საქართველოს მეცნიერებათა ეროვნული აკადემიის მოამბე, ტ. 8, №3, 2014
BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, vol. 8, no. 3, 2014

Mechanics

On the Investigation of Dynamical Hierarchical Models of Elastic Multi-Structures Consisting of Three-Dimensional Body and Multilayer Sub-Structure

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(Presented by Academy Member Guram Gabrichidze)

ABSTRACT. In this paper initial-boundary value problem for multi-structure consisting of three-dimensional body with general shape and multilayer part composed of plates with variable thickness is considered. A hierarchy of dynamical models defined on the union of three-dimensional and two-dimensional domains for dynamical three-dimensional model for the multi-structure is constructed. The pluridimensional initial-boundary value problems corresponding to the constructed hierarchical models are investigated in suitable function spaces. The convergence of the sequence of vector functions of three space variables, restored from the solutions of the constructed initial-boundary value problems defined on the union of three-dimensional and two-dimensional domains to the solution of the original three-dimensional problem is proved and under additional regularity conditions the rate of convergence is estimated. *© 2014 Bull. Georg. Natl. Acad. Sci.*

Key words: *dynamical models of elastic multi-structures, initial-boundary value problems, hierarchical modeling, Fourier-Legendre series.*

Elastic multi-structures are the bodies, which consist of several parts with different geometrical shapes. Many engineering constructions are multi-structures consisting of plates, shells, beams and other substructures and therefore mathematical modeling of them is important from practical as well as theoretical point of view. One of the first theoretical investigations of multi-structures was carried out by P.G. Ciarlet, H. Le Dret, R. Nzengwa [1]. Applying asymptotic method they constructed and investigated a mathematical model defined on the product of three-dimensional and two-dimensional domains for a multi-structure consisting of three-dimensional body with a plate clamped in it. Multi-structures consisting of plates and rods were considered by H. Le Dret [2]. Further, many works were devoted to mathematical modeling and numerical solution of problems for elastic multi-structures (see [3] and references given therein). Different approach for constructing two-dimensional models of elastic plates with variable thickness was suggested by I. Vekua [4], which was based on approximation of the components of the displacement vector-function of plate by partial sums of orthogonal Fourier-Legendre series with respect to the variable of plate thickness. Note that I. Vekua's hierarchical dimensional reduction method is one of spectral approximation methods. Moreover, classical Kirchhoff-Love and Mindlin-Reissner models can be incorporated into the hierarchy obtained by I. Vekua, and so it can be considered as an extension of the widely used engineering plate models. Later on, various investigations were devoted to the study of mathematical models constructed by I. Vekua's dimensional reduction method and its generalizations for elastic plates, shells and rods (see [5-7] and references given therein).

The present paper is devoted to the construction and investigation of hierarchical models of multi-structures consisting of three-dimensional body with general shape and multilayer part composed of plates with variable thickness, which may vanish on a part of the lateral surface, applying spectral dimensional reduction method. We consider variational formulation of three-dimensional initial-boundary value problem for dynamical linear model of elastic multi-structure and construct a hierarchy of models in Sobolev spaces defined on the union of three-dimensional and two-dimensional domains, when density of surface force is given on the upper and the lower faces of multilayer substructure, on a part of its lateral boundary and on a part of the boundary of elastic three-dimensional part with general shape, and the remaining part of the boundary of the multi-structure is clamped. We investigate the existence and uniqueness of solutions of the reduced pluridimensional problems in suitable weighted Sobolev spaces. Moreover, we prove convergence of the sequence of vector-functions of three space variables restored from the solutions of the constructed problems to the solution of the original three-dimensional initial-boundary value problem and if it possesses additional regularity we estimate the rate of convergence.

For any bounded domain $\Omega \subset \mathbf{R}^p, p \geq 1$, with Lipschitz boundary we denote by $L^2(\Omega)$ the space of square-integrable functions in Ω in the Lebesgue sense. $H^k(\Omega)$, $k \geq 1$, is the Sobolev space of order k based on $L^2(\Omega)$, $\mathbf{H}^k(\Omega) = (H^k(\Omega))^3$, $\mathbf{L}^2(\Omega) = (L^2(\Omega))^3$ and $\mathbf{L}^k(\hat{\Gamma}) = (L^k(\hat{\Gamma}))^3$, where $\hat{\Gamma}$ is a Lipschitz surface. For any Banach space X, $C^0([0,T];X)$ denotes the space of continuous functions on $[0,T]$ with values in X, $L^2(0,T;X)$ is the space of such functions $g:(0,T) \to X$ that $\|g(t)\|_X \in L^2(0,T)$. We denote by $g' = dg/dt$ the generalized derivative of $g \in L^2(0,T;X)$.

Let us consider an elastic multi-structure, which consists of three-dimensional part with general shape and multilayer substructure attached to it consisting of plates with variable thickness, i.e. elastic body with initial configuration $\bar{\Omega} = \overline{\Omega^{bd}} \cup \bigcup_{k=1}^{m} \overline{\Omega_k^{pl}}$, where $\Omega \subset \mathbf{R}^3$ and $\Omega^{bd} \subset \mathbf{R}^3$ are bounded Lipschitz domains,

$$\Omega_k^{pl} = \{(x_1, x_2, x_3) \in \mathbf{R}^3;\, h_k^-(x_1, x_2) < x_3 < h_k^+(x_1, x_2), \quad (x_1, x_2) \in \omega_k\},$$

$\omega_k \subset \mathbf{R}^2$, $k = 1,2,...,m$, are two-dimensional bounded Lipschitz domains with boundary $\partial\omega_k$, $\Omega^{bd} \cap \Omega_k^{pl} = \varnothing$, $k = 1,2,...,m$, $h_k^{\pm} \in C^0(\bar{\omega}_k) \cap C^{0,1}(\omega_k \cup \tilde{\gamma}_k)$ are continuous on $\bar{\omega}_k$ and Lipschitz continuous in ω_k and on $\tilde{\gamma}_k \subset \partial\omega$, $h_k^+(x_1,x_2) > h_k^-(x_1,x_2)$, for $(x_1,x_2) \in \omega_k \cup \tilde{\gamma}_k$, $\tilde{\gamma}_k \subset \partial\omega_k$ is a Lipschitz curve, $h_k^+(x_1,x_2) = h_k^-(x_1,x_2)$, for $(x_1,x_2) \in \partial\omega_k \setminus \tilde{\gamma}_k$, $k = 1,2,...,m$. The interfaces $\overline{\Omega^{bd}} \cap \overline{\Omega_k^{pl}} = \overline{\Gamma_k^{bd,pl}}$ between the three-dimensional part $\Omega^{bd} \subset \mathbf{R}^3$ and plates Ω_k^{pl} are parts of lateral surfaces of plates

$\Gamma_k^{bd,pl} = \{x \in \mathbf{R}^3; h_k^-(x_1,x_2) < x_3 < h_k^+(x_1,x_2)\ \ (x_1,x_2) \in \gamma_k^{bd,pl}\}$, $\gamma_k^{bd,pl} \subset \tilde{\gamma}_k$, $k = 1,2,...,m$. The upper and the lower faces of plate Ω_k^{pl}, defined by equations $x_3 = h_k^+(x_1,x_2)$ and $x_3 = h_k^-(x_1,x_2)$, $(x_1,x_2) \in \omega_k$, we denote by Γ_k^+ and Γ_k^-, respectively, and the lateral surface, where the thickness of Ω_k^{pl} is positive, we denote by $\tilde{\Gamma}_k = \partial\Omega_k^{pl} \setminus \overline{(\Gamma_k^+ \cup \Gamma_k^-)} = \{x \in \partial\Omega_k^{pl}; h_k^-(x_1,x_2) < x_3 < h_k^+(x_1,x_2),\ \ (x_1,x_2) \in \tilde{\gamma}_k\}$, $k = 1,2,...,m$. The interfaces $\overline{\Gamma_{k,k+1}^{pl}}$, $k = 1,2,...,m-1$, between neighbour plates Ω_k^{pl} and Ω_{k+1}^{pl} are the common parts of the upper and the lower faces of Ω_k^{pl} and Ω_{k+1}^{pl}, respectively, $\Gamma_{k,k+1}^{pl} = \partial\Omega_k^{pl} \cap \partial\Omega_{k+1}^{pl} = \{(x_1,x_2,x_3) \in \mathbf{R}^3;$ $x_3 = h_k^+(x_1,x_2) = h_{k+1}^-(x_1,x_2)$, when $(x_1,x_2) \in \omega_k \cap \omega_{k+1}$, $k = 1,2,...,m-1\}$.

