \partial\omega$ together with the first and second order derivatives, $h^+(x_1,x_2) > h^-(x_1,x_2)$, for $(x_1,x_2) \in \omega \cup \tilde{\gamma}$, $\tilde{\gamma} \subset \partial\omega$ is a Lipschitz curve, $h^+(x_1,x_2) = h^-(x_1,x_2)$, for $(x_1,x_2) \in \partial\omega \setminus \tilde{\gamma}$. The upper and the lower faces of Ω defined by the equations $x_3 = h^+(x_1,x_2)$ and $x_3 = h^-(x_1,x_2)$, $(x_1,x_2) \in \omega$, we denote by Γ^+ and Γ^-, respectively, and the lateral face, where thickness of Ω is positive, we denote by $\tilde{\Gamma} = \partial\Omega \setminus (\overline{\Gamma^+ \cup \Gamma^-}) = \{(x_1,x_2,x_3) \in \mathbf{R}^3; h^-(x_1,x_2) < x_3 < h^+(x_1,x_2),\ (x_1,x_2) \in \tilde{\gamma}\}$. We assume that plate is clamped and the temperature θ vanishes on a part $\tilde{\Gamma}_0 = \{(x_1,x_2,x_3) \in \mathbf{R}^3; h^-(x_1,x_2) < x_3 < h^+(x_1,x_2),\ (x_1,x_2) \in \tilde{\gamma}_0\}$ of the lateral face $\tilde{\Gamma}$, $\tilde{\gamma}_0 \subset \tilde{\gamma}$ is a Lipschitz curve, and on the remaining part $\Gamma_1 = \Gamma \setminus \overline{\tilde{\Gamma}_0}$ of the boundary the densities of surface force and normal component of heat flux are given, i.e. $\Gamma_0 = \Gamma_0^{\theta} = \tilde{\Gamma}_0$ and $\Gamma_1 = \Gamma_1^{\theta} = \Gamma \setminus \overline{\tilde{\Gamma}_0}$.

In order to construct an algorithm of approximation of Lord-Shulman nonclassical three-dimensional model for thermoelastic plates by a sequence of two-dimensional problems let us consider the subspaces $\mathbf{V}^3_{\mathbf{N}}(\Omega)$, $\mathbf{V}^2_{\mathbf{N}}(\Omega)$, $\mathbf{V}_{\mathbf{N}}(\Omega)$ and $\mathbf{H}_{\mathbf{N}}(\Omega)$ of $\mathbf{H}^3(\Omega) \cap \mathbf{V}(\Omega)$, $\mathbf{H}^2(\Omega) \cap \mathbf{V}(\Omega)$, $\mathbf{V}(\Omega)$ and $\mathbf{L}^2(\Omega)$, respectively, $\mathbf{N} = (N_1,N_2,N_3)$, consisting of vector-functions whose components are polynomials with respect to the variable of plate thickness x_3,

$$\mathbf{v_N} = (v_{\mathbf{N}i}),\quad v_{\mathbf{N}i} = \sum_{r_i=0}^{N_i} \frac{1}{h}\left(r_i + \frac{1}{2}\right) \overset{r_i}{v}_{\mathbf{N}i} P_{r_i}(z),\quad \overset{r_i}{v}_{\mathbf{N}i} \in L^2(\omega),\quad 0 \le r_i \le N_i,\quad i = 1,2,3,$$

where $z = \dfrac{x_3 - \bar{h}}{h}$, $h = \dfrac{h^+ - h^-}{2}$, $\bar{h} = \dfrac{h^+ + h^-}{2}$. In addition, we consider the subspaces $V^{\theta,2}_{N_\theta}(\Omega)$, $V^{\theta}_{N_\theta}(\Omega)$ and $H^{\theta}_{N_\theta}(\Omega)$ of $H^2(\Omega) \cap V^{\theta}(\Omega)$, $V^{\theta}(\Omega)$ and $L^2(\Omega)$, respectively, which consist of the following functions

$$\varphi_{N_\theta} = \sum_{r=0}^{N_\theta} \frac{1}{h}\left(r + \frac{1}{2}\right) \overset{r}{\varphi}_{N_\theta} P_r(z),\quad \overset{r}{\varphi}_{N_\theta} \in L^2(\omega),\quad 0 \le r \le N_\theta.$$

Since the functions h^+ and h^- are Lipschitz continuous together with their first and second order derivatives in the interior of the domain ω, from Rademacher's theorem [12] it follows that $h^{\pm}$, $\partial_\alpha h^{\pm}$ and $\partial_\alpha \partial_\beta h^{\pm}$ are differentiable almost everywhere in ω^* and $\partial_\alpha h^{\pm}, \partial_\alpha \partial_\beta h^{\pm}, \partial_\alpha \partial_\beta \partial_\delta h^{\pm} \in L^{\infty}(\omega^*)$ for all subdomains ω^*, $\overline{\omega^*} \subset \omega$, $\alpha,\beta,\delta = 1,2$. Therefore, the positiveness of h in ω implies that for any vector-function $\mathbf{v_N} = (v_{\mathbf{N}i})_{i=1}^3 \in \mathbf{V}^3_{\mathbf{N}}(\Omega)$ the corresponding functions $\overset{r_i}{v}_{\mathbf{N}i} \in H^3(\omega^*)$, for all ω^*, $\overline{\omega^*} \subset \omega$, i.e. $\overset{r_i}{v}_{\mathbf{N}i} \in H^3_{loc}(\omega)$, $0 \le r_i \le N_i$, $i = 1,2,3$. Similarly, if $\mathbf{v_N} = (v_{\mathbf{N}i})_{i=1}^3 \in \mathbf{V}^2_{\mathbf{N}}(\Omega)$ or $\mathbf{v_N} = (v_{\mathbf{N}i})_{i=1}^3 \in \mathbf{V}_{\mathbf{N}}(\Omega)$, then the corresponding functions $\overset{r_i}{v}_{\mathbf{N}i} \in H^2(\omega^*)$ or $\overset{r_i}{v}_{\mathbf{N}i} \in H^1(\omega^*)$ for all ω^*, $\overline{\omega^*} \subset \omega$, i.e. $\overset{r_i}{v}_{\mathbf{N}i} \in H^2_{loc}(\omega)$ or $\overset{r_i}{v}_{\mathbf{N}i} \in H^1_{loc}(\omega)$, $0 \le r_i \le N_i$, $i = 1,2,3$. For functions from the spaces $V^{\theta,2}_{N_\theta}(\Omega)$ and $V^{\theta}_{N_\theta}(\Omega)$ we also have

