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Informatics

A New Fuzzy Model of the Vehicle Routing Problem for Extreme Conditions

Gia Sirbiladze*, Bezhan Ghvaberidze*, Bidzina Matsaberidze*

* *Faculty of Exact and Natural Sciences, I. Javakhishvili Tbilisi State University, Tbilisi*

(Presented by Academy Member Mindia Salukvadze)

ABSTRACT. In the extreme and uncertainty conditions the difficulty of vehicle movement between different customers cause the imprecision of time of movement and the uncertainty of feasibility of movement. In this paper this uncertainty is presented by a possibility distribution. A new multiple criteria fuzzy optimization approach for the solution of the vehicle routing problem is constructed. A new subjective criterion – maximization of feasibility of movement on closed routes is constructed. The problem is reduced on the min-max bicriteria fuzzy partitioning problem for the so called promising routes. For the numerical solution of the scaling model Christofides exact algorithm is realized. To illustrate the results of the constructed new fuzzy approach a numerical example is presented.

Key words: *vehicle routing problem, multiple-criteria combinatorial optimization, fuzzy partitioning problem, possibility theory, Choquet integral*

Route distribution planning problems, also known as Vehicle Routing Problems (VRP), were thoroughly studied in a variety of areas, such as Operations Research, Artificial Intelligence, etc. The standard VRP was originally introduced by Dantzig and Ramser (1959) and is NP–hard, which is a complex combinatorial optimization problem [1]. Several variants of the basic problem were put forward and strong formulations were proposed. Most of these problems can be modeled as linear programming problems. The most common solution techniques are exact methods that guarantee finding an optimal solution, if it exists. These approaches are also applied together with numerous heuristics solution techniques developed with enough flexibility in optimization systems and can be adapted to various practical contexts.

The literature offers very little in terms of modeling VRP proposals, both from the standpoint of the solutions modeled as Fuzzy Mathematical Programming, and Fuzzy VRP (FVRP). Specifically, if we look at the FVRP models in the literature, the majority only assume vagueness for some of the following elements that are described in the model: 1) Fuzzy demands: customer demand is an imprecise variable and 2) Fuzzy times: service time and travel time can be imprecise variables. All of the above VRP can be formalized as problems of combinatorial optimization. An overview of literature in recent years for the FVRP and the ways of its solutions are presented in [2]. The systems approach and analysis play determining role in the vehicle routing problem (VRP) ([3-10] and others). The classical VRP is developed by many well-known authors ([5] and others). Here we present a new vision of the fuzzy vehicle routing problem, which is different from the approaches given in other researches. This new problem is connected with difficulties of optimal routing of vehicles in different extreme situations.

Construction of Bicriteria Partitioning Model for the VRP

We consider the following problem of optimal routing of vehicles as a main problem in extreme situations. Let the set of geographical points (customers) $I = \{1, 2, \ldots, n+1\}$ be given, where $n+1$-th node is depot. Other customers are supplied from depot by vehicles with uniform goods. The demand for goods from customers is known as well as the maximum load and mileage of the vehicles. The problem is the following (first criterion): *It is necessary to deliver the demanded goods to customers by vehicles in such a way that total distance traveled by vehicles to be minimal.* It is meant that the demand for goods by each customer is much less than the maximum load of vehicles.

Suppose $C = \| c_{ij} \|$, $i, j \in I$ is a matrix of positive real numbers and represents the distances between customers; Q and D are real numbers – the constraints of load and mileage of vehicles. P_i, $i = 1, 2, \ldots, n$ are also real numbers and represent the demand for goods by *i*-th customer $1 \le P_i < Q$, $i = 1, 2, \ldots, n$.

We have to find such closed set of routes $\bar{M} = \{ M_k \}$, $k = 1, 2, \ldots, m$, $M_k = \{ n+1, i_1^k, \ldots, i_{\ell_k}^k, n+1 \}$, $i_j^k \in \{ 1, 2, \ldots, n \}$, $j = 1, \ldots, \ell_k$, $1 \le \ell_k \le n$ (m and ℓ_k are not fixed in advance) that satisfy the constraints

$$\bigcup_{k=1}^{m} M_k = I; \quad M_k \cap M_q = \{ n+1 \},$$

$$k, q \in \{ 1, 2, \ldots, m \}, \quad k \ne q; \qquad \sum_{j=1}^{\ell_k} P_{i_j^k} \le Q; \tag{1}$$

$$c_k = c_{n+1, i_1^k} + c_{i_{\ell_k}^k, n+1} + \sum_{j=1}^{\ell_k - 1} c_{i_j^k, i_{j+1}^k} \le D, \quad k = 1, 2, \ldots, m;$$

and that have minimal total travel distance (objective function, first criterion)

$$\min \to \sum_{k=1}^{m} c_k = \sum_{k=1}^{m} \left(c_{n+1, i_1^k} + c_{i_{\ell_k}^k, n+1} + \sum_{j=1}^{\ell_k - 1} c_{i_j^k, i_{j+1}^k} \right). \tag{2}$$

