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Physics

Behaviour of Superfluid ^{3}He Polar and ABM Phases in Presence of Globally Stretched Aerogel

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ABSTRACT. The superfluid ^{3}He phases in the presence of uniaxially deformed aerogel are under an active investigation. The most valuable information on the properties of superfluid phases of ^{3}He is contained in the time-averaged and rapidly time-oscillating contributions to the dipole-dipole potential. In globally stretched aerogel the behavior of the Polar and ABMU(1)LIM models of superfluid ^{3}He are compared. In the time-averaged approximation these states look similar and apparently cannot be easily discriminated in the experiments exploring the pulsed NMR spin dynamics. On the other hand, our theoretical analysis of the spectrum of high-frequency spin oscillations superimposed on the time-averaged spin dynamics shows the pronounced difference between the behavior of the Polar and ABMU(1)LIM states. © *2015 Bull. Georg. Natl. Acad. Sci.*

Key words: *superfluidity of ^{3}He, properties of Polar and ABM phases.*

The behaviour of the superfluid ^{3}He in presence of uniaxially deformed aerogel is under the active investigation. Of a special interest is the possibility of the appearance of the Polar phase, which is not found among the stable bulk superfluid states.

In Ref. [1] it was argued that the Polar phase should be accommodated by an axially stretched aerogel in a narrow temperature stripe in the immediate vicinity of the transition to the superfluid state.

Supposedly the presence of the Polar phase has been found in Ref. [2] in the pulsed NMR regime, where the dipole shift of the NMR frequency from the Larmor value $\omega_L = gH$ was detected. On the other hand, it was argued that alternatively the same pulsed NMR data can be attributed to the ABM phase in axially stretched aerogel described by Volovik U(1)LIM model [3]. The firm discrimination between mentioned possibilities is obscured by the lack of reliable information about the Leggett dipole frequencies Ω_P and Ω_A of the Polar and ABM phases in "nematically ordered" aerogel used in Ref. [2].

In what follows our main concern will be to search for the possibilities to discriminate between the Polar and ABMU(1)LIM phases. We shall address to the case of a strong magnetic field $(\omega_L >> \Omega)$ where the dipole-dipole (spin-orbital) potential of considered states can be represented as the sums of the time-averaged (Van der Pol) and rapidly oscillating parts. It can be shown that in the Van der Pol approximation the results given for the dipole frequency shift (in the pulsed NMR regime) for the Polar and ABMU(1)LIM states are reproduced. In addition it is easy to show that the stability criterion of the coherent spin dynamics are identical for the Polar and ABMU(1)LIM states. This means that in the time-averaged approximation the firm discrimination between the two states is hardly possible. Finally we are left with the hope to find out the sufficiently pronounced difference between the rapidly time-oscillating parts which could discriminated between the Polar and ABMU(1)LIM states.

Analysis of the Rapidly Time-Oscillating Parts of the Polar and ABMU(1)LIM States

A. In order to realize the attempts to compare the rapidly oscillating parts of the dipole-dipole potentials of the Polar and ABMU(1)LIM states we are going to consider first the ABM phase in the presence of an axially stretched aerogel environment. Here we refer to the results obtained in Ref. [4]. According to this model the dipole-dipole potential is given as

$$U_D^{(A)} = -\frac{1}{2}\chi(\Omega_A / g)^2 \left(\hat{d}\cdot\hat{l}\right)^2, \quad \vec{H} = H\,\hat{z}, \tag{1}$$

where

$$\begin{aligned} &\hat{d} = \vec{R}\,(\alpha,\beta,\gamma)\,\hat{d}_0\,, \qquad \left(\hat{d}_0 = \hat{x}\right), \\ &\hat{l} = \left(\hat{\xi}\cos\Phi_l + \hat{\eta}\sin\Phi_l\right)\sin\lambda + \hat{\zeta}\cos\lambda\,. \end{aligned} \tag{2}$$

According to ABMU(1)LIM model the orbital $\hat{l}$ –axis is confined to (ξ,η) -plane. In keeping the magnetic field in (ξ,ζ) -plane, being inclined by an angle θ with respect to aerogel global deformation axis $\hat{\zeta} = \hat{\xi}\,x\,\hat{\eta}$, it is concluded that

$$\begin{cases} \hat{x}\cdot\hat{\xi} = \cos\theta, \\ \hat{x}\cdot\hat{\eta} = 0, \end{cases} \quad \begin{cases} \hat{y}\cdot\hat{\xi} = 0, \\ \hat{y}\cdot\hat{\eta} = 1, \end{cases} \quad \begin{cases} \hat{z}\cdot\hat{\xi} = \sin\theta, \\ \hat{z}\cdot\hat{\eta} = 0, \end{cases} \tag{3}$$

and

$$\hat{d}\cdot\hat{l} = d_x\cos\Phi_l\cos\theta + d_y\sin\Phi_l + d_z\cos\Phi_l\sin\theta\,. \tag{4}$$

