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Mathematics

Smirnov Classes of Analytic Functions with Variable Exponent in Multiply Connected Domains

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ABSTRACT. Let G be multiply connected domain with boundary, $\Gamma=\bigcup_{k=0}^{m}\Gamma_k$, where Γ_k, $k=\overline{0,m}$ are simple closed rectifiable curves such that $\Gamma_1,\dots,\Gamma_m$ lie outside each other, but all of them lie inside of Γ_0. The paper introduces the Smirnov classes $E^{\vec{p}(\cdot)}(G)$ with variable exponent $\vec{p}(t)=(p_0(t),p_1(t),\dots,p_m(t))$, where $p_k(t)$, $k=\overline{0,m}$ are given positive measurable functions on Γ_k. The properties of functions from these classes are established, in particular: an expansion theorem, representability by a Cauchy integral, generalizations of Smirnov's and Tumarkin's theorems, related to simply connected domains for multiply connected domains. Also, the question of belonging of Cauchy type integrals with a density from $L^{\vec{p}(\cdot)}(\Gamma)$ to the class $E^{\vec{p}(\cdot)}(G)$ is investigated. *© 2015 Bull. Georg. Natl. Acad. Sci.*

Key words: *Lebesgue space with variable exponent; Smirnov classes of analytic functions with variable exponent; multiply connected domain; Cauchy type integral.*

1. Introduction. Let G be a simply connected domain bounded by a closed rectifiable Jordan curve. In [1] V. I. Smirnov introduced the class $E^p(G)$, $p>0$ of analytic in G functions $\Phi(z)$ such that

$$\sup_{0<r<1}\int_{\Gamma_r}|\Phi(z)|^p|dz|<\infty,$$

where Γ_r is the image of the circumference $|w|=r$ for the conformal mapping of a circle $U=\{w:|w|<1\}$ on G.

When G is a unit circle, the class $E^p(G)$ coincides with the Hardy class H^p.

Vast literature is dedicated to the investigation of such classes.

For multiply connected domains, classes $E^p(G)$ were studied in [2-5] and other papers.

Classes E^p play an important role in mathematical analysis, which in particular concerns the representation of analytic functions by the Cauchy integral. Owing to this fact they found applications in the theory of boundary value problems for harmonic and analytic functions (see e.g. [6]).

Boundary value problems in various fields of mathematics have recently begun to be considered under the assumption that the sought function or its boundary values must be Lebesgue-summable with a variable exponent. The interest is due to the fact that such a formulation of problems enables one to investigate them more thoroughly taking into account the local behavior of given functions and their influence on the character of solvability.

Their various generalizations can be obtained by introducing Smirnov classes with variable exponents. The natural, as we think, generalizations of these classes were introduced and studied in [7-9]. Smirnov classes with variable exponent have found useful applications in the investigation of boundary value problems for analytic and harmonic functions and the cases were successfully investigated when the boundary of the domain, in which the problem is posed, admits cusps (see [10-12] and other papers).

In the present paper, we continue the investigation of Smirnov classes with variable exponent in arbitrary finitely connected domains *G*.

The properties of functions from these classes are established. In particular: it is proved that functions from the class $E^{\vec{p}(t)}(G)$ are representable by a Cauchy integral with a density from $L^{\vec{p}(t)}(\Gamma)$; an expansion theorem and its corollaries are obtained; the generalizations of Smirnov's and Tumarkin's theorems are generalized for the case of multiply connected domains; the conditions are found, under which a Cauchy type integral with a density from $L^{\vec{p}(t)}(\Gamma)$ belongs to the class $E^{\vec{p}(t)}(G)$.

2. Main Definitions and Notation

2.1. Standard domains and curves. Let $\Gamma = \bigcup_{k=0}^{m} \Gamma_k$ where $\Gamma_k, k = \overline{0,m}$ are simple closed rectifiable curves such that $\Gamma_1, \Gamma_2, \ldots, \Gamma_m$ lie outside each other, but all of them lie inside Γ_0 . Then Γ is the boundary of some domain *G* which we will call standard, and Γ a standard curve.

