Economics

On the Marketing Research of Consumer Prices

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ABSTRACT. Marketing research holds an important place in the analysis of modern market economy. Marketing information is used by financial and insurance institutions, business enterprises and companies for planning, control, monitoring and forecasting in business. One of the problems is the detection and investigation of factors, which influence the behavior of consumers. Such a basic factor is the consumer prices index. The significance of this factor periodically changes and depends on the values of main indexes of economy such as export, import, taxes, labor force, unemployment, inflation level, etc., and also on the behavior of consumers, their taste, living standard and style. For the marketing research of this dependence it is necessary to construct mathematical models of the evolution of consumer prices. In the paper, a new autoregression model with disturbances is constructed for consumer prices. The model includes monetary aggregate amount and control function. A new formula is derived for the solution of an equation for the consumer prices index, which can be used in forecasting the inflation process. Using the data on the consumer prices index in Georgia, a numerical example is given, which illustrates the estimates of the coefficients of the constructed model and the inflation process forecast.

Key words: *marketing information, consumer prices, consumer market, inflation, autoregression model*

The inflation process control is highly important for the stability of the consumer market. It is believed that inflation is the main destabilizing factor of market economy. The term “inflation” was used for the first time in Northern America during the civil war, and later came into use in Great Britain and France. In the economical literature, the notion of inflation became popular after World War I. The 20th century is called the epoch of inflation. The inflation process is typical of all countries. Hence it is important to have mathematical models adequately describing the character of inflation changes. The new auto-regression model is constructed in the paper and the explicit formula is derived for the consumer prices index. The formula depends on the time parameter and can be used for forecasting the inflation process.

A great number of monographs, text-books and scientific papers are dedicated to marketing research [1-3]. In these works essential use is made of statistical methods of investigation and, in particular, the method of regression analysis. In this context, a special reference should be made to the monograph [4] and the papers [5-7].

Let us consider the following autoregression model

$$p(k) = a_0 + a_1 p(k-1) + a_2 p(k-2) + \gamma m(k-1) + \beta\varepsilon(k), \tag{1}$$

where *p (k)* is the consumer prices index at the moment of time *k*; *m (k)* is monetary aggregate amount at the moment of time *k*, $\varepsilon(k)$ is a random component (random variable) with zero mean. It can be produced by various disturbing factors, for instance, by the irregularity of import, export or investment flows, the economy instability, consumers' behavior and other factors. The values a_0, a_1, a_2, γ and β are the numerical coefficients which are calculated by statistical data on *p (k)*.

We define the monetary aggregate as follows:

$$m(k) = \bar{m} + u(k), \tag{2}$$

where $\bar{m}$ is the average value of the monetary aggregate, *u (k)* is monetary aggregate increment. The function *u (k)* is used as control. From (1), (2) we obtain the following equation for *p (k)*:

$$p(k+2) - a_1 p(k+1) - a_2 p(k) = a_0' + \gamma u(k+1) + \beta\varepsilon(k+2), \tag{3}$$

where $a_0' = a_0 + \bar{m}$.

Let us consider the homogeneous equation

$$p(k+2) - a_1 p(k+1) - a_2 p(k) = 0, \tag{4}$$

whose solution has the form

$$p_0(k) = c_1 r_1^k + c_2 r_2^k,$$

where c_1 and c_2 are constants and the values

$$r_1 = \frac{a_1 + \sqrt{a_1^2 + 4a_2}}{2}, \quad r_2 = \frac{a_1 - \sqrt{a_1^2 + 4a_2}}{2}.$$

A particular solution of equation (3) will be sought in the form

$$p_1(k) = \mu_1(k) r_1^k + \mu_2(k) r_2^k. \tag{5}$$

Let

$$r_1^{k+1}\Delta\mu_1(k) + r_2^{k+1}\Delta\mu_2(k) = 0, \tag{6}$$

where

$$\Delta\mu_i(k) = \mu_i(k+1) - \mu_i(k), \quad i = 1,2.$$

Substituting (6) into equation (3) we obtain:

$$\begin{gathered}\left(r_1^{k+2}\Delta\mu_1(k) + r_2^{k+2}\Delta\mu_2(k)\right) + \left(r_1^{k+2}\Delta\mu_1(k+1) + r_2^{k+2}\Delta\mu_2(k+1)\right) - \\ -a_1\left(r_1^{k+1}\Delta\mu_1(k) + r_2^{k+1}\Delta\mu_2(k)\right) + \\ \mu_1(k)\left(r_1^{k+2} - a_1 r_1^{k+1} - a_2 r_1^k\right) + \mu_2(k)\left(r_2^{k+2} - a_1 r_2^{k+1} - a_2 r_2^k\right) = \\ = a_0' + \gamma u(k+1) + \beta\varepsilon(k+2).\end{gathered}$$

$$r_1^{k+2}\Delta\mu_1(k) + r_2^{k+2}\Delta\mu_2(k) = a_0' + \gamma u(k+1) + \beta\varepsilon(k+2). \tag{7}$$