According to Eq. (4) it is found that

$$\begin{aligned} &f_A = \left(\hat{d}\cdot\hat{l}\right)^2 = d_x^2\cos^2\Phi_l\cos^2\theta + d_y^2\sin^2\Phi_l + d_z^2\cos^2\Phi_l\sin^2\theta + \\ &d_x d_y\sin 2\Phi_l\cos\theta + d_x d_z\cos^2\Phi_l\sin 2\theta + d_y d_z\sin 2\Phi_l\sin\theta \end{aligned}\,. \tag{5}$$

In what follows the ABMU(1)LIM model will be adopted which is characterized by the spatial averages

$$\left\langle\sin^2\Phi_l\right\rangle = \left\langle\cos^2\Phi_l\right\rangle = \frac{1}{2}, \quad \left\langle\sin 2\Phi_l\right\rangle = 0\,. \tag{6}$$

After having performed these averaging it is found that

$$\begin{aligned} &f_A = \left(\hat{d}\cdot\hat{l}\right)^2 = \frac{1}{2}\left(d_x^2\cos^2\theta + d_y^2 + d_z^2\sin^2\theta + d_x d_z\sin 2\theta\right) = \\ &= \frac{1}{2}\left[\hat{d}^2 - \left(d_x\sin\theta - d_z\cos\theta\right)^2\right] = -\frac{1}{2}\left(d_x\sin\theta - d_z\cos\theta\right)^2 + const \end{aligned}\,. \tag{7}$$

After some trigonometry it is concluded that

$$\begin{aligned} f_A = -1/8\ \{ & 2\sin^2\beta(1+\cos 2\gamma)+\left[-1+3\cos^2\beta+1/2\ (1+\cos\beta)^2\cos 2(\alpha+\gamma)+\right.\\ & \left.+1/2(1-\cos\beta)^2\cos 2(\alpha-\gamma)-\sin^2\beta(\cos 2\alpha+3\cos 2\gamma)\right]\sin^2\theta+ \\ & +\sin\beta\left[2\cos\beta\cos\alpha+(1+\cos\beta)\cos(\alpha+2\gamma)-(1-\cos\beta)\cos(\alpha-2\gamma)\right]\sin 2\theta\} \end{aligned} \quad . \tag{8}$$

According to the Leggett spin-dynamics at $S = S_0 = \chi\ H/g$ the combination $\alpha+\gamma=\Phi$ is a slow variable and in the strong magnetic field case $(\omega_L = gH >> \Omega_A)$ the spin-orbital function f_A can be decomposed as

$$f_A = \bar{f}_A + \tilde{f}_A(t)\ , \tag{9}$$

where in the time-averaged (Van der Pol) approximation

$$\bar{f}_A(\beta,\Phi) = -\frac{1}{8}\left\{2\sin^2\beta+\left[-1+3\cos^2\beta+\frac{1}{2}(1+\cos\beta)^2\cos 2\Phi\right]\sin^2\theta\right\} \tag{10}$$

and

$$\begin{aligned} & \tilde{f}_A(t) = \\ & = -1/8\ \{2\sin^2\beta\cos 2\gamma+\left[1/2(1-\cos\beta)^2\cos 2(\alpha-\gamma)-\sin^2\beta(\cos 2\alpha+3\cos 2\gamma)\right]\sin^2\theta+ \\ & +\sin\beta\left[2\cos\beta\cos\alpha+(1+\cos\beta)\cos(\alpha+2\gamma)-(1-\cos\beta)\cos(\alpha-2\gamma)\right]\sin 2\theta\} \end{aligned} \tag{11}$$

The stationary value $\Phi=\Phi_{st}$ minimizes $\bar{U}_D^{(A)}(\beta,\Phi)$. This is achieved by maximizing $\bar{f}_A(\beta,\Phi)$ and gives $\Phi_{st} = (\pi/2,\ 3\pi/2)$. In this way it is established that

$$\bar{U}_D^{(A)}(\beta,\Phi_{st}) = 1/4\chi(\Omega_A/g)^2\left\{1/2\sin^2\beta-1/4\left[3/2+\cos\beta-5/2\cos^2\beta\right]\sin^2\theta\right\}. \tag{12}$$

In order to calculate the dipole shift $\delta\,\omega(\beta)$ from the Larmor value we use an equation

$$\delta\,\omega(\beta) = -1/S\,\frac{\partial\bar{U}_D^{(A)}(\beta,\Phi_{st})}{\partial\cos\beta} \tag{13}$$

from which it is found that

$$\delta\,\omega(\beta) = 1/2\left(\Omega_A^2/2\omega_L\right)\left[\cos\beta+1/4(1-5\cos\beta)\sin^2\theta\right]. \tag{14}$$

It will be shown below that the corresponding answer for the Polar phase is reproduced from Eq.(14) by substituting $\Omega_A \to \sqrt{2}\Omega_P$ (see Ref.[2]).