For the vehicle routing problem represented in this paper, when discussing extreme conditions on roads, exact or stochastic models may not work because some input parameters are unknown due to their vague nature. These parameters include transport movement time, representing a total value of customer-to-customer movement. The route turns to be feasible if the difference between the expected travel time and the planned one is minimal. Simply, delay time is the lowest. Therefore, it is planned to construct the second, subjective criterion within the framework of the paper – to maximizing the feasibility of vehicle movement removing on the routes.

A little about using possibility theory [11] in formation of the vehicle routing schemes: if the vehicle is in customer i of an allowable route, let us denote a set of the neighboring movement customers by $X^{(i)}$. What is a possibility of vehicle movement from customer i to customer j in planned approximate timeframe-$\tilde{t}_{ij}$?

Let us denote this value by π_{ij}, $0 \le \pi_{ij} \le 1$. $\left\{ \pi_{ij} \right\}_{j \in X^{(i)}}$ is a conditional possibility distribution on the set $X^{(i)}$. According to the possibility theory, possibility of any event is maximal among its supportive elementary events.

$$Pos\ (A) = \max_{j \in A} \pi_{ij}, \quad \forall\ A \subseteq X^{(i)},$$

The difficulty of movement between different customers and other problems cause the uncertainty and imprecision of vehicle movements on the routes. Suppose, that the expert evaluation of the movement time from customer *i* to customer *j* is represented by the positive triangular fuzzy number $\tilde{t}_{ij} = \left(t_{ij}^{(1)}, t_{ij}^{(2)}, t_{ij}^{(3)}\right)$ ([11], etc.), the membership function of which is defined by formula:

$$\mu_{\tilde{t}_{ij}}(t) = \begin{cases} 0, \quad t \le t_{ij}^{(1)}; \\ \dfrac{t - t_{ij}^{(1)}}{t_{ij}^{(2)} - t_{ij}^{(1)}}, \quad t_{ij}^{(1)} < t \le t_{ij}^{(2)}; \\ \dfrac{t_{ij}^{(3)} - t}{t_{ij}^{(3)} - t_{ij}^{(2)}}, \quad t_{ij}^{(2)} < t \le t_{ij}^{(3)}; \\ 0, \quad t > t_{ij}^{(3)}. \end{cases} \tag{3}$$

The possibilistic mean and standard deviation of a triangular fuzzy number is defined as:

Definition 1:

a) The possibilistic mean value of a triangular fuzzy number $\tilde{c} = (c_1, c_2, c_3)$ -

$E^{Pos}(\tilde{c}) = c_2 + (c_3 - 2c_2 + c_1)/6$

b) The possibilistic standard deviation of $\tilde{c}$ -

$\sigma^{(Pos)}(\tilde{c}) = \sqrt{Var^{(Pos)}(\tilde{c})} = \sqrt{(c_3 - c_2)^2 + (c_2 - c_1)^2 + (c_3 - c_2)(c_2 - c_1)}\,/\,(3\sqrt{2})$.

Let $\tilde{T} = \left\{\tilde{t}_{ij}\right\}$ be the matrix of fuzzy triangular numbers - times of movement from customer i to customer j and $\Pi = \left\{\pi_{ij}\right\}$ be the matrix of possibility levels of movements between $i \to j$ customers.