Equations (6) and (7) imply:

$$\Delta\mu_1(k) = -\frac{a_0' + \gamma u(k+1) + \beta\varepsilon(k+2)}{(r_2 - r_1) r_1^{k+1}},$$

$$\Delta\mu_2(k)=\frac{a_0'+\gamma u(k+1)+\beta\varepsilon(k+2)}{(r_2-r_1)r_2^{k+1}},$$

$$\mu_1(k)=-\frac{1}{r_2-r_1}\sum_{n=1}^{k}\frac{a_0'+\gamma u(n)+\beta\varepsilon(n+1)}{r_1^k},$$

$$\mu_2(k)=\frac{1}{r_2-r_1}\sum_{n=1}^{k}\frac{a_0'+\gamma u(n)+\beta\varepsilon(n+1)}{r_2^k}.$$

Thus, a particular solution of equation (3) has the form:

$$p_1(k)=\frac{a_0'}{1-a_1-a_2}+\frac{\gamma}{r_2-r_1}\sum_{n=1}^{k-1}\left(r_2^n-r_1^n\right)u(k-n)+$$
$$+\frac{\beta}{r_2-r_1}\sum_{n=1}^{k-1}\left(r_2^n-r_1^n\right)\varepsilon(k-n+1),$$

and a general solution of (3) is written as follows:

$$p(k)=c_1r_1^k+c_2r_2^k+\frac{a_0'}{1-a_1-a_2}+\frac{\gamma}{r_2-r_1}\sum_{n=1}^{k-1}\left(r_2^n-r_1^n\right)u(k-n)+$$
$$+\frac{\beta}{r_2-r_1}\sum_{n=1}^{k-1}\left(r_2^n-r_1^n\right)\varepsilon(k-n+1).$$

Using the initial conditions *p* (0) and *p* (1), the constants c_1 and c_2 are defined by the relations:

$$c_1=\frac{r_2}{r_2-r_1}\left[p(0)-\frac{a_0'}{1-a_1-a_2}\right]-\frac{1}{r_2-r_1}\left[p(1)-\frac{a_0'}{1-a_1-a_2}\right],$$
$$c_2=\frac{1}{r_2-r_1}\left[p(1)-\frac{a_0'}{1-a_1-a_2}\right]-\frac{r_1}{r_2-r_1}\left[p(0)-\frac{a_0'}{1-a_1-a_2}\right].$$

Thus, a general solution of equation (3) has the form:

$$p(k)=\frac{a_0'}{1-a_1-a_2}+\frac{\gamma}{r_2-r_1}\sum_{n=1}^{k-1}\left(r_2^n-r_1^n\right)u(k-n)+\frac{\beta}{r_2-r_1}\sum_{n=1}^{k-1}\left(r_2^n-r_1^n\right)\varepsilon(k-n+1)+$$
$$+\frac{r_1r_2\left(r_2^{k-1}-r_1^{k-1}\right)}{r_2-r_1}\left[p(0)-\frac{a_0'}{1-a_1-a_2}\right]+\frac{r_2^k-r_1^k}{r_2-r_1}\left[p(1)-\frac{a_0'}{1-a_1-a_2}\right]. \qquad (8)$$

The obtained formula (8) is used for forecasting inflation. Let us write this formula for the time unit *s*. We have:

$$p(k+s)=$$
$$=\frac{a_0'}{1-a_1-a_2}+\frac{\gamma}{r_2-r_1}\sum_{n=1}^{k-1}\left(r_2^n-r_1^n\right)u(k+s-n)+\frac{\beta}{r_2-r_1}\sum_{n=1}^{k-1}\left(r_2^n-r_1^n\right)\varepsilon(k+s-n+1)+$$
$$+\frac{r_1r_2\left(r_2^{s-1}-r_1^{s-1}\right)}{r_2-r_1}\left[p(k-1)-\frac{a_0'}{1-a_1-a_2}\right]+\frac{r_2^s-r_1^s}{r_2-r_1}\left[p(k)-\frac{a_0'}{1-a_1-a_2}\right], \qquad (9)$$