Let us assume that the three-dimensional part $\overline{\Omega^{bd}}$ consists of inhomogeneous anisotropic elastic material and the linear dynamical three-dimensional model of its stress-strain state is given by the following system

$$\rho^{bd} \frac{\partial^2 u_i^{bd}}{\partial t^2} = \sum_{j=1}^{3} \frac{\partial \sigma_{ij}^{bd}(\mathbf{u}^{bd})}{\partial x_j} + f_i^{bd} \quad \text{in} \quad \Omega^{bd} \times (0,T), \tag{1}$$

where $i = 1,2,3$, $\mathbf{u}^{bd} = (u_i^{bd})_{i=1}^3$ is the displacement vector-function of the three-dimensional part, $\rho^{bd} > 0$ denotes mass density of the three-dimensional part in reference configuration, $\mathbf{f}^{bd} = (f_i^{bd})_{i=1}^3$ is density of applied body force of the three-dimensional part, and $\boldsymbol{\sigma}^{bd} = (\sigma_{ij}^{bd})_{i,j=1}^3$ denotes linearized stress tensor of the three-dimensional part of the multi-structure, which is given by $\sigma_{ij}^{bd}(\mathbf{v}) = \sum_{p,q=1}^{3} a_{ijpq}^{bd} e_{pq}(\mathbf{v})$, $e_{pq}(\mathbf{v}) = (\partial v_p / \partial x_q + \partial v_q / \partial x_p)/2$, where a_{ijpq}^{bd} $(i,j,p,q = 1,2,3)$ are parameters characterizing mechanical properties of the three-dimensional part Ω^{bd}.

The remaining part $\bigcup_{k=1}^{m} \overline{\Omega_k^{pl}}$ of the multistructure consists of plates Ω_k^{pl}, $k = 1,2,...,m$, with variable thickness, which consist of anisotropic inhomogeneous elastic material, and their three-dimensional models are given by the following systems

$$\rho^{pl,k} \frac{\partial^2 u_i^{pl,k}}{\partial t^2} = \sum_{j=1}^{3} \frac{\partial \sigma_{ij}^{pl,k}(\mathbf{u}^{pl,k})}{\partial x_j} + f_i^{pl,k} \quad \text{in} \quad \Omega_k^{pl} \times (0,T), \tag{2}$$

where $i = 1,2,3$, $\mathbf{u}^{pl,k} = (u_i^{pl,k})_{i=1}^3$ denotes displacement of k-th plate Ω_k^{pl}, $\rho^{pl,k} > 0$ is the mass density of plate Ω_k^{pl} in reference configuration, $\mathbf{f}^{pl,k} = (f_i^{pl,k})_{i=1}^3$ is density of body force for plate Ω_k^{pl}, and $\boldsymbol{\sigma}^{pl,k} = (\sigma_{ij}^{pl,k})_{i,j=1}^3$ is linearized stress tensor of the plate Ω_k^{pl}, which is given by $\sigma_{ij}^{pl,k}(\mathbf{v}) = \sum_{p,q=1}^{3} a_{ijpq}^{pl,k} e_{pq}(\mathbf{v})$, $i,j = 1,2,3$, where $a_{ijpq}^{pl,k}$ are parameters characterizing mechanical properties of the plate Ω_k^{pl} of multi-structure.

Let us denote by $\mathbf{u}:\Omega\times(0,T)\to\mathbf{R}^3$ displacement vector-function of the entire multi-structure Ω, which equals to $\mathbf{u}^{bd}:\Omega^{bd}\times(0,T)\to\mathbf{R}^3$ in the three-dimensional part Ω^{bd}, $\mathbf{u}$ equals to $\mathbf{u}^{pl,k}:\Omega_k^{pl}\times(0,T)\to\mathbf{R}^3$ in the plate Ω_k^{pl} of the multilayer part, $k=1,2,...,m$. We assume that on the interfaces $\Gamma_k^{bd,pl}$ between the three-dimensional part Ω^{bd} and plates Ω_k^{pl}, $k=1,2,...,m$, rigid contact conditions, i.e. continuity of displacement vector-function and stress vector, are valid,

$$\mathbf{u}^{bd}=\mathbf{u}^{pl,k},\quad \sum_{j=1}^{3}\sigma_{ij}^{bd}(\mathbf{u}^{bd})n_j=\sum_{j=1}^{3}\sigma_{ij}^{pl,k}(\mathbf{u}^{pl,k})n_j \text{ on } \Gamma_k^{bd,pl}\times(0,T),\ i=1,2,3,\ k=1,2,...,m, \tag{3}$$

where $\mathbf{n}=(n_j)_{j=1}^3$ is a unit normal vector to the surface $\Gamma_k^{bd,pl}$. On the interfaces $\Gamma_{k,k+1}^{pl}$ between neighbour plates Ω_k^{pl}, $k=1,2,...,m-1$, conditions of continuity of displacement vector-function and stress vector are given

$$\mathbf{u}^{pl,k}=\mathbf{u}^{pl,k+1},\ \sum_{j=1}^{3}\sigma_{ij}^{pl,k}(\mathbf{u}^{pl,k})n_j=\sum_{j=1}^{3}\sigma_{ij}^{pl,k+1}(\mathbf{u}^{pl,k+1})n_j \text{ on } \Gamma_{k,k+1}^{pl}\times(0,T), \tag{4}$$

where $i=1,2,3,\ k=1,2,...,m-1$, $\mathbf{n}=(n_j)_{j=1}^3$ is a unit normal vector of the surface $\Gamma_{k,k+1}^{pl}$.

The multi-structure Ω, consisting of the three-dimensional part Ω^{bd} and elastic plates Ω_k^{pl}, $k=1,2,...,m$, is clamped along a part Γ_0 of its boundary, the density of the surface force is given on the remaining part of the boundary, and the initial values of the displacement and velocity are given

$$\begin{aligned}&\sum_{j=1}^{3}\sigma_{ij}(\mathbf{u})n_j=g_i \quad \text{on } \Gamma_1, \qquad \mathbf{u}=\mathbf{0} \quad \text{on } \Gamma_0,\\ &\mathbf{u}(x,0)=\boldsymbol{\varphi}(x), \qquad \frac{\partial\mathbf{u}}{\partial t}(x,0)=\boldsymbol{\psi}(x), \qquad x\in\Omega,\end{aligned} \tag{5}$$

where $\sigma_{ij}(\mathbf{u})=\sigma_{ij}^{bd}(\mathbf{u}^{bd})$ in Ω^{bd} and $\sigma_{ij}(\mathbf{u})=\sigma_{ij}^{pl,k}(\mathbf{u}^{pl,k})$ in Ω_k^{pl}, $k=1,2,...,m$ $i,j=1,2,3$, Γ_0 is an element of Lipschitz dissection of the boundary $\Gamma=\partial\Omega$ of the domain Ω, $\Gamma_1=\Gamma\setminus\overline{\Gamma_0}$, $\mathbf{n}=(n_j)_{j=1}^3$ is a unit outward normal vector of Γ_1, $\boldsymbol{\varphi}=(\varphi_i)_{i=1}^3$ and $\boldsymbol{\psi}=(\psi_i)_{i=1}^3$ are initial displacement and velocity vector-functions, $\mathbf{g}=(g_i)_{i=1}^3$ denotes density of surface force acting on the boundary Γ_1 of Ω, which equals to $\mathbf{g}^{bd}=(g_i^{bd})_{i=1}^3$ on the boundary of Ω^{bd} and equals to $\mathbf{g}^{pl,k}=(g_i^{pl,k})_{i=1}^3$ on the corresponding part of the boundary of Ω_k^{pl}. The clamped part of the three-dimensional part Ω^{bd} of the multi-structure we denote by Γ_0^{bd}, and the remaining part, where surface force is given, is denoted by $\Gamma_1^{bd}=\Gamma_1\cap\partial\Omega^{bd}$. The clamped parts of elastic plates are parts of the lateral boundaries $\Gamma_{k,0}^{pl}=\{(x_1,x_2,x_3)\in\partial\Omega_k^{pl};\ h_k^-(x_1,x_2)<x_3<h_k^+(x_1,x_2),\ (x_1,x_2)\in\gamma_{k,0}\subset\tilde{\gamma}_k\setminus\overline{\gamma_k^{bd,pl}}\}$, $k=1,2,...,m$, and the remaining part, where surface force is given, we denote by $\Gamma_{k,1}^{pl}=\Gamma_1\cap\partial\Omega_k^{pl}$.