It is obvious that in extreme conditions the feasibility of vehicle movement on routes is inversely proportional to the mean value of the movement time between customers - $E^{(Pos)}\left(\tilde{t}_{ij}\right)$ and also inversely proportional to the standard deviation of the same quantity – $\sigma^{(Pos)}\left(\tilde{t}_{ij}\right)$. Therefore it is inversely proportional to the product of these values - $E^{(Pos)}\left(\tilde{t}_{ij}\right) \cdot \delta^{(Pos)}\left(\tilde{t}_{ij}\right)$. Let us introduce a nonnegative fuzzy quantity "Small product" on the interval $[x_1; x_2]$, where $x_1 = \min\{E^{(Pos)}\left(\tilde{t}_{ij}\right) \cdot \delta^{(Pos)}\left(\tilde{t}_{ij}\right)\}$ and $x_2 = \max\{E^{(Pos)}\left(\tilde{t}_{ij}\right) \cdot \delta^{(Pos)}\left(\tilde{t}_{ij}\right)\}$. The graph of a "Small product" is a parabola of the function $y = ax^2 + bx + c$, where unknown parameters a, b, c are defined from the system of equations:

$$\begin{cases} -b/(2a) = x_1 \\ (4ac - b^2)/(4a) = 1 \\ ax_2^2 + bx_2 + c = 0\ . \end{cases}$$

Note: as calculations showed, choosing "small product" function does not have significant impact on the results of the problem. The main requirement is that it should be non-increasing function with values from 1 to 0 in $[x_1; x_2]$ interval.

Definition 2:

A feasibility level of movement from the customer i to the customer j is the value of "Small product" for the argument $E^{(Pos)}\left(\tilde{t}_{ij}\right)\cdot\delta^{(Pos)}\left(\tilde{t}_{ij}\right)$:

$$\delta_{ij} = Small\ product\left(E^{(Pos)}\left(\tilde{t}_{ij}\right)\cdot\delta^{(Pos)}\left(\tilde{t}_{ij}\right)\right). \tag{4}$$

Let us assume $M=\{n+1,i_1,i_2,...,i_k,n+1\}$ is some closed route and let us introduce the set of states (arcs) on the route M:

1	2	-	k
$n+1\longrightarrow i_1$	$i_1\longrightarrow i_2$	...	$i_k\longrightarrow n+1$

.

We can consider the possibility distribution on the states of the closed route M and feasibility levels of the vehicle movement on the closed route M:

$$\begin{pmatrix} 1 & 2 & ... & k+1 \\ \pi_1^0=\pi_{n+1,i_1}^0 & \pi_2^0=\pi_{i_1,i_2}^0 & ... & \pi_{k+1}^0=\pi_{i_k,n+1}^0 \\ \delta_1=\delta_{n+1,i_1} & \delta_2=\delta_{i_1,i_2} & ... & \delta_{k+1}=\delta_{i_k,n+1} \end{pmatrix}, \tag{5}$$

where $\pi_{ij}^0=\pi_{ij}/\left(\max\limits_{p,q\in M}\pi_{pq}\right)$.

For the construction of a feasibility of the movement on the closed route we use Choquete averaging aggregation operator [7].

Definition 3: A feasibility level of the movement on the closed route M is a Choquet integral of degrees $\delta_M\equiv\{\delta_1,\delta_2,...,\delta_{k+1}\}$ with respect to possibility distribution $\pi_M\equiv\{\pi_1^0,\pi_2^0,...,\pi_{k+1}^0\}$:

$$\begin{aligned} &feasibility(M)=(C)\int\delta_M\,d\left(\pi_M\right)= \\ &\int_0^1 Pos\left(\delta_M\geq\alpha\right)d\alpha=\sum_{j=1}^{k+1}[Pos\left(\{i(1),..i(j)\}\right)-Poss\left(\{i(1),..i(j-1)\}\right)]\times\delta_{i(j)} \\ &=\sum_{j=1}^{k+1}\left[\max_{l=1,j}\left(\pi_{i(l)}^0\right)-\max_{k=1,j-1}\left(\pi_{i(l)}^0\right)\right]\times\delta_{i(j)}, \end{aligned} \tag{6}$$

where $\delta_{i(1)}\geq\delta_{i(2)}\geq...\geq\delta_{i(k+1)}$ is the non-increasingly rearranged inversion of quantities δ_M; $\pi_{i(0)}\equiv 0$. Let $c\left(M_j\right)$ be the length of route M_j and $feasibility\left(M_j\right)$ - degree of feasibility of movement on M_j, for all admissible closed routes $\overline{M}=\{M_1,M_2,...,M_m\}$. For arbitrary chosen subset of routes $M'\subset\overline{M}$, let us introduce Boolean vector $\overline{x}=\{x_1,x_2,...,x_m\}$, where $\left(x_j=\begin{cases}1, if\ M_j\in M' \\ 0, if\ M_j\notin M'\end{cases}\right), j=1,...,m$. Then the feasibility measure of movement on arbitrary selected routes is defined as: $\sum\limits_{j=1}^{m} feasibility\left(M_j\right)\cdot x_j$ and