$\overset{r}{\varphi}_{N_\theta} \in H^2_{loc}(\omega)$, if $\varphi_{N_\theta} \in V^{\theta,2}_{N_\theta}(\Omega)$ and $\overset{r}{\varphi}_{N_\theta} \in H^1_{loc}(\omega)$, if $\varphi_{N_\theta} \in V^{\theta}_{N_\theta}(\Omega)$, $0 \le r \le N_\theta$. Moreover, the norms $\|\cdot\|_{\mathbf{H}^3(\Omega)}$, $\|\cdot\|_{\mathbf{H}^2(\Omega)}$, $\|\cdot\|_{\mathbf{H}^1(\Omega)}$ and $\|\cdot\|_{H^2(\Omega)}$, $\|\cdot\|_{H^1(\Omega)}$ in the spaces $\mathbf{H}^3(\Omega)$, $\mathbf{H}^2(\Omega)$, $\mathbf{H}^1(\Omega)$ and $H^2(\Omega)$, $H^1(\Omega)$ define weighted norms $\|\cdot\|_{***}$, $\|\cdot\|_{**}$, $\|\cdot\|_{*}$ and $\|\cdot\|_{\theta**}$, $\|\cdot\|_{\theta*}$ of vector-functions $\vec{v}_{\mathbf{N}} \in [H^3_{loc}(\omega)]^{N_{1,2,3}}$, $\vec{v}_{\mathbf{N}} \in [H^2_{loc}(\omega)]^{N_{1,2,3}}$, $\vec{v}_{\mathbf{N}} \in [H^1_{loc}(\omega)]^{N_{1,2,3}}$, $N_{1,2,3} = N_1 + N_2 + N_3 + 3$, with components $\overset{r_i}{v}_{\mathbf{N}i}$, $\vec{v}_{\mathbf{N}} = (\overset{r_i}{v}_{\mathbf{N}i})$, and $\vec{\varphi}_{N_\theta} \in [H^2_{loc}(\omega)]^{N_\theta+1}$, $\vec{\varphi}_{N_\theta} \in [H^1_{loc}(\omega)]^{N_\theta+1}$, with components $\overset{r}{\varphi}_{N_\theta}$, $\vec{\varphi}_{N_\theta} = (\overset{r}{\varphi}_{N_\theta})$, such that $\|\vec{v}_{\mathbf{N}}\|_{***} = \|\mathbf{v}_{\mathbf{N}}\|_{\mathbf{H}^3(\Omega)}$, $\|\vec{v}_{\mathbf{N}}\|_{**} = \|\mathbf{v}_{\mathbf{N}}\|_{\mathbf{H}^2(\Omega)}$, $\|\vec{v}_{\mathbf{N}}\|_{*} = \|\mathbf{v}_{\mathbf{N}}\|_{\mathbf{H}^1(\Omega)}$ and $\|\vec{\varphi}_{N_\theta}\|_{\theta**} = \|\varphi_{N_\theta}\|_{H^2(\Omega)}$, $\|\vec{\varphi}_{N_\theta}\|_{\theta*} = \|\varphi_{N_\theta}\|_{H^1(\Omega)}$. Using the properties of the Legendre polynomials, we can obtain explicit expressions for the norms $\|\cdot\|_{***}$, $\|\cdot\|_{**}$, $\|\cdot\|_{*}$, $\|\cdot\|_{\theta**}$ and $\|\cdot\|_{\theta*}$. Particularly, the norm $\|\cdot\|_{*}$ is given by

$$\|\vec{v}_{\mathbf{N}}\|_{*} = \left(\sum_{i=1}^{3} \sum_{r_i=0}^{N_i} (r_i + \frac{1}{2}) \left[\left\| \sum_{s_i=r_i}^{N_i} (s_i + \frac{1}{2})(1-(-1)^{r_i+s_i}) h^{-3/2} \overset{s_i}{v}_{\mathbf{N}i} \right\|^2_{L^2(\omega)} + \left\| h^{-1/2} \overset{r_i}{v}_{\mathbf{N}i} \right\|^2_{L^2(\omega)} + \right.\right.$$

$$\left.\left. + \sum_{\alpha=1}^{2} \left\| \sum_{s_i=r_i+1}^{N_i} (s_i + \frac{1}{2})(\partial_\alpha h^+ - (-1)^{r_i+s_i} \partial_\alpha h^-) h^{-3/2} \overset{s_i}{v}_{\mathbf{N}i} - h^{-1/2} \partial_\alpha \overset{r_i}{v}_{\mathbf{N}i} + (r_i+1) h^{-3/2} \partial_\alpha h \overset{r_i}{v}_{\mathbf{N}i} \right\|^2_{L^2(\omega)} \right] \right)^{1/2},$$

where we assume that the sum with the lower limit greater than the upper one equals to zero.