The variational formulation of the dynamical three-dimensional problem (1)-(5) for multi-structure Ω consisting of three-dimensional body and multilayer part is of the following form: find the unknown vector-function $\mathbf{u} \in C^0([0,T];\mathbf{V}(\Omega))$, $\mathbf{u}' \in C^0([0,T];\ \mathbf{L}^2(\Omega)) \cap L^2(0,T;\ \mathbf{V}(\Omega))$, which satisfies the equation

$$\frac{d}{dt}\sum_{i=1}^{3}\int_{\Omega}\rho u_i'(.)v_i\,dx+\sum_{i,j,p,q=1}^{3}\int_{\Omega}a_{ijpq}e_{pq}(\mathbf{u}(.))e_{ij}(\mathbf{v})dx=\sum_{i=1}^{3}\int_{\Omega}f_i\;v_i dx+\sum_{i=1}^{3}\int_{\Gamma_1}g_i v_i d\Gamma,\quad \mathbf{v}\in\mathbf{V}(\Omega), \tag{6}$$

in the sense of distribution on $(0,T)$ and the initial conditions

$$\mathbf{u}(0)=\boldsymbol{\varphi},\ \mathbf{u}'(0)=\boldsymbol{\psi}, \tag{7}$$

where $\boldsymbol{\varphi} \in \mathbf{V}(\Omega) = \{\mathbf{v} \in \mathbf{H}^1(\Omega) = (H^1(\Omega))^3; \mathbf{tr}(\mathbf{v}) = \mathbf{0}\ \text{ on } \Gamma_0\}$, $\boldsymbol{\psi} \in \mathbf{L}^2(\Omega)$, $\mathbf{tr}$ denotes the trace operator from Sobolev space $\mathbf{H}^1(\Omega)$ to $\mathbf{H}^{1/2}(\Gamma)$, $f_i \equiv f_i^{bd}$ in Ω^{bd}, $f_i \equiv f_i^{pl,k}$ in Ω_k^{pl} and

$$a_{ijpq}(x)=\begin{cases}a_{ijpq}^{bd}(x), & x\in\Omega^{bd},\\ a_{ijpq}^{pl,k}(x), & x\in\Omega_k^{pl}, k=1,2,...,m,\end{cases}\quad i,j,p,q=1,2,3.$$

The bilinear forms in the left and right parts of the equation (6) we denote by

$$R(\tilde{\mathbf{w}},\tilde{\mathbf{v}})=\sum_{i=1}^{3}\int_{\Omega}\rho\tilde{w}_i\,\tilde{v}_i\,dx,\ A(\mathbf{w},\mathbf{v})=\sum_{i,j,p,q=1}^{3}\int_{\Omega}a_{ijpq}e_{pq}(\mathbf{w})e_{ij}(\mathbf{v})dx,\ L(\mathbf{v})=\sum_{i=1}^{3}\int_{\Omega}f_i\;v_i dx+\sum_{i=1}^{3}\int_{\Gamma_1}g_i v_i d\Gamma,$$

where $\tilde{\mathbf{w}},\tilde{\mathbf{v}} \in \mathbf{L}^2(\Omega)$, $\mathbf{w},\mathbf{v} \in \mathbf{V}(\Omega)$, $\rho \equiv \rho^{bd}$ in Ω^{bd}, $\rho \equiv \rho^{pl,k}$ in Ω_k^{pl}, $k = 1,...,m$.

Note, that each vector-function $\mathbf{v} \in \mathbf{V}(\Omega)$ from the space $\mathbf{V}(\Omega)$ can be represented as $m+1$ vector-functions $\mathbf{v}^{bd}$ and $\mathbf{v}^{pl,1}$, $\mathbf{v}^{pl,2}$, . . . , $\mathbf{v}^{pl,m}$, which are restrictions of the vector-function $\mathbf{v}$ on the sets Ω^{bd} and Ω_1^{pl}, Ω_2^{pl}, . . . , Ω_m^{pl}, respectively. Consequently, the space $\mathbf{V}(\Omega)$ can be considered as a space of the vector-functions $(\mathbf{v}_{\mathbf{N}}^{bd},\mathbf{v}_{\mathbf{N}}^{pl,1},\mathbf{v}_{\mathbf{N}}^{pl,2},...,\mathbf{v}_{\mathbf{N}}^{pl,m})$ $\mathbf{H}^1(\Omega_1^{pl})\times...\times\mathbf{H}^1(\Omega_m^{pl})$, such that $\mathbf{v}^{bd} = 0$ on Γ_0^{bd}, $\mathbf{v}^{pl,1} = 0$ on $\Gamma_{1,0}^{pl}$, $\mathbf{v}^{pl,2} = 0$ on $\Gamma_{2,0}^{pl}$, . . . , $\mathbf{v}^{pl,m} = 0$ on $\Gamma_{m,0}^{pl}$, $\mathbf{v}^{bd} = \mathbf{v}^{pl,k}$ on $\Gamma_k^{bd,pl}$, $k = 1,2,...,m$, $\mathbf{v}^{pl,k} = \mathbf{v}^{pl,k+1}$ on $\Gamma_{k,k+1}^{pl}$, $k = 1,2,...,m-1$.

In order to construct dynamical model of multi-structure Ω let us consider subspaces $\mathbf{V}_{\mathbf{N}}(\Omega)$ and $\mathbf{H}_{\mathbf{N}}(\Omega)$ of $\mathbf{V}(\Omega)$ and $\mathbf{L}^2(\Omega)$, respectively, $\mathbf{N} = (N_1^{pl,1}, N_2^{pl,1}, N_3^{pl,1},..., N_1^{pl,m}, N_2^{pl,m},\ N_3^{pl,m})$ consisting of vector-functions with $m+1$ components $\mathbf{v}_{\mathbf{N}} = (\mathbf{v}_{\mathbf{N}}^{bd},\mathbf{v}_{\mathbf{N}}^{pl,1},\mathbf{v}_{\mathbf{N}}^{pl,2},...,\mathbf{v}_{\mathbf{N}}^{pl,m})$, where $\mathbf{v}_{\mathbf{N}}^{pl,k}$ is a vector-function the *i*-th component of which is a polynomial of order $N_i^{pl,k}$, $i = 1,2,3$, $k = 1,2,...,m$, with respect to the variable x_3, i.e.