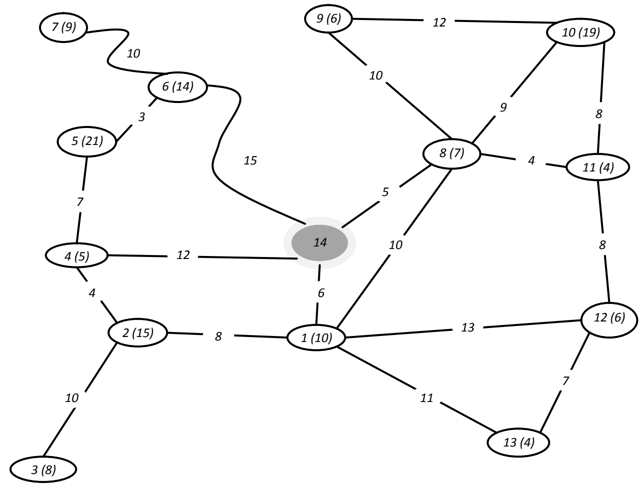

Fig. 1. Customers distribution network.

the measure of total length of the selected routes is - $\sum_{j=1}^{m} c\left(M_j\right)\cdot x_j$. Let us now define $\min-\max$ type bicriteria partitioning problem on $\overline{M}$. This problem considers the partitioning of closed routes, which satisfies two criteria: total length is minimal and feasibility of movement on these routes is maximal:

$$f_1 = \sum_{j=1}^{m} c(M_j)\cdot x_j \Rightarrow \min$$

$$f_2 = \sum_{j=1}^{m} feasibility(M_j)\cdot x_j \Rightarrow \max$$

$$\sum_{j=1}^{m} a_{ij}x_j = 1,\ \ i = 1,\dots,n;$$

$$x_j \in \{0,1\},\ j = 1,\dots,m, \tag{7}$$

where $a_{ij} = 1$, if customer belongs to admissible route M_j , otherwise $a_{ij} = 0$; $i = 1,\dots,n;\quad j = 1,\dots,m.$

Numerical Example

Our approach to solving the constructed bicriteria partitioning problem (7) belongs to the other specific heuristics as a two stage approach strategy (by the Laporte [12] classification).

First stage: At the first stage the so called “promising” routes are constructed. In practice, building all promising routes in the first stage of the algorithm is impossible because of their large number in real problems. Therefore we are considering only a limited number of promising routes based on heuristic arguments. We make this selection using the specially created algorithm based of “constructive” approach [13, 14]. In this algorithm construction of promising routes is performed by the analysis of individual customers’ demand values, their geographical locations and defined limits on the maximum route length and load capacity of the vehicles.

Second stage: We solve the bicriteria type partitioning problem (7) for the selected promising closed

Table 1 – Distance matrix (kms)

10	15	8	5	21	14	9	7	6	19	4	6	4		
	8						10				13	11	6	1
8		10	4											2
	10													3
	4			12									12	4
			12		3									5
				3		10							15	6
					10									7
10								10	9	4			5	8
							10		12					9
							9	12		8				10
							4		8		8			11
13										8		7		12
11											7			13
6			12		15		5							14

Table 2 – The possibility levels matrix

	0.8						0.8				0.6	0.8	1	1
0.8		0.7	0.9											2
	0.7													3
	0.9			0.8									1	4
			0.8		0.7									5
				0.7		0.9							1	6
					0.9									7
0.8								0.3	0.6	0.5			1	8
							0.3		0.9					9
							0.6	0.9		0.5				10
							0.5		0.5		0.7			11
0.6										0.7		0.6		12
0.8											0.6			13
1			1		1		1							14

Table 3 – The travel times matrix (minutes)