One more information for the ABMU(1)LIM model in the limits of Van der Pol approximation is the stability criterion (the concavity of $\bar{U}_D^{(A)}(\beta,\Phi_{st})$ with respect to $\cos\beta$) according to which

$$\frac{\partial^2\bar{U}_D^{(A)}(\beta,\Phi_{st})}{\partial(\cos\beta)^2} > 0 \ \Rightarrow\ \cos^2\theta < 1/5\,. \tag{15}$$

Now we pass to explore the frequency spectrum of the rapid spin - oscillations $\delta\,\vec{S}(t)$. This spectrum is to be constructed in using the amplitudes $U_i(\alpha)$ of the time-oscillating part $\tilde{f}_A(t)$ (see Eq. (11)). In particular the spectrum of the longitudinal spin-oscillations $\delta\,S_z(t)$ is found according the prescription (in what follows $\delta\,\vec{S}(t)$ will be measured in the units of $S=\chi H/g$):

$$\delta\,S_z(t) = \varepsilon\left[U_P(\alpha)+U_S(\alpha)\right],\qquad \varepsilon \sim \left(\Omega_A/\omega_L\right)^2, \tag{16}$$

$$U_P = \int d\alpha\left.\left(\frac{\partial\tilde{f}_A}{\partial\alpha}\right)\right|_{\Phi_{st}},\qquad U_S = \int d\alpha\left.\left(\frac{\partial\tilde{f}_A}{\partial\Phi}\right)\right|_{\Phi_{st}}. \tag{17}$$

In order to realize the prescriptions given in Eqs (16, 17) we have to use $(\alpha,\ \Phi)$ representation:

$$\begin{aligned}&\tilde{f}_A(\alpha,\Phi)=-1/8\left\{2\sin^2\beta\cos(2\alpha-2\Phi)+\right.\\&+\left[1/2(1-\cos\beta)^2\cos(4\alpha-2\Phi)-\sin^2\beta\left(\cos 2\alpha+3\cos(2\alpha-2\Phi)\right)\right]\sin^2\theta+\\&\left.+\sin\beta\left[2\cos\beta\cos\alpha+(1+\cos\beta)\cos(\alpha-2\Phi)-(1-\cos\beta)\cos(3\alpha-2\Phi)\right]\sin 2\theta\right\}.\end{aligned}\tag{18}$$

In using Eq. (18), from Eqs (16, 17) it is found that

$$\begin{aligned}&\delta S_z(t)=-1/8\left(\Omega_A/\omega_L\right)^2\left\{\left[\sin\beta(1+3\cos\beta)\sin 2\theta\right]\cos\omega_L t-\left[\sin^2\beta\sin^2\theta\right]\cos 2\omega_L t+\right.\\&\left.\left[1/3\sin\beta(1-\cos\beta)\sin 2\theta\right]\cos 3\omega_L t-\left[1/4(1-\cos\beta)^2\sin^2\theta\right]\cos 4\omega_L t\right\}.\end{aligned}\tag{19}$$

B. Now we turn to the analysis of the behaviour of the Polar phase in the aim of comparing it with the properties of the ABMU(1)LIM model. The starting point is the dipole-dipole potential of the Polar phase

$$U_D^{(P)}=\frac{1}{2}\chi_P\left(\Omega_P/g\right)^2\left(\hat{d}\cdot\hat{U}\right)^2,\quad \vec{H}=H\,\hat{z},\tag{20}$$

where $(U_x=0,\ U_y=-\sin\theta,\ U_z=\cos\theta)$ and the spin-orbital function

$$\begin{aligned}&f_P=\left(\hat{d}\cdot\hat{U}\right)^2=\left(-d_y\sin\theta+d_z\cos\theta\right)^2=\\&=1/4\left\{2\sin^2\beta(1+\cos 2\gamma)+\left[-1+3\cos^2\beta-1/2(1+\cos\beta)^2\cos 2(\alpha+\gamma)-\right.\right.\\&\left.-1/2(1-\cos\beta)^2\cos 2(\alpha-\gamma)+\sin^2\beta\left(\cos 2\alpha-3\cos 2\gamma\right)\right]\sin^2\theta+\\&\left.+\sin\beta\left[2\cos\beta\sin\alpha+(1+\cos\beta)\sin(\alpha+2\gamma)-(1-\cos\beta)\sin(\alpha-2\gamma)\right]\sin 2\theta\right\}\end{aligned}\tag{21}$$