$$\mathbf{V}_{\mathbf{N}}(\Omega)=\{\mathbf{v}_{\mathbf{N}}=(v_{\mathbf{N}i})_{i=1}^3;\ \mathbf{v}_{\mathbf{N}}=(\mathbf{v}_{\mathbf{N}}^{bd},\mathbf{v}_{\mathbf{N}}^{pl,1},\mathbf{v}_{\mathbf{N}}^{pl,2},...,\mathbf{v}_{\mathbf{N}}^{pl,m}),\ v_{\mathbf{N}i}^{bd}\in H^1(\Omega^{bd}),$$

$$v_{\mathbf{N}i}^{pl,k}=\sum_{r_i^{pl,k}=0}^{N_i^{pl,k}}\frac{1}{h_k}\left(r_i^{pl,k}+\frac{1}{2}\right)\overset{r_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}}P_{r_i^{pl,k}}(z^{pl,k})\in H^1(\Omega_k^{pl}),$$

$$v_{\mathbf{N}i}^{bd}=0\quad\text{on }\Gamma_0^{bd},\ h_k^{-1/2}\overset{r_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}}\in L^2(\omega_k),\ 0\le r_i^{pl,k}\le N_i^{pl,k},\ v_{\mathbf{N}i}^{pl,k}=0\quad\text{on }\Gamma_0^{pl,k},$$

$$v_{\mathbf{N}i}^{bd}=v_{\mathbf{N}i}^{pl,k}\quad\text{on }\Gamma_k^{bd,pl},\ k=1,...,m,\ v_{\mathbf{N}i}^{pl,\bar{k}+1}=v_{\mathbf{N}i}^{pl,\bar{k}}\quad\text{on }\Gamma_{\bar{k},\bar{k}+1}^{pl},\ i=1,2,3,\ \bar{k}=1,...,m-1\},$$

$$\mathbf{H_N}(\Omega)=\{\mathbf{v_N}=(v_{\mathbf{N}i})_{i=1}^{3};v_{\mathbf{N}i}^{bd}\in L^2(\Omega^{bd}),v_{\mathbf{N}i}^{pl,k}=\sum_{r_i^{pl,k}=0}^{N_i^{pl,k}}\frac{1}{h_k}\left(r_i^{pl,k}+\frac{1}{2}\right)\overset{r_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}}P_{r_i^{pl,k}}(z^{pl,k}),$$

$$h_k^{-1/2}\overset{r_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}}\in L^2(\omega_k),\ 0\le r_i^{pl,k}\le N_i^{pl,k},\ i=1,2,3,\ k=1,2,...,m\},$$

where $z^{pl,k}=\dfrac{x_3-\bar{h}_k}{h_k}$, $\bar{h}_k=\dfrac{h_k^{+}+h_k^{-}}{2}$, $h_k=\dfrac{h_k^{+}-h_k^{-}}{2}$, $k=1,...,m.$

Since functions h_k^+ and h_k^- ($k=1,...,m$) are Lipschitz continuous in ω_k, then from Rademacher's theorem [8] it follows that h_k^+ and h_k^- are differentiable almost everywhere in ω_k and $\partial_\alpha h_k^{\pm}\in L^\infty(\omega_k^*)$, for all subdomains ω_k^*, $\overline{\omega_k^*}\subset\omega_k$, $\alpha=1,2$, $k=1,...,m$. So, for any vector-function $\mathbf{v}_{\mathbf{N}}^{pl,k}=(v_{\mathbf{N}i}^{pl,k})_{i=1}^{3}\in\mathbf{H}^1(\Omega)$ the corresponding functions $\overset{r_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}}$ belong to $H^1(\omega_k^*)$, for all ω_k^*, $\overline{\omega_k^*}\subset\omega_k$, i.e. $\overset{r_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}}\in H_{loc}^1(\omega_k)$, $0\le r_i^{pl,k}\le N_i^{pl,k}$, $i=1,2,3$. Moreover, the norms $\|\cdot\|_{\mathbf{H}^1(\Omega_k^{pl})}$ in the spaces $\mathbf{H}^1(\Omega_k^{pl})$ define corresponding norms $\|\cdot\|_{k*}$ for vector-functions $\vec{v}_{\mathbf{N}}^{pl,k}=(\overset{r_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}})$ from the space $[H_{loc}^1(\omega_k)]^{N_{1,2,3}^{pl,k}}$, $N_{1,2,3}^{pl,k}=N_1^{pl,k}+N_2^{pl,k}+N_3^{pl,k}+3$, such that $\left\|\vec{v}_{\mathbf{N}}^{pl,k}\right\|_{k*}=\left\|\mathbf{v}_{\mathbf{N}}^{pl,k}\right\|_{\mathbf{H}^1(\Omega_k^{pl})}$, and applying properties of Legendre polynomials we can obtain their explicit expressions

$$\left\|\vec{v}_{\mathbf{N}}^{pl,k}\right\|_{k*}^2=\sum_{i=1}^{3}\sum_{r_i^{pl,k}=0}^{N_i^{pl,k}}\left(r_i^{pl,k}+\frac{1}{2}\right)\left[\left\|\sum_{s_i^{pl,k}=r_i^{pl,k}}^{N_i^{pl,k}}\left(s_i^{pl,k}+\frac{1}{2}\right)(1-(-1)^{r_i^{pl,k}+s_i^{pl,k}})h_k^{-3/2}\overset{s_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}}\right\|_{L^2(\omega_k)}^2+\right.$$

$$+\left\|h_k^{-1/2}\overset{r_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}}\right\|_{L^2(\omega_k)}^2+\sum_{\alpha=1}^{2}\left\|\sum_{s_i^{pl,k}=r_i^{pl,k}+1}^{N_i^{pl,k}}\left(s_i^{pl,k}+\frac{1}{2}\right)(\partial_\alpha h_k^{+}-(-1)^{r_i^{pl,k}+s_i^{pl,k}}\partial_\alpha h_k^{-})h_k^{-3/2}\overset{s_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}}-\right.$$

$$\left.\left.-h_k^{-1/2}\partial_\alpha\overset{r_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}}+(r_i^{pl,k}+1)h_k^{-3/2}\partial_\alpha h_k\overset{r_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}}\right\|_{L^2(\omega_k)}^2\right],$$

where we assume that the sum with the upper limit less than the lower one equals to zero.

Note, that for vector-function $\vec{v}_{\mathbf{N}}^{pl,k}\in[H_{loc}^1(\omega_k)]^{N_{1,2,3}^{pl,k}}$, which satisfies the conditon $\left\|\vec{v}_{\mathbf{N}}^{pl,k}\right\|_{k*}<\infty$, we can define the trace on $\gamma_{k,0}$. Indeed, the corresponding vector-function $\mathbf{v}_{\mathbf{N}}^{pl,k}=(v_{\mathbf{N}i}^{pl,k})_{i=1}^{3}$ of three space variables belongs to $\mathbf{H}^1(\Omega_k^{pl})$ and hence the trace $tr(v_{\mathbf{N}i}^{pl,k})$ of $v_{\mathbf{N}i}^{pl,k}$ on $\partial\Omega_k^{pl}$ belongs to $H^{1/2}(\partial\Omega_k^{pl})$ $(i=1,2,3)$. Therefore, for components $\overset{r_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}}$ of vector-function $\vec{v}_{\mathbf{N}}^{pl,k}$ the trace operator $tr_{\gamma_{k,0}}$ on $\gamma_{k,0}$ can be defined as follows

$$tr_{\gamma_{k,0}}(\overset{r_i^{pl,k}}{v}{}^{pl,k}_{\mathbf{N}i}) = \int_{h^-}^{h^+} tr(v^{pl,k}_{\mathbf{N}i})|_{\Gamma^{pl}_{k,0}} P_{r_i^{pl,k}}(z^{pl,k})dx_3, \quad 0 \le r_i^{pl,k} \le N_i^{pl,k}, \quad i = 1,2,3.$$

On the subspaces $\mathbf{V_N}(\Omega)$ and $\mathbf{H_N}(\Omega)$ from the original three-dimensional problem we obtain the following variational problem: find $\mathbf{w_N} \in C^0([0,T];\mathbf{V_N}(\Omega))$, $\mathbf{w}'_\mathbf{N} \in C^0([0,T];\mathbf{H_N}(\Omega))$, which satisfies equation

$$\frac{d}{dt}\sum_{i=1}^{3}\int_{\Omega} \rho w'_{\mathbf{N}i}(.)v_{\mathbf{N}i}dx + \sum_{i,j,p,q=1}^{3}\int_{\Omega} a_{ijpq}(x)e_{pq}(\mathbf{w_N}(.))e_{ij}(\mathbf{v_N})dx =$$
$$= \sum_{i=1}^{3}\int_{\Omega} f_i v_{\mathbf{N}i}dx + \sum_{i=1}^{3}\int_{\Gamma_1} g_i tr_{\Gamma_1}(v_{\mathbf{N}i})d\Gamma, \quad \forall \mathbf{v_N} \in \mathbf{V_N}(\Omega), \tag{8}$$

in the sense of distributions on $(0,T)$ and the initial conditions

$$\mathbf{w_N}(0) = \boldsymbol{\varphi}_\mathbf{N}, \qquad \mathbf{w}'_\mathbf{N}(0) = \boldsymbol{\psi}_\mathbf{N}, \tag{9}$$