	3,6,8						6,8,9				5,8,10	4,6,8	2,4,5	1
3,6,8		5,7,8	5,7,8											2
	5,7,8													3
	5,7,8			6,7,8									2,5,6	4
			6,7,8		4,5,6									5
				4,5,6		6,7,9							5,8,10	6
					6,7,9									7
6,8,9								4,6,8	4,5,6	4,5,6			2,4,6	8
							4,6,8		5,7,9					9
							4,5,6	5,7,9		4,6,7				10
							4,5,6		4,6,7		3,5,7			11
5,8,10										3,5,7		2,6,7		12
4,6,8											2,6,7			13
2,4,5			2,5,6		5,8,10		2,4,6							14

routes when the number of selected rational closed routes is much less than the real large number of admissible routes. The *scaling approach* is used for the numerical solution of the problem (7) for promising routes. For the scaling model:

$$\begin{aligned} &\alpha f_1 + (1-\alpha)(-f_2) \Rightarrow \ \min \\ &\sum_{j=1}^{k} a_{ij} x_j = 1, \ i = 1,\dots,n; \\ &x_j \in \{0,1\}, \ j = 1,\dots,m \end{aligned} \tag{8}$$

Table 4 – Optimal routes

Weight Parameter α	Optimal Routes in Scaling Model
Feasibility only ($\alpha = 0$) Total distance = 245 Feasibility level = 4.8	Vehicle 1: 14, 8, 11, 12, 13, 1, 14 Vehicle 2: 14, 2, 4, 5, 6, 14 Vehicle 3: 14, 9, 14 Vehicle 4: 14, 3, 14 Vehicle 5: 14, 7, 14 Vehicle 6: 14, 10, 14
$\alpha = 0.1, 0.2, 0.3$ Total distance = 318 Feasibility level = 7.87	Vehicle 1: 14, 8, 11, 12, 13, 1, 14 Vehicle 2: 14, 2, 4, 3, 2, 14 Vehicle 3: 14, 9, 10, 14 Vehicle 4: 14, 7, 14 Vehicle 5: 14, 6, 5, 14
$\alpha = 0.4, 0.5, 0.6$ Total distance = 318 Feasibility level = 8.73	Vehicle 1: 14, 8, 11, 12, 13, 1, 14 Vehicle 2: 14, 4, 3, 2, 14 Vehicle 3: 14, 9, 10, 14 Vehicle 4: 14, 7, 6, 14 Vehicle 5: 14, 5, 14
$\alpha = 0.7, 0.8, 0.9$ and $\alpha = 1$ (Distance only) Total distance = 218 Feasibility level = 3.36	Vehicle 1: 14, 8, 11, 12, 13, 1, 14 Vehicle 2: 14, 4, 3, 2, 14 Vehicle 3: 14, 9, 10, 14 Vehicle 4: 14, 7, 14 Vehicle 5: 14, 6, 5, 14

classical minimal partitioning problem is constructed. Where α, $0 \le \alpha \le 1$, is a weighted parameter and k is a number of selected promising routes. For the solving of the minimal partitioning problem (8) well known Christofides exact algorithm is used [15].

Example: For the illustration of the results we present the following customers distribution network (Fig. 1). It has 13 customers, labeled “1”, “2”, …, “13”, and a depot, labeled “14”. The amount of demand at each customer and the distances between customers (and the depot) are given in Table 1. The possibility levels matrix is presented in Table 2. The travel times are triangular fuzzy numbers shown in Table 3. Demands of each customer are presented in the first row of Table 1. A capacity of each vehicle is 55 units. Maximal travel distance for the vehicle is 55 km. These are all input data of the new FVRP model. The output data (optimal routes) shows the presence of extreme conditions on the roads. The solutions for different weight parameter α are shown in Table 4.

As shown in Table 4, we get different solutions for the scaling model (8), when $\alpha = 0$ (feasibility criterion domination) and $\alpha = 1$ (distance criterion domination). As we can observe solutions remain stable for intervals of α .

Conclusion

A new multiple criteria optimization model for the solution of the optimal vehicle routing problem is considered. Two stage scheme is used for the numerical solution of the FVRP. This problem is reduced on the min-max bicriteria fuzzy partitioning problem. At the first stage based on the constructive approach the so called “promising” closed routes are constructed. On the second stage we solve the bicriteria partitioning problem

(7) for the selected promising closed routes when the number of selected rational closed routes is much less than the real large number of admissible routes. The scaling model (8) is used for the numerical solution of the problem (7) for promising routes. A numerical example is considered for the illustration of the results of the new fuzzy approach in FVRP.