For components $\overset{r_i}{v}_{\mathbf{N}i}$ and $\overset{r}{\varphi}_{N_\theta}$ of vector-functions $\vec{v}_{\mathbf{N}} \in [H^1_{loc}(\omega)]^{N_{1,2,3}}$ and $\vec{\varphi}_{N_\theta} \in [H^1_{loc}(\omega)]^{N_\theta+1}$, which satisfy the conditions $\|\vec{v}_{\mathbf{N}}\|_{*} < \infty$ and $\|\vec{\varphi}_{N_\theta}\|_{\theta*} < \infty$ we can define the traces on $\tilde{\gamma}$. Indeed, the corresponding vector-function $\mathbf{v}_{\mathbf{N}} = (v_{\mathbf{N}i})_{i=1}^3$ and function φ_{N_θ} of three space variables belong to the spaces $\mathbf{V}_{\mathbf{N}}(\Omega) \subset \mathbf{H}^1(\Omega)$ and $V^{\theta}_{N_\theta}(\Omega) \subset H^1(\Omega)$, respectively. Consequently, applying the trace operator on the space $H^1(\Omega)$ we define the traces on $\tilde{\gamma}$ for functions $\overset{r_i}{v}_{\mathbf{N}i}$ and $\overset{r}{\varphi}_{N_\theta}$,

$$tr_{\tilde{\gamma}}(\overset{r_i}{v}_{\mathbf{N}i}) = \int_{h^-}^{h^+} tr(v_{\mathbf{N}i})|_{\tilde{\Gamma}} P_{r_i}(z) dx_3, \quad tr_{\tilde{\gamma}}(\overset{r}{\varphi}_{N_\theta}) = \int_{h^-}^{h^+} tr(\varphi_{N_\theta})|_{\tilde{\Gamma}} P_r(z) dx_3, \quad r_i = \overline{0, N_i}, \ i = \overline{1,3}, \ r = \overline{0, N_\theta}.$$

Since the vector-functions $\mathbf{v}_{\mathbf{N}} = (v_{\mathbf{N}i})$ from the subspaces $\mathbf{v}_{\mathbf{N}}(\Omega)$ and $\mathbf{H}_{\mathbf{N}}(\Omega)$, and the functions φ_{N_θ} from $V^{\theta}_{N_\theta}(\Omega)$ and $H^{\theta}_{N_\theta}(\Omega)$ are uniquely defined by functions $\overset{r_i}{v}_{\mathbf{N}i}$ and $\overset{r}{\varphi}_{N_\theta}$ of two space variables, therefore considering the original three-dimensional problem (6)-(8) on these subspaces, we obtain the following hierarchy of two-dimensional initial-boundary value problems: Find $\vec{w}_{\mathbf{N}} \in C^0([0,T];\ \vec{V}_{\mathbf{N}}(\omega))$, $\vec{w}'_{\mathbf{N}} \in C^0([0,T];\ \vec{V}_{\mathbf{N}}(\omega))$, $\vec{w}''_{\mathbf{N}} \in L^\infty(0,T;\ \vec{V}_{\mathbf{N}}(\omega))$, $\vec{w}'''_{\mathbf{N}} \in L^\infty(0,T;\ \vec{H}_{\mathbf{N}}(\omega))$ and $\vec{\zeta}_{N_\theta} \in C^0([0,T];\vec{V}^{\theta}_{N_\theta}(\omega))$, $\vec{\zeta}'_{N_\theta} \in L^\infty(0,T;\vec{V}^{\theta}_{N_\theta}(\omega))$, $\vec{\zeta}''_{N_\theta} \in L^\infty(0,T;\vec{H}^{\theta}_{N_\theta}(\omega))$, which satisfy the following equations in the sense of distributions on $(0,T)$,

$$\frac{d}{dt}R_{\mathbf{N}}(\vec{w}'_{\mathbf{N}},\vec{v}_{\mathbf{N}})+a_{\mathbf{N}}(\vec{w}_{\mathbf{N}},\vec{v}_{\mathbf{N}})-b_{\mathbf{N}N_\theta}(\vec{\zeta}_{N_\theta},\vec{v}_{\mathbf{N}})=L_{\mathbf{N}}(\vec{v}_{\mathbf{N}}),\ \forall\vec{v}_{\mathbf{N}}\in\vec{V}_{\mathbf{N}}(\omega), \tag{10}$$

$$\frac{d}{dt}R^\theta_{N_\theta}(\vec{\zeta}_{N_\theta}+\tau_0\vec{\zeta}'_{N_\theta},\vec{\varphi}_{N_\theta})+a^\theta_{N_\theta}(\vec{\zeta}_{N_\theta},\vec{\varphi}_{N_\theta})+\Theta_0\frac{d}{dt}b^\theta_{\mathbf{N}N_\theta}(\vec{w}_{\mathbf{N}}+\tau_0\vec{w}'_{\mathbf{N}},\vec{\varphi}_{N_\theta})=L^\theta_{N_\theta}(\vec{\varphi}_{N_\theta}), \tag{11}$$

for all $\vec{\varphi}_{N_\theta}\in\vec{V}^\theta_{N_\theta}(\omega)$, together with the initial conditions

$$\vec{w}_{\mathbf{N}}(0)=\vec{w}_{\mathbf{N}0},\qquad \vec{w}'_{\mathbf{N}}(0)=\vec{w}_{\mathbf{N}1},\qquad \vec{\zeta}_{N_\theta}(0)=\vec{\zeta}_{N_\theta 0},\qquad \vec{\zeta}'_{N_\theta}(0)=\vec{\zeta}_{N_\theta 1}, \tag{12}$$

where $\vec{V}_{\mathbf{N}}(\omega)=\{\vec{v}_{\mathbf{N}}=(\overset{r_i}{v}_{\mathbf{N}i})\in[H^1_{loc}(\omega)]^{N_{1,2,3}};\|\vec{v}_{\mathbf{N}}\|_*<\infty, tr_{\tilde{\gamma}}(\overset{r_i}{v}_{\mathbf{N}i})=0 \text{ on } \tilde{\gamma}_0, r_i=\overline{0,N_i}, i=\overline{1,3}\}$,

$$\vec{H}^1_{\mathbf{N}}(\omega)=\{\vec{v}_{\mathbf{N}}=(\overset{r_i}{v}_{\mathbf{N}i})\in[H^1_{loc}(\omega)]^{N_{1,2,3}};\|\vec{v}_{\mathbf{N}}\|_*<\infty\},$$

$$\vec{V}^2_{\mathbf{N}}(\omega)=\{\vec{v}_{\mathbf{N}}=(\overset{r_i}{v}_{\mathbf{N}i})\in[H^2_{loc}(\omega)]^{N_{1,2,3}}\cap\vec{V}_{\mathbf{N}}(\omega);\ \|\vec{v}_{\mathbf{N}}\|_{**}<\infty\},$$