where $\boldsymbol{\varphi}_\mathbf{N} \in \mathbf{V_N}(\Omega)$, $\boldsymbol{\psi}_\mathbf{N} \in \mathbf{H_N}(\Omega)$. Since the unknown vector-function $\mathbf{w_N}$ of the problem (8), (9) in the multilayer part of multi-structure is of the following form

$$\mathbf{w_N} = (w_{\mathbf{N}i})_{i=1}^3, \; w_{\mathbf{N}i} = w^{pl,k}_{\mathbf{N}i} = \sum_{r_i^{pl,k}=0}^{N_i^{pl,k}} \frac{1}{h_k}\left(r_i^{pl,k} + \frac{1}{2}\right)\overset{r_i^{pl,k}}{w}{}^{pl,k}_{\mathbf{N}i} P_{r_i^{pl,k}}(z^{pl,k}) \quad \text{in } \Omega_k^{pl}, \; i = 1,2,3, k = 1,...,m,$$

hence in the sub-structures Ω_k^{pl} the vector-function $\mathbf{w_N}$ is defined by functions of two space variables $\overset{r_i^{pl,k}}{w}{}^{pl,k}_{\mathbf{N}i}$. Therefore, problem (8), (9) is equivalent to the following problem defined on the union of three-dimensional and two-dimensional domains: find the unknown vector-function $\vec{w}_\mathbf{N} \in C^0([0,T];\vec{V}_\mathbf{N}(\Omega^{bd,pl}))$, $\vec{w}'_\mathbf{N} \in C^0([0,T];\vec{H}_\mathbf{N}(\Omega^{bd,pl}))$, which satisfies equation

$$\frac{d}{dt}R_\mathbf{N}(\vec{w}'_\mathbf{N}(.),\vec{v}_\mathbf{N}) + A_\mathbf{N}(\vec{w}_\mathbf{N}(.),\vec{v}_\mathbf{N}) = L_\mathbf{N}(\vec{v}_\mathbf{N}), \qquad \forall \vec{v}_\mathbf{N} \in \vec{V}_\mathbf{N}(\Omega^{bd,pl}), \tag{10}$$

in sense of distributions on $(0,T)$ and the initial conditions

$$\vec{w}_\mathbf{N}(0) = \vec{\varphi}_\mathbf{N}, \qquad \vec{w}'_\mathbf{N}(0) = \vec{\psi}_\mathbf{N}, \tag{11}$$

where $\vec{\varphi}_\mathbf{N} \in \vec{V}_\mathbf{N}(\Omega^{bd,pl}) = \{\vec{v}_\mathbf{N} = (\mathbf{v}^{bd}_\mathbf{N}, \vec{v}^{pl,1}_\mathbf{N}, ..., \vec{v}^{pl,m}_\mathbf{N}) \in [H^1(\Omega^{bd})]^3 \times \prod_{k=1}^{m}[H^1_{loc}(\omega_k)]^{N^{pl,k}_{1,2,3}}; \; \mathbf{v}^{bd}_\mathbf{N} = (v^{bd}_{\mathbf{N}i}),$

$$tr_{\Gamma_0^{bd}}(v^{bd}_{\mathbf{N}i}) \equiv 0, \; \left\|\vec{v}^{pl,k}_\mathbf{N}\right\|_{k*} < \infty, \; \vec{v}^{pl,k}_\mathbf{N} = (\overset{r_i^{pl,k}}{v}{}^{pl,k}_{\mathbf{N}i}), \; tr_{\gamma_{k,0}}(\overset{r_i^{pl,k}}{v}{}^{pl,k}_{\mathbf{N}i}) \equiv 0, \; r_i^{pl,k} = 0,...,N_i^{pl,k}, \; tr_{\Gamma_k^{bd,pl}}(v^{bd}_{\mathbf{N}i}) =$$

$$tr_{\Gamma_k^{bd,pl}}\left(\sum_{r_i^{pl,k}=0}^{N_i^{pl,k}} \frac{1}{h_k}\left(r_i^{pl,k} + \frac{1}{2}\right)\overset{r_i^{pl,k}}{v}{}^{pl,k}_{\mathbf{N}i} P_{r_i^{pl,k}}(z^{pl,k})\right), \; k = 1,2,...,m, \; \sum_{r_i^{pl,\bar{k}}=0}^{N_i^{pl,\bar{k}}} \frac{1}{h_{\bar{k}}}\left(r_i^{pl,\bar{k}} + \frac{1}{2}\right)\overset{r_i^{pl,\bar{k}}}{v}{}^{pl,\bar{k}}_{\mathbf{N}i} =$$

$$\sum_{r_i^{pl,\bar{k}+1}=0}^{N_i^{pl,\bar{k}+1}} \frac{1}{h_{\bar{k}+1}}\left(r_i^{pl,\bar{k}+1} + \frac{1}{2}\right)\overset{r_i^{pl,\bar{k}+1}}{v}{}^{pl,\bar{k}+1}_{\mathbf{N}i}(-1)^{r_i^{pl,\bar{k}+1}} \quad \text{in } \omega_{\bar{k}} \cap \omega_{\bar{k}+1}, \; i = 1,2,3, \; \bar{k} = 1,2,...,m-1\},$$

$$N_{1,2,3}^{pl,k}=N_1^{pl,k}+N_2^{pl,k}+N_3^{pl,k}+3\,,\ k=1,2,...,m\,,\ \vec{\psi}_{\mathbf{N}}\in\vec{H}_{\mathbf{N}}(\Omega^{bd,pl})=\{\vec{v}_{\mathbf{N}}=(v_{\mathbf{N}i}^{bd},\vec{v}_{\mathbf{N}}^{pl,1},\vec{v}_{\mathbf{N}}^{pl,2},...,$$

$$\vec{v}_{\mathbf{N}}^{pl,m})\in[L^2(\Omega^{bd})]^3\times\prod_{k=1}^{m}[L^2(\omega_k)]^{N_{1,2,3}^k};\|\vec{v}_{\mathbf{N}}\|^2_{\vec{H}_{\mathbf{N}}(\Omega^{bd,pl})}=\sum_{i=1}^{3}(\|v_{\mathbf{N}i}^{bd}\|^2_{L^2(\Omega^{bd})}+\sum_{k=1}^{m}\sum_{r_i^{pl,k}=0}^{N_i^{pl,k}}\|h_k^{-1/2}\overset{r_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}}\|^2_{L^2(\omega_k)})<\infty\},$$

$\vec{\varphi}_{\mathbf{N}}$ and $\vec{\psi}_{\mathbf{N}}$ correspond to $\boldsymbol{\varphi}_{\mathbf{N}}\in\mathbf{V}_{\mathbf{N}}(\Omega)$ and $\boldsymbol{\psi}_{\mathbf{N}}\in\mathbf{H}_{\mathbf{N}}(\Omega)$, respectively. The bilinear $R_{\mathbf{N}}(\vec{y}_{\mathbf{N}},\vec{v}_{\mathbf{N}})$, $A_{\mathbf{N}}(\vec{y}_{\mathbf{N}},\vec{v}_{\mathbf{N}})$ and linear $L_{\mathbf{N}}(\vec{v}_{\mathbf{N}})$ forms are defined by corresponding forms $R(.,.)$, $A(.,.)$ and $L(.)$, and applying properties of Legendre polynomials we obtain their explicit expressions

$$R_{\mathbf{N}}(\vec{y}_{\mathbf{N}},\vec{v}_{\mathbf{N}})=\sum_{i=1}^{3}\int_{\Omega^{bd}}\rho^{bd}y_{\mathbf{N}i}^{bd}v_{\mathbf{N}i}^{bd}dx+\sum_{k=1}^{m}\sum_{i=1}^{3}\sum_{r_i^{pl,k},\tilde{r}_i^{pl,k}=0}^{N_i^{pl,k}}\left(r_i^{pl,k}+\frac{1}{2}\right)\left(\tilde{r}_i^{pl,k}+\frac{1}{2}\right)\int_{\omega_k}\frac{1}{h_k^2}\overset{r_i^{pl,k}\tilde{r}_i^{pl,k}}{\rho^{pl,k}}\overset{r_i^{pl,k}}{y_{\mathbf{N}i}^{pl,k}}\overset{\tilde{r}_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}}d\omega_k,$$