If $\gamma = \bigcup_{j=0}^{m} \gamma_j$, where γ_j, $j = \overline{0,m}$ are circumferences, is the boundary of the standard domain *K*, then we will call *K* a standard circular domain.

For any standard domain *G* of the plane *z* there exists a standard circular domain *K* of the plane ζ that contains the points $\zeta = 0$, ζ_j, $j = \overline{1,m}$, and has the standard boundary $\tilde{\gamma} = \bigcup_{k=0}^{m} \gamma_k$, where $\gamma_0 = \{\zeta : |\zeta| = 1\}$, $\gamma_j = \{\zeta : |\zeta - \zeta_j| = \rho_j\}$, $0 < |\zeta_j| + \rho_j < 1$, $j = \overline{1,m}$, which can be conformally mapped onto *G* [13: p. 235].

2.2. Exponent classes. We will say that a positive measurable function $p(t)$ given on Γ belongs to the class $\mathcal{P}(\Gamma)$, if:

1) there exists a constant $c(p)$ such that for any $t_1, t_2 \in \Gamma$

$$|p(t_1) - p(t_2)| < \frac{c(p)}{|\ln|t_1 - t_2||} \quad ;$$

2) $\min\limits_{t\in\Gamma} p(t) = \underline{p} > 1$.

2.3. Lebesgue classes with variable exponent. Let f be a measurable function on Γ, $p(t) \in \mathcal{P}(\Gamma)$, and

$$\| f \|_{L^{p(\cdot)}(\Gamma)} = \| f \|_{p(\cdot)} = \inf \left\{ \lambda : \lambda > 0,\ \int_0^l \left| \frac{f(t(s))}{\lambda} \right|^{p(t(s))} ds \le 1 \right\},$$

where $t = t(s)$, $0 \le s \le l$, be the equation of Γ with respect to the arc abscissa.

Assume $L^{p(\cdot)}(\Gamma) = \{ f : \| f \|_{p(\cdot)} < \infty \}$.

If $\Gamma = \bigcup\limits_{k=0}^{m} \Gamma_k$, $\vec{p}(t) = (p_0(t), p_1(t), \dots, p_m(t))$, $f \in L^{p_k(\cdot)}(\Gamma_k)$, then we denote by $L^{\vec{p}(\cdot)}(\Gamma)$ the set of those collections $(f_0, f_1, \dots, f_m)$, for which

$$\left(\| f_0 \|_{L^{p_0(\cdot)}(\Gamma_0)} + \| f_1 \|_{L^{p_1(\cdot)}(\Gamma_1)} + \dots + \| f_m \|_{L^{p_m(\cdot)}(\Gamma_m)} \right) < \infty.$$

2.4. Smirnov class $E^{p(\cdot)}(G)$ in simply connected domains. Let Γ be a simple rectifiable closed curve bounding the domain G and $z = z(\zeta)$ conformally map U onto G. Denote $p(\vartheta) = p(z(e^{i\vartheta}))$. We will say that an analytic function Φ in G belongs to the class $E^{p(\cdot)}(G)$ if

$$C(\Phi) = \sup_{0<r<1} \int_0^{2\pi} | \Phi(z(re^{i\vartheta})) |^{p(\vartheta)} | z'(re^{i\vartheta}) | d\vartheta < \infty.$$

Proposition 1. *If* $\inf\limits_{t\in\Gamma} p(t) = \underline{p} > 0$, $\Phi \in E^{p(\cdot)}(G)$, *then for almost all* $t \in \Gamma$ *there exists an angular limit* $\Phi^+(t)$ *and*

$$\int_\Gamma | \Phi^+(t) |^{p(t)} | dt | \le \sup_r \int_0^{2\pi} | \Phi(z(re^{i\vartheta})) |^{p(\vartheta)} | z'(re^{i\vartheta}) | d\vartheta = C(\Phi) < \infty.$$

3. Smirnov Classes with Variable Exponent in Standard Domains

Let $\Gamma = \bigcup\limits_{k=0}^{m} \Gamma_k$ be the standard curve bounding the domain G and K be the domain of the plane ζ with boundary $\gamma = \bigcup\limits_{k=0}^{m} \gamma_k$, which is conformally mapped by the function $z = z(\zeta)$ onto G.