$\vec{V}^3_{\mathbf{N}}(\omega)=\{\vec{v}_{\mathbf{N}}=(\overset{r_i}{v}_{\mathbf{N}i})\in[H^3_{loc}(\omega)]^{N_{1,2,3}}\cap\vec{V}_{\mathbf{N}}(\omega);\|\vec{v}_{\mathbf{N}}\|_{***}<\infty\}$, $\vec{H}_{\mathbf{N}}(\omega)=\{\vec{v}_{\mathbf{N}}=(\overset{r_i}{v}_{\mathbf{N}i})\in$ $[L^2(\omega)]^{N_{1,2,3}};\ \|\vec{v}_{\mathbf{N}}\|^2_{\vec{H}_{\mathbf{N}}(\omega)}=\sum_{i=1}^3\sum_{r_i=0}^{N_i}\left\|h^{-1/2}\overset{r_i}{v}_{\mathbf{N}i}\right\|^2_{L^2(\omega)}<\infty\}$, $\vec{V}^\theta_{N_\theta}(\omega)=\{\vec{\varphi}_{N_\theta}=(\overset{r}{\varphi}_{N_\theta})\in[H^1_{loc}(\omega)]^{N_\theta+1};$

$$\left\|\vec{\varphi}_{N_\theta}\right\|_{\theta*}<\infty,\ tr_{\tilde{\gamma}}(\overset{r}{\varphi}_{N_\theta})=0 \text{ on } \tilde{\gamma}_0, r=\overline{0,N_\theta}\},$$

$\vec{V}^{\theta,2}_{N_\theta}(\omega)=\{\vec{\varphi}_{N_\theta}=(\overset{r}{\varphi}_{N_\theta})\in[H^2_{loc}(\omega)]^{N_\theta+1}\cap\vec{V}^\theta_{N_\theta}(\omega);\ \left\|\vec{\varphi}_{N_\theta}\right\|_{\theta**}<\infty\}$, $\vec{H}^\theta_{N_\theta}(\omega)=\{\vec{\varphi}_{N_\theta}=(\overset{r}{\varphi}_{N_\theta})\in[L^2(\omega)]^{N_\theta+1};$

$\left\|\vec{\varphi}_{N_\theta}\right\|^2_{\vec{H}^\theta_{N_\theta}(\omega)}=\sum_{r=0}^{N_\theta}\left\|h^{-1/2}\overset{r}{\varphi}_{N_\theta}\right\|^2_{L^2(\omega)}<\infty\}$, the bilinear forms $R_{\mathbf{N}}$, $R^\theta_{N_\theta}$, $a_{\mathbf{N}}$, $a^\theta_{N_\theta}$, $b_{\mathbf{N}N_\theta}$, $b^\theta_{\mathbf{N}N_\theta}$ are defined by the corresponding forms in the left-hand sides of the equations (6), (7) and by taking into account the properties of Legendre polynomials, we obtain the following explicit expressions

$$R_{\mathbf{N}}(\vec{y}_{\mathbf{N}},\vec{v}_{\mathbf{N}})=\sum_{i=1}^3\sum_{r_i=0}^{N_i}\left(r_i+\frac{1}{2}\right)\rho\int_\omega\frac{1}{h}\overset{r_i}{y}_{\mathbf{N}i}\overset{r_i}{v}_{\mathbf{N}i}\,d\omega,\ R^\theta_{N_\theta}(\vec{\psi}_{N_\theta},\vec{\varphi}_{N_\theta})=\sum_{r=0}^{N_\theta}\left(r+\frac{1}{2}\right)\chi\int_\omega\frac{1}{h}\overset{r}{\psi}_{N_\theta}\overset{r}{\varphi}_{N_\theta}\,d\omega,$$

$$a_{\mathbf{N}}(\vec{y}_{\mathbf{N}},\vec{v}_{\mathbf{N}})=\sum_{r=0}^{N_{\max}}\left(r+\frac{1}{2}\right)\int_\omega\frac{1}{h}\left(\lambda\sum_{p=1}^3\overset{r}{e}_{pp}(\vec{y}_{\mathbf{N}})\sum_{q=1}^3\overset{r}{e}_{qq}(\vec{v}_{\mathbf{N}})+2\mu\sum_{i,j=1}^3\overset{r}{e}_{ij}(\vec{y}_{\mathbf{N}})\overset{r}{e}_{ij}(\vec{v}_{\mathbf{N}})\right)d\omega,$$

$$a^\theta_{N_\theta}(\vec{\psi}_{N_\theta},\vec{\varphi}_{N_\theta})=\sum_{r=0}^{N_\theta}\left(r+\frac{1}{2}\right)\kappa\int_\omega\left[\frac{1}{h^3}\left(\sum_{s=r}^{N_\theta}(s+\frac{1}{2})\overset{s}{\psi}_{N_\theta}(1-(-1)^{r+s})\right)\left(\sum_{\hat{s}=r}^{N_\theta}(\hat{s}+\frac{1}{2})\overset{\hat{s}}{\varphi}_{N_\theta}(1-(-1)^{r+\hat{s}})\right)+\right.$$

$$+\sum_{\alpha=1}^2\frac{1}{h}\left(\partial_\alpha\overset{r}{\psi}_{N_\theta}-(r+1)\frac{\partial_\alpha h}{h}\overset{r}{\psi}_{N_\theta}-\sum_{s=r+1}^{N_\theta}\frac{\overset{s}{\psi}_{N_\theta}}{h}\left(s+\frac{1}{2}\right)\left(\partial_\alpha h^+-(-1)^{r+s}\partial_\alpha h^-\right)\right)\times$$