where $\overset{r_i^{pl,k}\tilde{r}_i^{pl,k}}{\rho^{pl,k}}=\int_{h_k^-}^{h_k^+}\rho^{pl,k}P_{r_i^{pl,k}}(z_k)P_{\tilde{r}_i^{pl,k}}(z_k)dx_3$, $0\le r_i^{pl,k},\tilde{r}_i^{pl,k}\le N_i^{pl,k}$, $i=1,2,3$, $k=1,2,...,m$, the bilinear form $A_{\mathbf{N}}(\vec{y}_{\mathbf{N}},\vec{v}_{\mathbf{N}})$ is given by

$$A_{\mathbf{N}}(\vec{y}_{\mathbf{N}},\vec{v}_{\mathbf{N}})=\sum_{i,j,p,q=1}^{3}\int_{\Omega^{bd}}a_{ijpq}^{bd}e_{pq}(\mathbf{y}_{\mathbf{N}}^{bd})e_{ij}(\mathbf{v}_{\mathbf{N}}^{bd})dx+$$

$$+\sum_{k=1}^{m}\sum_{r^{pl,k},\tilde{r}^{pl,k}=0}^{N_{\max}^{pl,k}}\left(r^{pl,k}+\frac{1}{2}\right)\left(\tilde{r}^{pl,k}+\frac{1}{2}\right)\int_{\omega_k}\frac{1}{h_k^2}\sum_{i,j,p,q=1}^{3}\overset{r^{pl,k}\tilde{r}^{pl,k}}{a_{ijpq}^{pl,k}}\overset{\tilde{r}^{pl,k}}{e_{pq}^{k}}(\vec{y}_{\mathbf{N}})\overset{r^{pl,k}}{e_{ij}^{k}}(\vec{v}_{\mathbf{N}})d\omega_k,$$

where $N_{\max}^{pl,k}=\max\limits_{1\le i\le 3}\{N_i^{pl,k}\}$,

$$\overset{r}{e_{ij}^{k}}(\vec{v}_{\mathbf{N}})=\frac{1}{2}\left(\partial_i(\overset{r}{v_{\mathbf{N}j}^{pl,k}})+\partial_j(\overset{r}{v_{\mathbf{N}i}^{pl,k}})+\sum_{s=r}^{N_{\max}^{pl,k}}(\overset{r}{d_{is}^{pl,k}}\overset{s}{v_{\mathbf{N}j}^{pl,k}}+\overset{r}{d_{js}^{pl,k}}\overset{s}{v_{\mathbf{N}i}^{pl,k}})\right),\ r\in\mathbf{N}\cup\{0\},$$

$$\overset{r}{d_{ir}^{pl,k}}=-\frac{r+1}{h_k}\partial_i h_k,\quad \overset{r}{d_{is}^{pl,k}}=-\frac{1}{h_k}\left(s+\frac{1}{2}\right)(\partial_i h_k^+-(-1)^{r+s}\partial_i h_k^-)+\frac{1}{h_k}\left(s+\frac{1}{2}\right)(1-(-1)^{r+s})\frac{(i-1)(i-2)}{2},$$

for $s>r$, and $\overset{r_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}}=\overset{r_i^{pl,k}}{y_{\mathbf{N}i}^{pl,k}}=0$, for $N_i^{pl,k}<r_i^{pl,k}\le N_{\max}^{pl,k}$, $\overset{r^{pl,k}\tilde{r}^{pl,k}}{a_{ijpq}^{pl,k}}=\int_{h_k^-}^{h_k^+}a_{ijpq}^{pl,k}P_{r^{pl,k}}(z_k)P_{\tilde{r}^{pl,k}}(z_k)dx_3$, $i,j,p,q=1,2,3,\ \ 0\le r^{pl,k},\tilde{r}^{pl,k}\le N_{\max}^{pl,k}$, $k=1,2,...,m$. The linear form $L_{\mathbf{N}}(\vec{v}_{\mathbf{N}})$ is of the following form

$$L_{\mathbf{N}}(\vec{v}_{\mathbf{N}})=\sum_{i=1}^{3}\int_{\Omega^{bd}}f_i v_{\mathbf{N}i}^{bd}dx+\sum_{i=1}^{3}\int_{\Gamma_1^{bd}}g_i tr_{\Gamma_1}(v_{\mathbf{N}i}^{bd})d\Gamma+$$

$$+\sum_{k=1}^{m}\sum_{i=1}^{3}\sum_{r_i^{pl,k}=0}^{N_i^{pl,k}}\left(r_i^{pl,k}+\frac{1}{2}\right)\left[\int_{\omega_k}\frac{1}{h_k}\overset{r_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}}\left(\overset{r_i^{pl,k}}{f_i^{pl,k}}+g_i^{pl,k,+}\lambda_{k,+}+g_i^{pl,k,-}\lambda_{k,-}(-1)^{r_i^{pl,k}}\right)d\omega_k+\int_{\gamma_{k,1}}\frac{1}{h_k}\overset{r_i^{pl,k}}{v_{\mathbf{N}i}^{pl,k}}\overset{r_i^{pl,k}}{g_i^{pl,k}}d\gamma_{k,1}\right],$$

where $\gamma_{k,1}=\tilde{\gamma}_k \setminus \overline{\gamma_k^{bd,pl}\cup\gamma_{k,0}}$, $\lambda_{k,\pm}=\sqrt{1+(\partial_1 h_k^{\pm})^2+(\partial_2 h_k^{\pm})^2}$, $\overset{r_i^{pl,k}}{f_i^{pl,k}}=\int\limits_{h_k^-}^{h_k^+} f_i^{pl,k} P_{r_i^{pl,k}}(z_k)dx_3$, $g_i^{pl,k,+}(x_1,x_2)$

$=g_i(x_1,x_2,x_3)$, for $(x_1,x_2,x_3)\in\Gamma_k^+\cap\Gamma_1\neq\varnothing$ and $g_i^{pl,k,+}(x_1,x_2)=0$, for $(x_1,x_2,x_3)\in\Gamma_k^+\setminus\Gamma_1$, $g_i^{pl,k,-}(x_1,x_2)=g_i(x_1,x_2,x_3)$, for $(x_1,x_2,x_3)\in\Gamma_k^-\cap\Gamma_1\neq\varnothing$ and $g_i^{pl,k,-}(x_1,x_2)=0$, for

$(x_1,x_2,x_3)\in\Gamma_k^-\setminus\Gamma_1$, $\overset{r_i^{pl,k}}{g_i^{pl,k}}=\int\limits_{h_k^-}^{h_k^+} g_i^{pl,k} P_{r_i^{pl,k}}(z_k)dx_3$, $r_i^{pl,k}=0,\dots,N_i^{pl,k}$, $i=1,2,3$, $k=1,2,\dots,m$.

The obtained hierarchical models (10), (11) are pluridimensional models written in variational form of multi-structure consisting of three-dimensional body with general shape and multilayer part composed of plates, and for corresponding initial-boundary value problems the following existence and uniqueness theorem is valid.