A number $\varepsilon > 0$ will be assumed admissible if the curve $\gamma^\varepsilon = \bigcup\limits_{j=0}^{m} \gamma_{j\varepsilon}$, where $\gamma_{0\varepsilon} = \{ \zeta : | \zeta | = 1 - \varepsilon \}$, $\gamma_{j\varepsilon} = \{ \zeta : | \zeta - \zeta_j | = \rho_j + \varepsilon \}$, $j = \overline{1,m}$, and the domain K^ε bounded by it lies in K.

Let $\vec{p}(t) = (p_0(t), p_1(t), \dots, p_m(t))$, $p_j(t) \in \mathcal{P}(\Gamma_j)$, $j = \overline{0,m}$. Denote $p_0(\vartheta) = p_0(z(e^{i\vartheta}))$, $p_j(\vartheta) = p_j(z(\zeta_j + \rho_j e^{i\vartheta}))$, $j = \overline{1,m}$.

Definition 1. An analytic function Φ in the domain G belongs to the class $E^{\vec{p}(\cdot)}(G)$, if

$$C(\Phi)=\sup_{\varepsilon}\int_{\gamma^{\varepsilon}}|\Phi(z(re^{i\vartheta}))|^{\vec{p}(e^{i\vartheta})}|z'(re^{i\vartheta})|d\vartheta$$

$$=\sup_{\varepsilon}[\int_{0}^{2\pi}|\Phi(z((1-\varepsilon)e^{i\vartheta}))|^{p_0(\vartheta)}|z'((1-\varepsilon)e^{i\vartheta})|d\vartheta$$

$$+\sum_{j=0}^{m}\int_{0}^{2\pi}|\Phi(z(\zeta_j+(\rho_j+\varepsilon)e^{i\vartheta}))|^{p_j(\vartheta)}|z'(\zeta_j+(\rho_j+\varepsilon)e^{i\vartheta})|d\vartheta]<\infty, \quad (1)$$

where the upper bound is taken with respect to all admissible ε.

If (1) holds for some conformal mapping, then it will also hold for any other conformal mappings. This fact proves the correctness of Definition 1.

Proposition 2. *If $\Phi(z)\in E^{\vec{p}(\cdot)}(G)$, then for almost all $t\in\Gamma$ there exists a boundary value $\Phi^+(t)$ and the function Φ^+ belongs to the class $L^{\vec{p}(\cdot)}(\Gamma)$.*

By virtue of Propositions 1 and 2, the function Φ^+ belongs to $L^{\vec{p}(\cdot)}(\Gamma)$. Moreover, from the inclusions $E^{\vec{p}(\cdot)}(G)\subset E^1(G)$ it follows that $\Phi(z)$ is representable by a Cauchy integral [5: 73]. We can finally conclude that the following statement is valid.

Theorem 1. *If G is a standard domain with boundary $\Gamma=\bigcup_{k=0}^{m}\Gamma_k$, $p_k(t)\in\mathcal{P}(\Gamma_k)$, $k=\overline{0,m}$, then every function Φ from $E^{\vec{p}(\cdot)}(G)$, $\vec{p}(t)=(p_0(t),p_1(t),\ldots,p_m(t))$, has boundary values $\Phi^+(t)$ for almost all $t\in\Gamma$, $\Phi^+\in L^{\vec{p}(\cdot)}(\Gamma)$, and*

$$\Phi(z)=(K_\Gamma\Phi^+)(z)=\frac{1}{2\pi i}\int_\Gamma\frac{\Phi^+(t)dt}{t-z}, \quad z\in G. \quad (2)$$

4. On the Belonging of Cauchy Type Integrals with a Density from the Class $L^{\vec{p}(\cdot)}(\Gamma)$ to the Class $E^{\vec{p}(\cdot)}(G)$