$$\left.\times\left(\partial_\alpha\overset{r}{\varphi}_{N_\theta}-(r+1)\frac{\partial_\alpha h}{h}\overset{r}{\varphi}_{N_\theta}-\sum_{\hat{s}=r+1}^{N_\theta}\frac{\overset{\hat{s}}{\varphi}_{N_\theta}}{h}\left(\hat{s}+\frac{1}{2}\right)\left(\partial_\alpha h^+-(-1)^{r+\hat{s}}\partial_\alpha h^-\right)\right)\right]d\omega,$$

$$b_{\mathbf{N}N_\theta}(\vec{\varphi}_{N_\theta},\vec{v}_{\mathbf{N}})=b^\theta_{\mathbf{N}N_\theta}(\vec{v}_{\mathbf{N}},\vec{\varphi}_{N_\theta})=\eta\sum_{r=0}^{N_\theta}\left(r+\frac{1}{2}\right)\int_\omega\left[\frac{1}{h^2}\left(\sum_{s=r}^{N_3}(s+\frac{1}{2})\overset{s}{v}_{\mathbf{N}3}(1-(-1)^{r+s})\right)+\right.$$

$$\left.+\sum_{\alpha=1}^{2}\frac{1}{h}\left(\partial_\alpha\overset{r}{v}_{\mathbf{N}\alpha}-(r+1)\frac{\partial_\alpha h}{h}\overset{r}{v}_{\mathbf{N}\alpha}-\sum_{s=r+1}^{N_\alpha}\frac{\overset{s}{v}_{\mathbf{N}\alpha}}{h}\left(s+\frac{1}{2}\right)\left(\partial_\alpha h^+-(-1)^{r+s}\partial_\alpha h^-\right)\right)\right]\overset{r}{\varphi}_{N_\theta}\,d\omega,$$

where $N_{\max}=\max\{N_1,N_2,N_3\}$, $\overset{r}{e}_{ij}(\vec{v}_{\mathbf{N}})=\frac{1}{2}\left(\partial_i(\overset{r}{v}_{\mathbf{N}j})+\partial_j(\overset{r}{v}_{\mathbf{N}i})+\overset{r}{\tilde{e}}_{ij}(\vec{v}_{\mathbf{N}})\right)$, $i,j=1,2,3$,

$$\overset{r}{\tilde{e}}_{ij}(\vec{v}_{\mathbf{N}})=-\frac{r+1}{h}\left(\partial_i h\overset{r}{v}_{\mathbf{N}j}+\partial_j h\overset{r}{v}_{\mathbf{N}i}\right)-\sum_{s=r+1}^{N_{\max}}\frac{1}{h}\left(s+\frac{1}{2}\right)\left(\overset{s}{v}_{\mathbf{N}j}(\partial_i h^+-(-1)^{r+s}\partial_i h^-)+\right.$$

$$\left.+\overset{s}{v}_{\mathbf{N}i}(\partial_j h^+-(-1)^{r+s}\partial_j h^-)\right)+\sum_{s=r}^{N_{\max}}\frac{1}{h}\left(s+\frac{1}{2}\right)(1-(-1)^{r+s})\left(\frac{(i-1)(i-2)}{2}\overset{s}{v}_{\mathbf{N}j}+\frac{(j-1)(j-2)}{2}\overset{s}{v}_{\mathbf{N}i}\right).$$

The linear forms $L_{\mathbf{N}}$, $L^\theta_{N_\theta}$ are defined by the right-hand sides of the equations (6), (7) and are given by

$$L_{\mathbf{N}}(\vec{v}_{\mathbf{N}})=\sum_{i=1}^{3}\sum_{r_i=0}^{N_i}\left(r_i+\frac{1}{2}\right)\left[\int_\omega\frac{1}{h}\overset{r_i}{v}_{\mathbf{N}i}\left(\overset{r_i}{f}_i+\overset{+}{g}_{\mathbf{N}i}\lambda_++\overset{-}{g}_{\mathbf{N}i}\lambda_-(-1)^{r_i}\right)d\omega+\int_{\gamma_1}\frac{1}{h}\overset{r_i}{v}_{\mathbf{N}i}\overset{r_i}{g}_{\mathbf{N}i}\,d\gamma_1\right],$$

$$L^\theta_{N_\theta}(\vec{\varphi}_{N_\theta})=\sum_{r=0}^{N_\theta}\left(r+\frac{1}{2}\right)\left[\int_\omega\frac{1}{h}\overset{r}{\varphi}_{N_\theta}\left(\overset{r}{f}{}^\theta+g^{\theta+}_{N_\theta}\lambda_++g^{\theta-}_{N_\theta}\lambda_-(-1)^r\right)d\omega+\int_{\gamma_1}\frac{1}{h}\overset{r}{\varphi}_{N_\theta}\overset{r}{g}{}^\theta_{N_\theta}\,d\gamma_1\right],$$

where $\gamma_1=\tilde{\gamma}\setminus\tilde{\gamma}_0$, $\lambda_\pm=\sqrt{1+(\partial_1 h^\pm)^2+(\partial_2 h^\pm)^2}$, $\overset{r}{v}=\int_{h^-}^{h^+}vP_r(z)dx_3$, for all functions $v\in L^2(\Omega)$, $r\in\mathbf{N}\cup\{0\}$,