Theorem 1. *Let the three-dimensional part* Ω^{bd} *with general shape and plates* Ω_k^{pl}, $k=1,2,\dots,m$, *are such that the domain* Ω *occupied by multi-structure is a Lipschitz domain, the parameters characterizing mechanical properties of three-dimensional part* $a_{ijpq}^{bd}\in L^\infty(\Omega^{bd})$, $i,j,p,q=1,2,3$, *satisfy symmetry and positive definiteness conditions*

$$a_{ijpq}^{bd}(x)=a_{jipq}^{bd}(x)=a_{pqij}^{bd}(x),\ \sum_{i,j,p,q=1}^{3} a_{ijpq}^{bd}(x)\varepsilon_{ij}\varepsilon_{pq}\geq c_a^{bd}\sum_{i,j=1}^{3}(\varepsilon_{ij})^2,\quad \forall x\in\Omega^{bd},\varepsilon_{ij}\in\mathbf{R},\varepsilon_{ij}=\varepsilon_{ji},\tag{12}$$

$i,j,p,q=1,2,3$, $c_a^{bd}=const>0$, *and* $\rho^{bd}\in L^\infty(\Omega^{bd})$, $\rho^{bd}\geq c=const>0$, *and coefficients* $\overset{r^{pl,k}\tilde{r}^{pl,k}}{a_{ijpq}^{pl,k}}/h_k\in L^\infty(\omega_k)$, $r^{pl,k},\tilde{r}^{pl,k}=0,\dots,N_{\max}^{pl,k}$, $k=1,2,\dots,m$, *defining two-dimensional models of plates satisfy the following conditions*

$$\overset{r^{pl,k}\tilde{r}^{pl,k}}{a_{ijpq}^{pl,k}}=\overset{r^{pl,k}\tilde{r}^{pl,k}}{a_{jipq}^{pl,k}}=\overset{r^{pl,k}\tilde{r}^{pl,k}}{a_{ijqp}^{pl,k}}=\overset{\tilde{r}^{pl,k}r^{pl,k}}{a_{ijpq}^{pl,k}},\ \overset{r^{pl,k}\tilde{r}^{pl,k}}{\rho^{pl,k}}=\overset{\tilde{r}^{pl,k}r^{pl,k}}{\rho^{pl,k}},\ i,j,p,q=1,2,3,\ r^{pl,k},\tilde{r}^{pl,k}=0,\dots,N_{\max}^{pl,k},$$

$$\sum_{r^{pl,k},\tilde{r}^{pl,k}=0}^{N_{\max}^{pl,k}}\sum_{i,j,p,q=1}^{3}\frac{1}{h_k}\left(r^{pl,k}+\frac{1}{2}\right)\left(\tilde{r}^{pl,k}+\frac{1}{2}\right)\overset{r^{pl,k}\tilde{r}^{pl,k}}{a_{ijpq}^{pl,k}}\overset{r^{pl,k}}{\varepsilon_{ij}}\overset{\tilde{r}^{pl,k}}{\varepsilon_{pq}}\geq\alpha\sum_{r^{pl,k}=0}^{N_{\max}^{pl,k}}\sum_{i,j=1}^{3}\left(r^{pl,k}+\frac{1}{2}\right)\overset{r^{pl,k}}{\varepsilon_{ij}}\overset{r^{pl,k}}{\varepsilon_{ij}},\qquad \text{in } \omega_k,$$

for all $\overset{r^{pl,k}}{\varepsilon_{ij}}\in\mathbf{R}$, $\overset{r^{pl,k}}{\varepsilon_{ij}}=\overset{r^{pl,k}}{\varepsilon_{ji}}$, $i,j,p,q=1,2,3$, $\alpha=const>0$, $\overset{r^{pl,k}\tilde{r}^{pl,k}}{\rho^{pl,k}}/h_k\in L^\infty(\omega_k)$,

$\overset{r^{pl,k}\tilde{r}^{pl,k}}{\rho^{pl,k}}/h_k\geq c=const>0$, $r^{pl,k},\tilde{r}^{pl,k}=0,\dots,N_{\max}^{pl,k}$, $k=1,2,\dots,m$. If $\vec{\varphi}_{\mathbf{N}}\in\vec{V}_{\mathbf{N}}(\Omega^{bd,pl})$, $\vec{\psi}_{\mathbf{N}}\in\vec{H}_{\mathbf{N}}(\Omega^{bd,pl})$

and given functions f_i, g_i, $\overset{r_i^{pl,k}}{f_i^{pl,k}}$, $g_i^{pl,k,\pm}$ *and* $\overset{r_i^{pl,k}}{g_i^{pl,k}}$ *are such that*

$$f_i\in L^2(0,T;L^2(\Omega^{bd})),\ g_i,g_i'\in L^2(0,T;L^{4/3}(\Gamma_1^{bd})),\ i=1,2,3,$$

$$h_k^{-1/2}\overset{r_i^{pl,k}}{f_i^{pl,k}}\in L^2(0,T;L^2(\omega_k)),\lambda_{k,\pm}^{3/4}g_i^{pl,k,\pm},\lambda_{k,\pm}^{3/4}(g_i^{pl,k,\pm})'\in L^2(0,T;L^{4/3}(\omega_k)),$$

$$h_k^{-1/4} \overset{r_i^{pl,k}}{g_i^{pl,k}}, h_k^{-1/4} (\overset{r_i^{pl,k}}{g_i^{pl,k}})' \in L^2(0,T; L^{4/3}(\gamma_{k,1})), r_i^{pl,k} = 0,...,N_i^{pl,k}, i = 1,2,3, k = 1,...,m,$$

then initial-boundary value problem (10), (11) *defined on the union of three-dimensional and two-dimensional domains possesses a unique solution.*

For the considered multi-structure consisting of three-dimensional part with general shape and multilayer substructure composed of plates with variable thickness the relationship between the original three-dimensional model and the constructed hierarchy of pluridimensional ones is investigated. In order to formulate the corresponding theorem let us define the following anisotropic weighted Sobolev spaces

$$\mathbf{H}^{0,0,s}_{h_k^\pm}(\Omega_k^{pl}) = \{\mathbf{v}; h_k^p \partial_3^p \mathbf{v} \in \mathbf{L}^2(\Omega_k^{pl}), p = 0,1,...,s\}, \quad s \in \mathbf{N},$$

$$\mathbf{H}^{1,1,s}_{h_k^\pm}(\Omega_k^{pl}) = \{\mathbf{v}; \partial_3^{p-1}\mathbf{v} \in \mathbf{H}^1(\Omega_k^{pl}), p = 1,...,s,\ \partial_\alpha h_k^\pm\ \partial_3^r \mathbf{v} \in \mathbf{L}^2(\Omega_k^{pl}), \alpha = 1,2, r = 1, s, \min\{2,s\}\},\ s \in \mathbf{N},$$

where $k = 1,2,...,m,$ which are Hilbert spaces with respect to the following norms

$$\|\mathbf{v}\|_{\mathbf{H}^{0,0,s}_{h_k^\pm}(\Omega_k^{pl})} = \left(\sum_{p=0}^{s} \left\| h_k^p \partial_3^p \mathbf{v} \right\|^2_{\mathbf{L}^2(\Omega_k^{pl})} \right)^{1/2},$$

$$\|\mathbf{v}\|_{\mathbf{H}^{1,1,s}_{h_k^\pm}(\Omega_k^{pl})} = \left(\sum_{p=1}^{s} \left\| \partial_3^{p-1} \mathbf{v} \right\|^2_{\mathbf{H}^1(\Omega_k^{pl})} + \sum_{\alpha=1}^{2} \left(\left\| \partial_\alpha h_k^+ \partial_3 \mathbf{v} \right\|^2_{\mathbf{L}^2(\Omega_k^{pl})} + \left\| \partial_\alpha h_k^- \partial_3 \mathbf{v} \right\|^2_{\mathbf{L}^2(\Omega_k^{pl})} + \right.\right.$$

$$\left.\left. + \left\| \partial_\alpha h_k^+ \partial_3^s \mathbf{v} \right\|^2_{\mathbf{L}^2(\Omega_k^{pl})} + \left\| \partial_\alpha h_k^- \partial_3^s \mathbf{v} \right\|^2_{\mathbf{L}^2(\Omega_k^{pl})} + \left\| \partial_\alpha h_k^+ \partial_3^{\min\{2,s\}} \mathbf{v} \right\|^2_{\mathbf{L}^2(\Omega_k^{pl})} + \left\| \partial_\alpha h_k^- \partial_3^{\min\{2,s\}} \mathbf{v} \right\|^2_{\mathbf{L}^2(\Omega_k^{pl})} \right) \right)^{1/2}.$$

The following theorem is valid.