Definition 2. Let $\Gamma=\bigcup_{k=0}^{m}\Gamma_k$ be the boundary of the domain G and $\vec{p}(t)=(p_0(t),p_1(t),\ldots,p_m(t))$. We will say that a pair $(\Gamma,\vec{p}(\cdot))$ belongs to the class $C^{\vec{p}(\cdot)}$ if for any function $f\in L^{\vec{p}(\cdot)}(\Gamma)$ a Cauchy type integral $(K_\Gamma f)(z)\in E^{\vec{p}(\cdot)}(G)$. If G is a simply connected domain, we will write $(\Gamma,p(\cdot))\in C^{p(\cdot)}$.

If $(\Gamma,\vec{p}(\cdot))\in C^{\bar{p}(\cdot)}$, then the singular Cauchy operator

$$S_{\Gamma_k}:\varphi\to S_{\Gamma_k}\varphi, \quad (S_{\Gamma_k}\varphi)(t)=\frac{1}{\pi i}\int_{\Gamma_k}\frac{\varphi(\tau)d\tau}{\tau-t}, \quad t\in\Gamma_k, \quad \varphi\in L^{p_k(\cdot)}(\Gamma_k), \quad k=\overline{0,m},$$

maps $L^{p_k(\cdot)}(\Gamma_k)$ into itself, thus $\Gamma_k\in R$ [12, p. 101] and therefore Γ_k is a Smirnov curve.

In the case of a simply connected domain *a pair $(\Gamma,p(\cdot))$ belongs to the class $C^{p(\cdot)}$ if:*

I. Γ is a piecewise-smooth curve without zero cusps that bounds the finite domain G and $p(t)\in\mathcal{P}(\Gamma)$ [12: 84];

or

II. Γ is a Lavrent'yev curve and $p(t)$ belongs to the Hölder class and $\underline{p}>1$ [8].

In the case of multiply connected domains we have

Theorem 2. *If* $\Gamma=\bigcup_{k=0}^{m}\Gamma_k$ *is the boundary of a standard domain G and* $\vec{p}(t)=(p_0(t),p_1(t),\dots,p_m(t))$, *then a pair* $(\Gamma,\vec{p}(\cdot))$ *belongs to* $C^{\vec{p}(\cdot)}$ *if pairs* $(\Gamma_j,p_j(\cdot))$ *belong to* $C^{p_j(\cdot)}$, $j=\overline{0,m}$.

5. Expansion Theorem

If a standard domain G is bounded by the curve $\Gamma=\bigcup_{k=0}^{m}\Gamma_k$, then we denote by G_k the domain bounded by Γ_k and containing G. Therefore if Γ_k partitions the plane into domains G_k^+ and G_k^-, then $G_0=G_0^+$, and $G_k=G_k^-$, $k=\overline{1,m}$.

Theorem 3. *Let G be a standard domain with boundary* Γ. *If*

$$\vec{p}(t)=(p_0(t),p_1(t),\dots,p_m(t)),\quad p_k(t)\in\mathcal{P}(\Gamma_k),\quad k=\overline{0,m},\quad (\Gamma,\vec{p}(\cdot))\in C^{\vec{p}(\cdot)},\tag{3}$$

and $\Phi\in E^{\vec{p}(\cdot)}(G)$, then $\Phi(z)$ is representable as

$$\Phi(z)=\Phi_0(z)+\Phi_1(z)+\cdots+\Phi_m(z),\quad \Phi_k(z)\in E^{p_k(\cdot)}(G_k),\quad k=\overline{0,m}.\tag{4}$$

Conversely, if (3) *holds and an analytic function* Φ *in G is representable as* (4)*, then* $\Phi\in E^{\vec{p}(\cdot)}(G)$.

6. Generalization of Smirnov's One Theorem

6.1. Smirnov's Theorem for Classes $E^p(G)$, $p=const$. For constant p the following Smirnov's theorem is well known (see [1]).

If G is a simply connected Smirnov domain with boundary Γ, $\Phi\in E^p(G)$ *and* $\Phi^+\in L^{p_1}(\Gamma)$, $p<p_1$, *then* $\Phi\in E^{p_1}(G)$.