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გ. სირბილაძე,* ბ. ღვაბერიძე,* ბ. მაცაბერიძე*

**ი. ჯავახიშვილის სახ. თბილისის სახელმწიფო უნივერსიტეტი, ზუსტ და საბუნებისმეტყველო მეცნიერებათა ფაკულტეტი, თბილისი*

(წარმოდგენილია აკადემიის წევრის მ. სალუქვაძის მიერ)

სატრანსპორტო მარშრუტიზაციის ამოცანების გადაწყვეტა განსაკუთრებით დიდ მნიშვნელობას იძენს ბოლო დროს გზებზე შექმნილი გართულებული სიტუაციების გამო: ტრანსპორტით გადატვირთული გზები; ექსტრემალური მოვლენები, როგორიცაა: კატასტროფები, მიწისძვრები, სხვადასხვა მიზეზებით გამოწვეული ცუდი ხილვადობა გზებზე და სხვ. ყოველივე ეს ართულებს სატრანსპორტო მარშრუტიზაციის ამოცანების გადაწყვეტას. ასეთ შემთხვევაში გზებზე სატრანსპორტო საშუალებებით გადაადგილების მინიმიზაციის ამოცანასთან ერთად ჩნდება ისეთი კრიტერიუმი, როგორიცაა მარშრუტებზე გადაადგილების განხორციელების შესაძლებლობის მაქსიმიზაცია. ამ უკანასკნელის აგება დაკავშირებულია საექსპერტო ცოდნის ფორმირებისა, ინჟინერიის მეთოდებსა და ტექნოლოგიებთან. მეორე კრიტერიუმი წარმოშობს სატრანსპორტო მარშრუტიზაციის ბიკრიტერიალურ ამოცანას. როგორც ცნობილია, კარგადაა შესწავლილი კლასიკური სატრანსპორტო მარშრუტიზაციის ამოცანები, რომლებიც, სტაციონარული პირობების შეზღუდვებში ძირითადად გზებზე სატრანსპორტო საშუალებებით გადაადგილების მინიმიზაციის ამოცანის გადაწყვეტას გულისხმობს. ამოცანა დისკრეტული ოპტიმიზაციის ამოცანათა კლასს მიეკუთვნება და NP -სირთულისაა. დიდი განზომილებების შემთხვევაში მის გადასაწყვეტად შექმნილია ევრისტიკული თუ სხვა ტიპის მიახლოებითი მიდგომები. ნაშრომში განხილული ამოცანის რეალიზაცია წარმოადგენს ჩვენ მიერ შექმნილი ფაზი-სატრანსპორტო მარშრუტიზაციის ამოცანის გადაწყვეტის ახალ მიდგომას. ექსპერტული მონაცემების ბაზაზე შექმნილი მეთოდოლოგია დაფუძნებულია შესაძლებლობის თეორიის ბოლო შედეგებზე. სტატიაში განხილულია მინ-მაქს ტიპის ბიკრიტერიალური სატრანსპორტო მარშრუტიზაციის ამოცანა ექსტრემალური გარემოსთვის.

აგებულია ახალი სუბიექტური კრიტერიუმი - ჩაკეტილ გზებზე გადაადგილების განხორციელების შესაძლებლობის მაქსიმიზაცია. კრიტერიუმის ასაგებად გამოყენებულია შოკეს სასრული ინტეგრალი. წარმოდგენილია ამ ამოცანის რეალიზაციის ორეტაპიანი სქემა. პირველ ეტაპზე კონსტრუქციული მიდგომით ფორმირდება ე.წ. „იმედის მომცემი“ ჩაკეტილი მარშრუტები. მეორე ეტაპზე შერჩეული „იმედის მომცემი“ ჩაკეტილი მარშრუტებისთვის ფორმირებული მინ-მაქს ტიპის ბიკრიტერიალური კომბინატორული ოპტიმიზაციის ამოცანა, რომელიც სკალირებულ მოდელში დაყვანილია კლასიკურ მინიმალური დაფარვის ამოცანაზე. ამ უკანასკნელის რიცხვითი ამოხსნისთვის გამოყენებულია შტოებისა და საზღვრების ზუსტი ალგორითმი. შექმნილია შესაბამისი პროგრამული უზრუნველყოფა. ახალი მიდგომის შედეგების შესაფასებლად განხილულია რიცხვითი მაგალითი. მიღებული შედეგები აჩვენებს, რომ სადისტრიბუციო ქსელებზე განუზღვრელობისა და ექსტრემალური სიტუაციების წარმოშობის პირობებში ოპტიმალური მარშრუტები იცვლება იმ ხარისხით, რა ხარისხითაც განუზღვრელობა წარმოადგენს სატრანსპორტო საშუალებების გზებზე დროული გადაადგილების ხელის შემშლელ ფაქტორს.

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