$\overset{+}{g}_{\mathbf{N}i}$, $g^{\theta+}_{N_\theta}$ and $\overset{-}{g}_{\mathbf{N}i}$, $g^{\theta-}_{N_\theta}$ are restrictions of

$$g_{\mathbf{N}i}(t)=g_i(t)+\sum_{j=1}^{3}\left(\lambda\sum_{p=1}^{3}e_{pp}(\mathbf{w}_{\mathbf{N}0})\delta_{ij}+2\mu e_{ij}(\mathbf{w}_{\mathbf{N}0})-\eta\zeta_{N_\theta 0}\delta_{ij}\right)v_j\Bigg|_{\Gamma_1}+$$

$$+\sum_{j=1}^{3}\left(\lambda\sum_{p=1}^{3}e_{pp}(\mathbf{w}_{\mathbf{N}1})\delta_{ij}+2\mu e_{ij}(\mathbf{w}_{\mathbf{N}1})-\eta\zeta_{N_\theta 1}\delta_{ij}\right)v_j\Bigg|_{\Gamma_1}t-g_i(0)-g_i'(0)t,\ \ i=1,2,3,$$

$$g^\theta_{N_\theta}(t)=g^\theta(t)+\sum_{j=1}^{3}\kappa\frac{\partial\zeta_{N_\theta 0}}{\partial x_j}v_j\Bigg|_{\Gamma_1}-g^\theta(0)$$

on the upper Γ^+ and the lower Γ^- faces of the plate, respectively, $\mathbf{w}_{\mathbf{N}0}\in\mathbf{V}^3_{\mathbf{N}}(\Omega)$, $\mathbf{w}_{\mathbf{N}1}\in\mathbf{V}^2_{\mathbf{N}}(\Omega)$, $\zeta_{N_\theta 0}\in V^{\theta,2}_{N_\theta}(\Omega)$, $\zeta_{N_\theta 1}\in V^\theta_{N_\theta}(\Omega)$ correspond to the initial data $\vec{w}_{\mathbf{N}0}$, $\vec{w}_{\mathbf{N}1}$, $\vec{\zeta}_{N_\theta 0}$, $\vec{\zeta}_{N_\theta 1}$ of the two-dimensional problem.

For the two-dimensional initial-boundary value problem (10)-(12) for thermoelastic plates constructed within the framework of Lord-Shulman theory the following existence and uniqueness theorem is proved.

Theorem 2. *If two-dimensional domain* ω *and functions* h^+, h^- *are such that* $\Omega \subset \mathbf{R}^3$ *is a Lipschitz domain,* $\rho > 0$, $\Theta_0 > 0$, $\mu > 0$, $3\lambda + 2\mu > 0$, $\chi > 0$, $\kappa > 0$, $\tau_0 > 0$, *the functions* $\overset{r_i}{f}_i$, $\overset{r_i}{g}_{\mathbf{N}i}$, $g^{\pm}_{\mathbf{N}i}$ $(r_i = 0,...,N_i, i = 1,2,3)$, $\overset{r}{f}{}^{\theta}$, $\overset{r}{g}{}^{\theta}_{N_\theta}$ $(r = 0,...,N_\theta)$, $g^{\theta\pm}_{N_\theta}$ *satisfy the following conditions*

$$\vec{f}_{\mathbf{N}} = (\overset{r_i}{f}_i) \in C^0([0,T]; \vec{H}^1_{\mathbf{N}}(\omega)),\ h^{-1/2}(\overset{r_i}{f}_i)', h^{-1/2}(\overset{r_i}{f}_i)'' \in L^2(0,T; L^2(\omega)),$$

$$\lambda_{\pm}^{3/4} g^{\pm}_{\mathbf{N}i}, \lambda_{\pm}^{3/4}(g^{\pm}_{\mathbf{N}i})', \lambda_{\pm}^{3/4}(g^{\pm}_{\mathbf{N}i})'', \lambda_{\pm}^{3/4}(g^{\pm}_{\mathbf{N}i})''' \in L^2(0,T; L^{4/3}(\omega)),$$

$$h^{-1/4}\overset{r_i}{g}_{\mathbf{N}i}, h^{-1/4}(\overset{r_i}{g}_{\mathbf{N}i})', h^{-1/4}(\overset{r_i}{g}_{\mathbf{N}i})'', h^{-1/4}(\overset{r_i}{g}_{\mathbf{N}i})''' \in L^2(0,T; L^{4/3}(\gamma_1)), \quad r_i = 0,...,N_i,\ i = 1,2,3,$$

$$h^{-1/2}\overset{r}{f}{}^{\theta}, h^{-1/2}(\overset{r}{f}{}^{\theta})' \in L^2(0,T; L^2(\omega)), \lambda_{\pm}^{3/4} g^{\theta\pm}_{N_\theta}, \lambda_{\pm}^{3/4}(g^{\theta\pm}_{N_\theta})', \lambda_{\pm}^{3/4}(g^{\theta\pm}_{N_\theta})'' \in L^2(0,T; L^{4/3}(\omega)),$$

$$h^{-1/4}\overset{r}{g}{}^{\theta}_{N_\theta}, h^{-1/4}(\overset{r}{g}{}^{\theta}_{N_\theta})', h^{-1/4}(\overset{r}{g}{}^{\theta}_{N_\theta})'' \in L^2(0,T; L^{4/3}(\gamma_1)),\ r = 0,...,N_\theta,$$

and $\vec{w}_{\mathbf{N}0} \in \vec{V}^3_{\mathbf{N}}(\omega)$, $\vec{w}_{\mathbf{N}1} \in \vec{V}^2_{\mathbf{N}}(\omega)$, $\vec{\zeta}_{N_\theta 0} \in \vec{V}^{\theta,2}_{N_\theta}(\omega)$, $\vec{\zeta}_{N_\theta 1} \in \vec{V}^{\theta}_{N_\theta}(\omega)$, *then the dynamical two-dimensional problem* (10)-(12) *possesses a unique solution.*