6.2. Smirnov's theorem for classes $E^{p(t)}(G)$ in the case of simply connected domains. Various generalizations of the aforementioned Smirnov's theorem are obtained in [7].

The following assertions are valid:

I. If G is a bounded domain with boundary $\Gamma\in C^1(A_1,\dots,A_n;\nu_1,\dots,\nu_n)$, $0<\nu_k\le 2$, then if $\Phi\in E^{p(\cdot)}(G)$, $\underline{p}>0$, $\Phi^+\in L^{\mu(\cdot)}(\Gamma)$, $\mu\in\mathcal{P}(\Gamma)$, we have $\Phi\in E^{\lambda(\cdot)}(G)$, where $\lambda(t)=\max(p(t),\mu(t))$ ([12, p. 85]).

II. Let a domain G with boundary Γ is such that the conformal mapping $z=z(\zeta)$ of the circle U onto G possesses the properties

$$z'(\zeta)\in\bigcup_{\alpha>1}H^{\alpha},\qquad \frac{1}{z'(\zeta)}\in\bigcup_{\alpha>0}H^{\alpha},\tag{5}$$

where H^{α} are Hardy classes of analytic functions in U. Then if $\Phi(z)\in E^{p(\cdot)}(G)$, $\Phi^+\in L^{\mu(\cdot)}(\Gamma)$ and μ is a function of the Hölder class on Γ, we have $\Phi \in E^{\lambda(\cdot)}(G)$ ([8]).

Note that condition (5) is fulfilled for Lavrent'yev curves (see [15, p.170]).

6.3. Smirnov's theorem in the case of finitely connected domains.

Theorem 4. *If G is a standard domain with boundary* $\Gamma = \bigcup_{j=0}^{m} \Gamma_j$, $\vec{p}(t) = (p_0(t), p_1(t), \dots, p_m(t))$, $p_j(t) \in \mathcal{P}(\Gamma_j)$, $j = \overline{0,m}$, $(\Gamma, \vec{p}(\cdot)) \in C^{\vec{p}(\cdot)}$, $\Phi \in E^{\vec{p}(\cdot)}(G)$, *and* $\Phi^{+} \in L^{\vec{\mu}(\cdot)}(\Gamma)$, *where* $\vec{\mu}(t) = (\mu_0(t), \mu_1(t), \dots, \mu_m(t))$, *then*

$$\Phi(z) = E^{\vec{\lambda}(\cdot)}(G), \qquad \vec{\lambda}(t) = (\lambda_0(t), \lambda_1(t), \dots, \lambda_m(t)),$$

where $\lambda_j(t) = \max(p_j(t), \mu_j(t))$, $j = \overline{0,m}$.

7. On the Convergence of a Sequence from $E^{\vec{p}(\cdot)}(G)$

7.1. On the convergence of a sequence of functions from Smirnov's class $E^{p}(G)$. The following theorem holds true in the case of a bounded simply connected domain *G* with boundary Γ, a rectifiable closed Jordan curve.

Theorem (G. Tumarkin). *If the sequence* $\{f_n(t)\}$ *of boundary values of functions* $f_n(z)$ *from the class* $E^{p}(G)$ *converges in measure on the set* ℓ, $m(\ell) > 0$ (*m is a Lebesgue measure*), *of the boundary* Γ *of the domain G and* $\int_{\Gamma} | f_n(t) |^{p} | dt | < C$, *where C does not depend on n, then the sequence* $\{f_n(z)\}$ *uniformly converges in the domain G to a function* $f(z)$ *of the class* $E^{p}(G)$ *and the sequence* $\{f_n^{+}(t)\}$ *converges in measure on the set* ℓ *to a function* $f^{+}(t)$, *the angular boundary values of a function* $f(z)$ (see [14, p. 268]).