where *p* (k - 1) and *p* (k) are the initial conditions at the moment of time *k*. Therefore, a forecast function for step *s* has the form:

$$\hat{p}(k+s) = E_k\left[p(k+s)\right] = \frac{a_0'}{1-a_1-a_2} + \frac{\gamma}{r_2-r_1}\sum_{n=1}^{s}\left(r_2^n - r_1^n\right)u(k+s-n) +$$

$$+\frac{r_1 r_2\left(r_2^{s-1}-r_1^{s-1}\right)}{r_2-r_1}\left[p(k-1)-\frac{a_0'}{1-a_1-a_2}\right]+\frac{r_2^s-r_1^s}{r_2-r_1}\left[p(k)-\frac{a_0'}{1-a_1-a_2}\right]. \quad (10)$$

In particular, if $s = 1$, then from (10) we obtain:

$$\hat{p}(k+1) = \hat{p}(k) + \gamma\, u(k),$$

if $s = 2$, then

$$\hat{p}(k+2) = \frac{a_0'}{1-a_1-a_2} + \frac{\gamma}{r_2-r_1}\left\{(r_2-r_1)u(k+1) + \left(r_2^2-r_1^2\right)u(k)\right\} +$$

$$+\frac{r_1 r_2^2 - r_2 r_1^2}{r_2-r_1}\left[p(k-1)-\frac{a_0'}{1-a_1-a_2}\right]+(r_2+r_1)\left[p(k)-\frac{a_0'}{1-a_1-a_2}\right],$$

and if $n = 3$, then

$$\hat{p}(k+3) = \frac{a_0'}{1-a_1-a_2} + \frac{\gamma}{r_2-r_1}\left\{(r_2-r_1)u(k+2) + \left(r_2^2-r_1^2\right)u(k+1) + \left(r_2^3-r_1^3\right)u(k)\right\} +$$

$$+r_1 r_2(r_1+r_2)\left[p(k-1)-\frac{a_0'}{1-a_1-a_2}\right]+\left(r_1^2+r_1r_2+r_2^2\right)\left[p(k)-\frac{a_0'}{1-a_1-a_2}\right].$$

Example. Let us consider data on the consumer prices index in Georgia in 2002-2013 (Table).

Table. Data on the consumer prices index in Georgia [5].

Year	2002	2003	2004	2005	2006	2007
Index	104.4	105.0	5106.3	108.4	106.9	110.7
Year	2008	2009	2010	2011	2012	2013
Index	108.3	111.3	101.6	111.4	101.3	99.1

The problem consists in the estimate of the coefficients a_0, a_1, a_2 of model (1). We have the following system of equations:

$$\begin{cases}
a_0 + 105.6a_1 + 104.6a_2 = 104.4 \\
a_0 + 104.4a_1 + 105.6a_2 = 105.5 \\
a_0 + 105.5a_1 + 104.4a_2 = 106.3 \\
a_0 + 106.3a_1 + 105.5a_2 = 108.4 \\
a_0 + 108.4a_1 + 106.3a_2 = 106.9 \\
a_0 + 106.9a_1 + 108.4a_2 = 110.7 \\
a_0 + 110.7a_1 + 106.9a_2 = 108.8 \\
a_0 + 108.8a_1 + 110.7a_2 = 111.3 \\
a_0 + 111.3a_1 + 108.8a_2 = 101.6 \\
a_0 + 101.6a_1 + 111.3a_2 = 111.4 \\
a_0 + 111.4a_1 + 101.6a_2 = 101.3 \\
a_0 + 101.3a_1 + 111.4a_2 = 99.10
\end{cases}$$

We have to find the maximum point of the function:

$$g(a_0,a_1,a_2)=(a_0+105.6a_1+104.6a_2-104.4)^2+$$
$$+(a_0+104.4a_1+105.6a_2-105.5)^2+(a_0+105.5a_1+104.4a_2-106.3)^2+$$
$$+(a_0+106.3a_1+105.5a_2-108.4)^2+(a_0+108.4a_1+106.3a_2-106.9)^2+$$
$$+(a_0+106.9a_1+108.4a_2-110.7)^2+(a_0+110.7a_1+106.9a_2-108.8)^2+$$
$$+(a_0+108.8a_1+110.7a_2-111.3)^2+(a_0+111.3a_1+108.8a_2-101.6)^2+$$
$$+(a_0+101.6a_1+111.3a_2-111.4)^2+(a_0+111.4a_1+101.6a_2-101.3)^2+$$
$$+(a_0+101.3a_1+111.4a_2-99.10)^2.$$