Thus, we have constructed a hierarchical algorithm of approximation of Lord-Shulman non-classical three-dimensional model for thermoelastic plates with variable thickness by two-dimensional problems. In the following theorem we present the results on the relationship between the obtained two-dimensional and original three-dimensional initial-boundary value problems, but in order to formulate the corresponding theorem let us define the following anisotropic weighted Sobolev spaces

$$H^{0,0,s}_{h^{\pm}}(\Omega) = \{v; h^k \partial_3^k v \in L^2(\Omega), 0 \le k \le s\},\ s \in \mathbf{N},$$

$$H^{1,1,s}_{h^{\pm}}(\Omega) = \{v; h^{k-1}\partial_3^{k-1}\partial_i^r v \in L^2(\Omega), h^{k-1}\partial_\alpha h^{\pm}\partial_3^k v \in L^2(\Omega), 1 \le k \le s, r = 0,1, i = 1,2,3, \alpha = 1,2\},$$

$$\tilde{H}^{1,1,s+1}_{h^{\pm}}(\Omega) = \{v; h^{k-1}\partial_3^{k-1}\partial_i^r\partial_j^{\tilde{r}} v \in L^2(\Omega), h^{k-1}\partial_\alpha h^{\pm}\partial_3^k\partial_i^r v \in L^2(\Omega), h^{k-1}\partial_\alpha\partial_\beta h^{\pm}\partial_3^k v \in L^2(\Omega),$$
$$1 \le k \le s, h^{\tilde{k}-2}\partial_\alpha h^{\pm}\partial_\beta h^{\pm}\partial_3^{\tilde{k}} v \in L^2(\Omega), 1 \le \tilde{k} \le s+1, \alpha,\beta = 1,2, r,\tilde{r} = 0,1, 1 \le i,j \le 3\},$$

$$\hat{H}^{3,3,4}_{h^{\pm}}(\Omega) = \{v \in H^3(\Omega); \partial_\alpha h^{\pm}\partial_3^p v \in L^2(\Omega), p = 1,...,4, \partial_\alpha\partial_\beta h^{\pm}\partial_3^k\partial_i^{r-1} v \in L^2(\Omega), \partial_\alpha h^{\pm}\partial_3\partial_\beta\partial_i v \in L^2(\Omega),$$
$$h^{r-1}\partial_\alpha h^{\pm}\partial_\beta h^{\pm}\partial_\delta h^{\pm}\partial_3^{r+2} v \in L^2(\Omega), h^{r-1}\partial_\alpha\partial_\beta\partial_\delta h^{\pm}\partial_3^r v \in L^2(\Omega), h\partial_\delta\partial_\beta^{r-1}h^{\pm}\partial_3\partial_3\partial_\alpha\partial_\beta^{r-2} v \in L^2(\Omega),$$
$$h^{r-1}\partial_\alpha\partial_\beta^{\hat{r}-1}h^{\pm}\partial_\delta h^{\pm}\partial_3\partial_3^r\partial_i^{2-\hat{r}} v \in L^2(\Omega), h\partial_\alpha\partial_\beta\partial_\delta\partial_3 v \in L^2(\Omega), r,\hat{r},\alpha,\beta,\delta = 1,2, i = 1,2,3\}$$

which are Hilbert spaces with respect to the corresponding norms.

Theorem 3. *If* $\rho > 0$, $\Theta_0 > 0$, $\mu > 0$, $3\lambda + 2\mu > 0$, $\chi > 0$, $\kappa > 0$, $\tau_0 > 0$, $\mathbf{u}_0 \in \mathbf{H}^3(\Omega) \cap \mathbf{V}(\Omega)$, $\mathbf{u}_1 \in \mathbf{H}^2 \cap \mathbf{V}(\Omega)$, $\theta_0 \in H^2(\Omega) \cap V^{\theta}(\Omega)$, $\theta_1 \in V^{\theta}(\Omega)$, $\mathbf{f} \in C^0([0,T];\ \mathbf{H}^1(\Omega))$, $\mathbf{f}', \mathbf{f}'' \in L^2(0,T; \mathbf{L}^2(\Omega))$, $\mathbf{g}, \mathbf{g}', \mathbf{g}'', \mathbf{g}''' \in L^2(0,T; \mathbf{L}^{4/3}(\Gamma_1))$, $f^{\theta}, f^{\theta'} \in L^2(0,T; L^2(\Omega))$, $g^{\theta}, g^{\theta'}, g^{\theta''} \in L^2(0,T; L^{4/3}(\Gamma_1))$ *and the compatibility conditions* (9) *are valid, vector-functions of three space variables* $\mathbf{w}_{\mathbf{N}0} \in \mathbf{V}^3_{\mathbf{N}}(\Omega)$, $\mathbf{w}_{\mathbf{N}1} \in \mathbf{V}^2_{\mathbf{N}}(\Omega)$, $\zeta_{N_\theta 0} \in V^{\theta,2}_{N_\theta}(\Omega)$, $\zeta_{N_\theta 1} \in V^{\theta}_{N_\theta}(\Omega)$, *corresponding to the initial conditions* $\vec{w}_{\mathbf{N}0} \in \vec{V}^3_{\mathbf{N}}(\omega)$, $\vec{w}_{\mathbf{N}1} \in \vec{V}^2_{\mathbf{N}}(\omega)$, $\vec{\zeta}_{N_\theta 0} \in \vec{V}^{\theta,2}_{N_\theta}(\omega)$, $\vec{\zeta}_{N_\theta 1} \in \vec{V}^{\theta}_{N_\theta}(\omega)$ *of the two-dimensional problems* (10)-(12)*, tend to* $\mathbf{u}_0$, $\mathbf{u}_1$, θ_0 *and* θ_1 *in* *the spaces* $\mathbf{H}^3(\Omega)$, $\mathbf{H}^2(\Omega)$, $H^2(\Omega)$ *and* $H^1(\Omega)$, *respectively, as* $N_{\min} = \min_{1\le i\le 3}\{N_i, N_\theta\} \to \infty$, *then the sequences of vector-functions* $(\mathbf{w_N}(t))$ *and functions* $(\zeta_{N_\theta}(t))$ *restored from the solutions* $\vec{w}_\mathbf{N}$ *and* $\vec{\zeta}_{N_\theta}$ *of the reduced two-dimensional problem* (10)-(12), *tend to the solutions* $\mathbf{u}(t)$ *and* $\theta(t)$ *of the original three-dimensional problem* (6)-(8),

$$\begin{aligned}
&\mathbf{w_N}(t) \to \mathbf{u}(t) && \text{in } \mathbf{H}^1(\Omega), \\
&\mathbf{w}'_\mathbf{N}(t) \to \mathbf{u}'(t) && \text{in } \mathbf{H}^1(\Omega), \\
&\mathbf{w}''_\mathbf{N}(t) \to \mathbf{u}''(t) && \text{in } \mathbf{L}^2(\Omega), \qquad \textit{for all } t\in[0,T], \ \textit{as } N_{\min}\to\infty. \\
&\zeta_{N_\theta}(t) \to \theta(t) && \text{in } H^1(\Omega), \\
&\zeta'_{N_\theta}(t) \to \theta'(t) && \text{in } L^2(\Omega),
\end{aligned}$$