7.2. The case of a simply connected domain and a variable exponent.

Theorem 5. *If* 1) $(\Gamma, p(\cdot)) \in C^{p(\cdot)}$, $p(t) \in \mathcal{P}(\Gamma)$; 2) *the sequence* $\{\Phi_n(\zeta)\}$, $\zeta \in \Gamma$, *converges in measure on* $\ell \in \Gamma$, $m(\ell) > 0$; 3) $\Phi_n(z) \in E^{p(\cdot)}(G)$ *and there exists a number C not depending on n, such that*

$$\sup_{n} C(\Phi_n) = \sup_{n} \sup_{0<r<1} \int_0^{2\pi} | \Phi_n(z(re^{i\vartheta})) |^{p(z(e^{i\vartheta}))} | z'(r(e^{i\vartheta})) | d\vartheta = C. \tag{6}$$

Then the sequence $\{\Phi_n(z)\}$ uniformly converges in *G* to a function $\Phi \in E^{p(\cdot)}(G)$ and $\{\Phi_n(\zeta)\}$ converges in measure on ℓ to a function $\Phi^{+}(\zeta)$.

7.3. The case of a finitely connected domain.

Theorem 6. *If* 1) $\Gamma = \bigcup_{k=0}^{m} \Gamma_k$ *is the boundary of a standard domain G;* 2) $p_k(t) \in \mathcal{P}(\Gamma_k)$ *and* $\vec{p}(t) = (p_0(t), p_1(t), \dots, p_m(t))$; 3) $(\Gamma_k, p_k(\cdot)) \in C^{p_k(\cdot)}$, $k = \overline{0,m}$; 4) $\Phi_n(z) \in E^{\vec{p}(\cdot)}(G)$ *and there exists a number C not depending on n, such that*

$$C(\Phi_n) < C;$$

5) the sequence $\{\Phi_n(\zeta)\}$ *converges in measure on a set* $\ell \subset \Gamma$ *such that* $m(\ell_k) > 0$, *where* $\ell_k = \ell \cap \Gamma_k$, $k = \overline{0,m}$, *then* $\{\Phi_n(z)\}$ *uniformly converges in G to a function* $\Phi(z) \in E^{\vec{p}(\cdot)}(G)$ *and the sequence* $\{\Phi_n(\zeta)\}$ *converges in measure on* ℓ *to a function* $\Phi^+(\zeta)$.

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ანალიზურ ფუნქციათა სმირნოვის ცვლადმაჩვენებლიანი კლასები მრავლადბმულ არეებში

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(წარმოდგენილია აკადემიკოს ვ. კოკილაშვილის მიერ)

ვთქვათ, $\Gamma_0, \Gamma_1, \ldots, \Gamma_m$ ერთმანეთის გარეთ მდებარე მარტივი, შეკრული გაწრფევადი წირებია, რომელთაგან $\Gamma_1, \ldots, \Gamma_m$ მდებარეობენ Γ_0-ის შიგნით, ხოლო $\vec{p}(t) = (p_0(t), p_1(t), \ldots, p_m(t))$, სადაც $p_k(t)$, $k = \overline{0,m}$ არის Γ_k-ზე განსაზღვრული ზომადი ფუნქცია. ნაშრომში შემოღებულია $\Gamma = \bigcup_{k=0}^{m} \Gamma_k$ წირით შემოსაზღვრულ მრავლადბმულ G არეში ანალიზურ ფუნქციათა სმირნოვის ცვლადმაჩვენებლიანი $E^{\vec{p}(\cdot)}(G)$ კლასები. დადგენილია ამ კლასების ფუნქციათა რიგი თვისებები, მათ შორის, გაშლის თეორემა, კოშის ინტეგრალით წარმოდგენადობა, განზოგადებულია მარტივად ბმული არისა და მუდმივი p მაჩვენებლის შემთხვევისათვის ცნობილი სმირნოვისა და ტუმარკინის თეორემები მრავლადბმული არის შემთხვევისთვის. განსაკუთრებული ყურადღება ეთმობა $L^{\vec{p}(\cdot)}(\Gamma)$ სიმკვრივის მქონე კოშის ტიპის ინტეგრალის $E^{\vec{p}(\cdot)}(G)$ კლასისადმი მიკუთვნების საკითხს.

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