In decomposing f_P as

$$f_P=\bar{f}_P+\tilde{f}_P(t),\tag{22}$$

it is found that

$$\bar{f}_P=1/4\left\{2\sin^2\beta+\left[-1+3\cos^2\beta-1/2(1+\cos\beta)^2\cos 2\Phi\right]\sin^2\theta\right\}\tag{23}$$

and

$$\begin{aligned}&\tilde{f}_P=1/4\left\{2\sin^2\beta\cos 2\gamma+\left[-1/2(1-\cos\beta)^2\cos 2(\alpha-\gamma)+\sin^2\beta\left(\cos 2\alpha-3\cos 2\gamma\right)\right]\sin^2\theta+\right.\\&\left.+\sin\beta\left[2\cos\beta\sin\alpha+(1+\cos\beta)\sin(\alpha+2\gamma)-(1-\cos\beta)\sin(\alpha-2\gamma)\right]\sin 2\theta\right\}\end{aligned}\tag{24}$$

According to Eq. (23) it can be shown that for the Polar phase in the Van der Pol approximation the stationary value of $\Phi=\Phi_{st}=(0,\pi)$, the dipole shift from the Larmor value ω_L is

$$\delta\omega(\beta)=\left(\Omega_P^2/2\omega_L\right)\left[\cos\beta+1/4(1-5\cos\beta)\sin^2\theta\right],\tag{25}$$

and the criterion of the coherent spin dynamics stability is given as $\cos^2\theta<1/5$.

Now we are going to explore the frequency spectrum of the rapid spin-oscillations. As in the case of the ABMU(1)LIM model this task is to be realized in using for the longitudinal spin-oscillations $\delta S_z(t)$ the prescription given in Eqs (16, 17). In order to perform this procedure it will be needed to use (α,Φ) representation of $\tilde{f}_P$. From Eq. (24) it follows that for the Polar phase

$$\tilde{f}_P(\alpha,\Phi) = 1/4\Big\{2\sin^2\beta\cos(2\alpha-2\Phi) + \Big[-1/2\big(1-\cos\beta\big)^2\cos\big(4\alpha-2\Phi\big) + \\ +\sin^2\beta\big(\cos 2\alpha - 3\cos(2\alpha-2\Phi)\big)\Big]\sin^2\theta + \\ +\sin\beta\Big[2\cos\beta\sin\alpha - \big(1+\cos\beta\big)\sin\big(\alpha-2\Phi\big) - \big(1-\cos\beta\big)\sin\big(3\alpha-2\Phi\big)\Big]\sin 2\theta\Big\} \tag{26}$$

After having used Eq. (26), from Eqs (16, 17) it is found that for the Polar phase

$$\delta S_z(t) = 1/4\,\left(\Omega_P/\omega_L\right)^2\Big\{\big[\sin\beta(1+3\cos\beta)\sin 2\theta\big]\sin\omega_L t + \big[\sin^2\beta\sin^2\theta\big]\cos 2\omega_L t - \\ -\big[1/3\sin\beta(1-\cos\beta)\sin 2\theta\big]\sin 3\omega_L t - 1/4\big[(1-\cos\beta)^2\sin^2\theta\big]\cos 4\omega_L t\Big\}\ . \tag{27}$$

This answer is to be compared with Eq. (19) appropriate to ABMU(1)LIM model. The comparison shows that the structure of the frequency spectra of $\delta S_z(t)$ for the Polar and ABMU(1)LIM states are sufficiently different in order to discriminate experimentally between these two superfluid states in pulsed NMR regime.

ფიზიკა

ზედენადი ^{3}He-ის პოლარული და ABM ფაზების თვისებები გლობალურად დეფორმირებული აეროგელის თანდასწრებით

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შედარებულია ზედენადი ^{3}He-ის პოლარული და ABMU(1)LIM ფაზების მოდელები გლობალურად გაჭიმული აეროგელის პირობებში. დროში გასაშუალებულ მიახლოებაში ორივე ფაზა მსგავსია და მათი ექსპერიმენტული განცალება იმპულსური ბირთვული მაგნიტური რეზონანსის მეთოდით რთულია. მეორე მხრივ, როგორც ჩვენმა თეორიულმა გამოკვლევამ გვიჩვენა, მაღალი სიხშირის სპინური ოსცილაციების სპექტრზე დაკვირვება საშუალებას მოგვცემს განვაცალოთ ზემოხსენებული ზედენადი ფაზები.

REFERENCES

1. *Aoyama K.* and *Ikeda R.* (2006) Phys. Rev. **B73**: 060504.
2. *Askhadullin R.Sh., Dmitriev V.V. et al.* (2012) Pisma v JETF, **95**: 355.
3. *Elbs J., Bunkov Yu. et al.* (2008) Phys. Rev. Lett. **100**: 215304.
4. *Baramidze G.* and *Kharadze G.* (2011) J. Low Temp. Phys. **162**.

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