In addition, if $d^r\mathbf{u}/dt^r \in L^2(0,T;\ (H_{h^\pm}^{1,1,s_r}(\Omega))^3)$, $r=0,1,2$, $\mathbf{u}''' \in L^2(0,T;\ (H_{h^\pm}^{0,0,s_3}(\Omega))^3)$, $s_0,\ s_1, s_2, s_3 \in \mathbf{N}$, $d^r\theta/dt^r \in L^2(0,T;\ H_{h^\pm}^{1,1,s_r^\theta}(\Omega))$, $r=0,1$, $\theta'' \in L^2(0,T;\ H_{h^\pm}^{0,0,s_2^\theta}(\Omega))$, $s_0^\theta, s_1^\theta, s_2^\theta \in \mathbf{N}$, $s_0, s_0^\theta, s_1, s_1^\theta, s_2 \ge 2$, and $\mathbf{u}_0 \in (\tilde{H}_{h^\pm}^{1,1,\tilde{s}_0+1}(\Omega))^3 \cap (\hat{H}_{h^\pm}^{3,3,4}(\Omega))^3$, $\mathbf{u}_1 \in (\tilde{H}_{h^\pm}^{1,1,\tilde{s}_1+1}(\Omega))^3$, $\theta_0 \in \tilde{H}_{h^\pm}^{1,1,\tilde{s}_0^\theta+1}(\Omega)$, $\theta_1 \in H_{h^\pm}^{1,1,\tilde{s}_1^\theta}(\Omega)$, $\tilde{s}_0, \tilde{s}_1, \tilde{s}_0^\theta, \tilde{s}_1^\theta \in \mathbf{N}$, $\tilde{s}_0, \tilde{s}_1, \tilde{s}_0^\theta, \tilde{s}_1^\theta \ge 2$, then for appropriate initial conditions $\vec{w}_{\mathbf{N}0}$, $\vec{w}_{\mathbf{N}1}$, $\vec{\zeta}_{N_\theta 0}$, $\vec{\zeta}_{N_\theta 1}$ the following estimate is valid

$$\begin{aligned}
&\|\mathbf{u}-\mathbf{w_N}\|_{C^0([0,T];\mathbf{H}^1(\Omega))} + \|\mathbf{u}'-\mathbf{w}'_\mathbf{N}\|_{C^0([0,T];\mathbf{H}^1(\Omega))} + \|\mathbf{u}''-\mathbf{w}''_\mathbf{N}\|_{C^0([0,T];\mathbf{L}^2(\Omega))} + \\
&+\|\theta'-\zeta'_{N_\theta}\|_{C^0([0,T];L^2(\Omega))} + \|\theta-\zeta_{N_\theta}\|_{C^0([0,T];H^1(\Omega))} \le \frac{1}{(N_{\min})^s} o(T,\Omega,\tilde{\Gamma}_0,h^\pm,\mathbf{N},N_\theta),
\end{aligned}$$

where $s=\min\{s_0-1, s_1-1, s_2-1, s_3, s_0^\theta-1, s_1^\theta-1, s_2^\theta, \tilde{s}_0-3/2, \tilde{s}_1-3/2, \tilde{s}_0^\theta-3/2, \tilde{s}_1^\theta-1\}$ *and* $o(T,\Omega,\tilde{\Gamma}_0,h^\pm,\ \mathbf{N},N_\theta) \to 0$, *as* $N_{\min}\to\infty$.

მათემატიკა

ცვალებადი სისქის თერმოდრეკადი ფირფიტებისათვის ლორდ-შულმანის მოდელის ორგანზომილებიანი ამოცანებით მიახლოების შესახებ

გ. ავალიშვილი*, მ. ავალიშვილი**

** ი. ჯავახიშვილის სახ. თბილისის სახელმწიფო უნივერსიტეტი, ზუსტ და საბუნებისმეტყველო მეცნიერებათა ფაკულტეტი, თბილისი*
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(წარმოდგენილია აკადემიის წევრის ე. ნადარაიას მიერ)

წარმოდგენილ ნაშრომში განხილულია თერმოდრეკადი სხეულები ლორდ-შულმანის თერმოდრეკადობის არაკლასიკური თეორიის ფარგლებში. ვარიაციული მიდგომის გამოყენებით სამგანზომილებიანი მოდელის შესაბამისი საწყის-სასაზღვრო ამოცანა გამოკვლეულია სათანადო ვექტორული მნიშვნელობების მქონე განაწილებების სივრცეებში მნიშვნელობებით სობოლევის სივრცეებში. აგებულია ცვალებადი სისქის ფირფიტის დინამიკური სამგანზომილებიანი მოდელის ორგანზომილებიანი ამოცანებით აპროქსიმაციის ალგორითმი, როცა ფირფიტის ზედა და ქვედა პირით ზედაპირებზე მოცემულია ზედაპირული ძალის და სითბოს ნაკადის ნორმალური მდგენელის სიმკვრივეები. მიღებული ორგანზომილებიანი საწყის-სასაზღვრო ამოცანები გამოკვლეულია სათანადო ფუნქციონალურ სივრცეებში. ამავე დროს, დამტკიცებულია აგებული ორგანზომილებიანი ამოცანების ამონახსნებიდან აღდგენილი სამი სივრცითი ცვლადის ვექტორ-ფუნქციების მიმდევრობის კრებადობა საწყისი სამგანზომილებიანი საწყის-სასაზღვრო ამოცანის ამონახსნისაკენ და დამატებით პირობებში მიღებულია კრებადობის რიგის შეფასება.

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