Using the least square method, we obtain the following system of algebraic equations:

$$\begin{cases}12a_0+1282.2a_1+1285.5a_2=1275.7\\1282.2a_0+137132.66a_1+137308.7a_2=136322.87\\1285.5a_0+125585.57a_1+137812.12a_2=136694.66\end{cases}.$$

$$\begin{cases}a_0=141.3122843\\a_1=-0.0058061\\a_2=-0.3209668\end{cases}.$$

Therefore, an autoregression model of the consumer prices index in Georgia has the form:

$$p(k)=141.3122843-0.0058061p(k-1)-0.3209668p(k-2)+u(k)+\varepsilon(k).$$

Neglecting the control $u(k)$ and a random factor $\varepsilon(k)$ $(u(k)=\varepsilon(k)=0)$, we obtain the expected value of the consumer prices index of Georgia in 2014 as compared with that of the preceding year:

$$p(2014)=141.3122843-0.0058061\cdot 99.1-0.3209668\cdot 101.3\approx 108\%.$$

Thus the inflation forecast for 2014 in Georgia is about 8% as compared with the last year.

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სამომხმარებლო ფასების მარკეტინგული კვლევის შესახებ

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მარკეტინგულ კვლევას მნიშვნელოვანი ადგილი უკავია თანამედროვე საბაზრო ეკონომიკის ანალიზში. მარკეტინგულ ინფორმაციას იყენებენ ფინანსური და სადაზღვევო ინსტიტუტები, ფირმები და კომპანიები ბიზნესის დაგეგმვაში, მართვაში, კონტროლსა და პროგნოზირებაში. ერთ-ერთი ამოცანაა იმ ფაქტორების გამოვლენა და შესწავლა, რომლებიც გავლენას ახდენს მომხმარებლის ქცევაზე. ერთ-ერთ ასეთ ძირითად ფაქტორს წარმოადგენს, მაგალითად, სამომხმარებლო ფასების ინდექსი. ამ ინდექსის მნიშვნელობა პერიოდულად იცვლება და დამოკიდებულია ეკონომიკის ძირითადი მაჩვენებლების მნიშვნელობებზე, როგორიცაა ექსპორტი, იმპორტი, ინვესტიცია, გადასახადები, სამუშაო ძალა, უმუშევრობა, ინფლაციის დონე და ა. შ., აგრეთვე მომხმარებლის ქცევაზე, მათ გემოვნებაზე, ცხოვრების დონეზე და სტილზე. ამ კავშირის მარკეტინგული კვლევისთვის აუცილებელია სამომხმარებლო ფასების ევოლუციის მათემატიკური მოდელების აგება. სტატიაში მოცემულია ახალი ავტორეგრესიული მოდელი შეშფოთებებით სამომხმარებლო ფასებისთვის. ამ მოდელში ჩართულია ფულის მასის მოცულობა და მართვის ფუნქცია. სამომხმარებლო ფასების ინდექსისთვის მიღებულია განტოლების ამონახსნის ცხადი ფორმულა, რომელიც შეიძლება გამოვიყენოთ ინფლაციური პროცესის პროგნოზირებაში. საქართველოში სამომხმარებლო ფასების ინდექსის შესახებ მონაცემების გამოყენებით ნაშრომში მოტანილია აგებული მოდელის კოეფიციენტების და ინფლაციური პროცესის შეფასების რიცხვითი მაგალითი.

REFERENCES

1. *P. Kotler, G. Armstrong* (2009), Principles of Marketing. *Pearson Education, New Jersey*.
2. *N. K. Malhotra* (2008), Marketing Research: An Applied Orientation. *New Jersey*.
3. *P. Doyle, P. Stern* (2006), Marketing Management and Strategy. *Prentice Hall, New York*.
4. *N. Todua* (2012), Marketing Research of Consumer Behavior in Georgian Market. *Universali, Tbilisi* (in Georgian).
5. *GEOSTAT* (2014), Consumer Process Index, National Statistics Office of Georgia.
6. *B. Hansen* (2013), Econometrics. *University of Wilconsin*.
7. *N. Todua, P. Babilua, T. Dochviri* (2013), Bull. Georg. Natl. Acad. Sci., **7**, 3: 18-26.

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