Theorem 2. *Let the parts* Ω^{bd} *and* Ω_k^{pl}, $k = 1,2,...,m$, *of the multi-structure are such that* Ω *is a Lipschitz domain, the parameters characterizing mechanical properties of the general three-dimensional and multilayer parts* $a_{ijpq}^{bd} \in L^\infty(\Omega^{bd})$, $a_{ijpq}^{pl,k} \in L^\infty(\Omega_k^{pl})$, $i,j,p,q = 1,2,3,$ *satisfy symmetry and positive definiteness conditions* (12) and

$$a_{ijpq}^{pl,k}(x) = a_{jipq}^{pl,k}(x) = a_{pqij}^{pl,k}(x), \sum_{i,j,p,q=1}^{3} a_{ijpq}^{pl,k}(x)\varepsilon_{ij}\varepsilon_{pq} \ge c_a^{pl,k} \sum_{i,j=1}^{3} (\varepsilon_{ij})^2, \quad \forall x \in \Omega_k^{pl}, \varepsilon_{ij} \in \mathbf{R}, \varepsilon_{ij} = \varepsilon_{ji},$$

$i,j,p,q = 1,2,3$, $c_a^{pl,k} = const > 0$, $\rho^{bd} \in L^\infty(\Omega^{bd})$, $\rho^{pl,k} \in L^\infty(\Omega_k^{pl})$, $\rho^{bd} \ge c = const > 0$, $\rho^{pl,k} \ge c = const > 0$, $k = 1,...,m$, *and* $\mathbf{f} \in L^2(0,T; \mathbf{L}^2(\Omega)),$ $\mathbf{g}, \mathbf{g}' \in L^2(0,T; \mathbf{L}^{4/3}(\Gamma_1)),$ $\boldsymbol{\varphi} \in \mathbf{V}(\Omega)$, $\boldsymbol{\psi} \in \mathbf{L}^2(\Omega)$. *If vector-functions* $\boldsymbol{\varphi}_\mathbf{N} \in \mathbf{V}_\mathbf{N}(\Omega)$ *and* $\boldsymbol{\psi}_\mathbf{N} \in \mathbf{H}_\mathbf{N}(\Omega)$, *which correspond to* $\vec{\varphi}_\mathbf{N} \in \vec{V}_\mathbf{N}(\Omega^{bd,pl})$ *and* $\vec{\psi}_\mathbf{N} \in \vec{H}_\mathbf{N}(\Omega^{bd,pl})$ *tend to* $\boldsymbol{\varphi}$ *and* $\boldsymbol{\psi}$ *in the spaces* $\mathbf{H}^1(\Omega)$ *and* $\mathbf{L}^2(\Omega)$, *respectively, as* $N_{\min} = \min_{1 \le k \le m,\ 1 \le i \le 3} \{N_i^{pl,k}\} \to \infty$ *or* $h_{\max} = \max\{h_k(x_1,x_2); (x_1,x_2) \in \overline{\omega}_k, k = 1,...,m\} \to 0$, *then the sequence of vector-functions* $\mathbf{w}_\mathbf{N}(t) = (w_{\mathbf{N}i}(t))_{i=1}^3 \in C^0([0,T]; \mathbf{V}_\mathbf{N}(\Omega))$ *restored from the solutions* $\vec{w}_\mathbf{N} \in C^0([0,T]; \vec{V}_\mathbf{N}(\Omega^{bd,pl}))$ *of the constructed problems* (10), (11) *defined on the union of three-dimensional and two-dimensional domains converges to the solution* $\mathbf{u}(t)$ *of the three-dimensional problem* (6), (7),

$$\begin{array}{lll} \mathbf{w_N}(t) \to \mathbf{u}(t) & \text{in } \mathbf{V}(\Omega), & \\ \mathbf{w}'_\mathbf{N}(t) \to \mathbf{u}'(t) & \text{in } \mathbf{L}^2(\Omega), & \end{array} \quad \forall t \in [0,T], \text{ as } N_{\min} \to \infty \text{ or } h_{\max} \to 0 .$$

Moreover, if $\mathbf{u}$ satisfies additional regularity conditions $d^r(\mathbf{u}|_{\Omega_k^{pl}})/dt^r \in L^2(0,T;\ \mathbf{H}^{1,1,s_r}_{h_k^\pm}(\Omega_k^{pl}))$, $r=0,1$, $d^2(\mathbf{u}|_{\Omega_k^{pl}})/dt^2 \in L^2(0,T;\ \mathbf{H}^{0,0,s_2}_{h_k^\pm}(\Omega_k^{pl}))$, $s_0, s_1, s_2 \in \mathbf{N}$, $s_0 \ge 2$, $s_1 \ge 2$, $\boldsymbol{\varphi}|_{\Omega_k^{pl}} \in \mathbf{H}^{1,1,\tilde{s}_0}_{h_k^\pm}(\Omega_k^{pl})$, $\tilde{s}_0 \in \mathbf{N}$, $\tilde{s}_0 \ge 2$, $\boldsymbol{\psi}|_{\Omega_k^{pl}} \in \mathbf{H}^{1,1,1}_{h_k^\pm}(\Omega_k^{pl})$, $k=1,2,...,m$, then for suitable initial conditions $\vec{\varphi}_\mathbf{N}$ and $\vec{\psi}_\mathbf{N}$ of the initial-boundary value problems corresponding to the hierarchical models the following estimate is valid

$$\|\mathbf{u}'(t) - \mathbf{w}'_\mathbf{N}(t)\|^2_{\mathbf{L}^2(\Omega)} + \|\mathbf{u}(t) - \mathbf{w_N}(t)\|^2_{\mathbf{H}^1(\Omega)} \le \left(\frac{h_{\max}}{N_{\min}}\right)^{2s} \delta(T,\Omega,\Gamma_0,\mathbf{N}), \quad \forall t \in [0,T],$$

where $s = \min\{s_2, s_1 - 1, s_0 - 1, \tilde{s}_0 - 1\}$ and $\delta(T,\Omega,\Gamma_0,\mathbf{N}) \to 0$, as $N_{\min} \to \infty$.

Acknowledgement. The present research is supported by State Science Grant in applied research of Shota Rustaveli National Science Foundation (Contract No. 30/28).

მექანიკა

სამგანზომილებიანი სხეულისა და ფენოვანი ქვესტრუქტურისაგან შემდგარი დრეკადი მულტისტრუქტურების დინამიკური იერარქიული მოდელების გამოკვლევის შესახებ

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ნაშრომში განხილულია საწყის-სასაზღვრო ამოცანა მულტისტრუქტურისათვის, რომელიც შედგება ზოგადი ფორმის მქონე სამგანზომილებიანი სხეულისა და ცვალებადი სისქის ფირფიტებისაგან შედგენილი მრავალფენიანი ნაწილისაგან. აგებულია სამგანზომილებიანი და ორგანზომილებიანი არეების ერთობლიობაზე განსაზღვრულ დინამიკურ მოდელთა იერარქია. აგებული იერარქიული მოდელების შესაბამისი პლურიგანზომილებიანი საწყის-სასაზღვრო ამოცანები გამოკვლეულია სათანადო ფუნქციონალურ სივრცეებში. დამტკიცებულია აგებული სამგანზომილებიანი და ორგანზომილებიანი არეების ერთობლიობაზე განსაზღვრული საწყის-

სასაზღვრო ამოცანების ამონახსნებიდან აღდგენილი სამი სივრცითი ცვლადის ვექტორ-ფუნქციების მიმდევრობის კრებადობა საწყისი სამგანზომილებიანი ამოცანის ამონახსნისაკენ და დამატებით რეგულარობის პირობებში შეფასებულია კრებადობის რიგი.

REFERENCES

1. *P.G. Ciarlet, H. Le Dret, R. Nzengwa* (1989) J. Math. Pures Appl., **68**: 261-295.
2. *H. Le Dret* (1991) Problèmes Variationnels dans les Multi-Domaines: Modélisation des Jonctions et Applications, Masson, Paris (French).
3. *P.G. Ciarlet* (1990) Plates and Junctions in Elastic Multi-Structures: An Asymptotic Analysis, Masson, Paris.
4. *I.N. Vekua* (1955) Trudi Tbil. Matem. Inst., **21**: 191-259 (Russian).
5. *G . Avalishvili, M. Avalishvili, D. Gordeziani, B. Miara* (2010) Anal. Appl., **8**: 125-159.
6. *M. Dauge, E. Faou, Z. Yosibash* (2004) Encyclopedia of Computational Mechanics, **1**: 199-236.
7. *G.V. Jaiani* (2001) Z. Angew. Math. Mech., **81**, 3: 147-173.
8. *H. Whitney* (1957) Geometric integration theory: Princeton University Press, Princeton.

Received July, 2014